## OUTCOME 4

## STEAM AND GAS TURBINE POWER PLANT

## TUTORIAL No. 9 - GAS TURBINE THEORY

## 4 Understand the operation of steam and gas turbine power plant

Principles of operation: impulse and reaction turbines; condensing; pass-out and back pressure steam turbines;
single and double shaft gas turbines; regeneration and re-heat in gas turbines; combined heat and power plants

Circuit and property diagrams: circuit diagrams to show boiler/heat exchanger; superheater; turbine; condenser; condenser cooling water circuit; hot well; economiser/feedwater heater; condensate extraction and boiler feed pumps; temperature entropy diagram of Rankine cycle

Performance characteristics: Carnot, Rankine and actual cycle efficiencies; turbine isentropic efficiency; power output; use of property tables and enthalpy-entropy diagram for steam

When you have completed tutorial 9 you should be able to do the following.

- Describe the basic gas turbine power cycle.
- Describe advanced gas turbine power cycles.
- Solve problems concerning gas turbine power plant.

In this section we will examine how practical gas turbine engine sets vary from the basic Joule cycle.

The efficiency of gas turbine engines increases with pressure compression ratio. In practice this is limited, as the type of compressor needed to produce very large flows of air cannot do so at high pressures. 6 bar is a typical pressure for the combustion chamber.

The efficiency of gas turbines may be improved by the use of inter-cooling and heat exchangers.

### 1.1 GAS CONSTANTS

The first point is that in reality, although air is used in the compressor, the gas going through the turbine contains products of combustion so the adiabatic index and specific heat capacity is different in the turbine and compressor.

### 1.2 FREE TURBINES

Most designs used for gas turbine sets use two turbines, one to drive the compressor and a free turbine. The free turbine drives the load and it is not connected directly to the compressor. It may also run at a different speed to the compressor. Fig. 1 shows the layouts for parallel turbines.


Fig. 1 PARALLEL TURBINES
Figure 2 shows the layout for series turbines.


Figure 2 SERIES TURBINES

### 1.3 INTER-COOLING AND REHEATING

Basically, if the air is compressed in stages and cooled between each stage, then the work of compression is reduced and the efficiency increased.

The reverse theory also applies. If several stages of turbine expansions are used and the gas reheated between stages, the power output and efficiency is increased.

Figure 3 shows the layout for intercoolers and re-heaters.


Fig. 3 INTER-COOLER and RE-HEATER

## WORKED EXAMPLE No. 1

A gas turbine draws in air from atmosphere at 1 bar and $10{ }^{\circ} \mathrm{C}$ and compresses it to 5 bar. The air is heated to 1200 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 80 kW of power.

Draw the circuit.
Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

Assume $\gamma=1.4$ and $\mathrm{c}_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ for both the turbines and the compressor.
Neglect the increase in mass due to the addition of fuel for burning.
Compare the efficiency to the air standard efficiency.

## SOLUTION



Fig. 4

## COMPRESSOR

$\mathrm{T}_{2}=\mathrm{T}_{1} r_{p}^{\left(1-\frac{1}{\gamma}\right)}=283 \times 5^{0.286}=448.4 \mathrm{~K}$

Power input to compressor $=\mathrm{mc}_{\mathrm{p}}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)$
Power output of h.p. turbine $=\mathrm{mc}_{\mathrm{p}}\left(\mathrm{T}_{3}-\mathrm{T}_{4}\right)$
Since these are equal we may equate them.
$1.005(448.4-283)=1.005\left(1200-\mathrm{T}_{4}\right) \quad \mathbf{T}_{4}=\mathbf{1 0 3 4 . 6} \mathrm{K}$

## HIGH PRESSURE TURBINE

$\frac{\mathrm{T}_{4}}{\mathrm{~T}_{3}}=\left(\frac{\mathrm{p}_{4}}{\mathrm{p}_{5}}\right)^{1-\frac{1}{\gamma}}$
$\frac{\mathrm{T}_{4}}{\mathrm{~T}_{5}}=\frac{1034.6}{1200}=0.595=\left(\frac{\mathrm{p}_{4}}{5}\right)^{0.286} \mathrm{p}_{4}=5 \times 0.595^{\frac{1}{0.286}}=2.977 \mathrm{bar}$

## LOW PRESSURE TURBINE

$\frac{\mathrm{T}_{5}}{\mathrm{~T}_{4}}=\left(\frac{1}{2.977}\right)^{1-\frac{1}{\gamma}}=(0.336)^{0.286}=0.732 \quad \mathrm{~T}_{5}=757.3 \mathrm{~K}$

## NET POWER

The net power is 80 kW .
$80=\mathrm{mcp}\left(\mathrm{T}_{4}-\mathrm{T}_{5}\right)=\mathrm{mx} 1.005(1034.6-757.3)$
$\mathbf{m}=\mathbf{0 . 2 8 8} \mathrm{kg} / \mathrm{s}$
HEAT INPUT
$\Phi(\mathrm{in})=\operatorname{mcp}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)=0.288 \times 1.005(1200-448.4)=217.5 \mathrm{~kW}$

## THERMAL EFFICIENCY

$\eta_{\text {th }}=\mathrm{P}($ nett $) / \Phi(\mathrm{in})=80 / 219.9=0.367$ or $36.7 \%$
The air standard efficiency is the Joule efficiency.
$\eta=1-r_{p}^{-0.286}=1-5^{-0.286}=0.369$ or $36.9 \%$

## SELF ASSESSMENT EXERCISE No. 1

1. A gas turbine draws in air from atmosphere at 1 bar and $15^{\circ} \mathrm{C}$ and compresses it to 4.5 bar. The air is heated to 1100 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 100 kW of power.

For the compressor $\gamma=1.4$ and for the turbines $\gamma=1.3$.
The gas constant is $0.287 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ for both.
Neglect the increase in mass due to the addition of fuel for burning. Assume the specific heat of the gas in the combustion chamber is the same as that for the turbines.

Calculate the following.
i. The specific heat $\mathrm{c}_{\mathrm{p}}$ of the air and the burned mixture. (1.005 and 1.243)
ii. The mass flow of air. ( $0.407 \mathrm{~kg} / \mathrm{s}$ )
iii. The inter-stage pressure of the turbines. (2.67 bar)
iv. The thermal efficiency of the cycle. (30\%)
2. A gas turbine draws in air from atmosphere at 1 bar and 300 K and compresses it to 6 bar. The air is heated to 1300 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it.
$\gamma=1.4$ and $\mathrm{R}=0.287 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ for both the turbine and compressor. Neglect the increase in mass due to the addition of fuel for burning.

Calculate the following for a mass flow of $1 \mathrm{~kg} / \mathrm{s}$.
i. The inter-stage pressure. (3.33 bar)
ii. The net power output. ( 322 kW )
iii. The thermal efficiency of the cycle. (40\%)

### 1.4 EXHAUST GAS HEAT EXCHANGER

The exhaust gas from a turbine is hotter than the air leaving the compressor. If heat is passed to the air from the exhaust gas, then less fuel is needed in the combustion chamber to raise the air to the operating temperature. This requires an exhaust heat exchanger. Fig. 5 shows the layout required.


Fig. 5
In order to solve problems associated with this cycle, it is necessary to determine the temperature prior to the combustion chamber (T3).

A perfect heat exchanger would heat up the air so that $\mathrm{T}_{3}$ is the same as T 5 . It would also cool down the exhaust gas so that $\mathrm{T}_{6}$ becomes $\mathrm{T}_{2}$. In reality this is not possible so the concept of THERMAL RATIO is used. This is defined as the ratio of the enthalpy given to the air to the maximum possible enthalpy lost by the exhaust gas. The enthalpy lost by the exhaust gas is

$$
\Delta H=m_{g c} \operatorname{pg}^{\left(T_{5}-T_{6}\right)}
$$

This would be a maximum if the gas is cooled down such that $\mathrm{T}_{6}=\mathrm{T}_{2}$. Of course in reality this does not occur and the maximum is not achieved and the gas turbine does not perform as well as predicted by this idealisation.

$$
\Delta H(\text { maximum })=m g c p g\left(\mathrm{~T}_{5}-\mathrm{T}_{2}\right)
$$

The enthalpy gained by the air is

$$
\Delta H(\text { air })=\mathrm{m}_{\mathrm{a}} \mathrm{c}_{\mathrm{pa}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)
$$

Hence the thermal ratio is

$$
\mathrm{T} . \mathrm{R} .=\mathrm{m}_{\mathrm{a}} \mathrm{c}_{\mathrm{pa}}\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right) / \mathrm{mg}_{\mathrm{g}} \mathrm{ppg}\left(\mathrm{~T}_{5}-\mathrm{T}_{2}\right)
$$

The suffix a refers to the air and $g$ to the exhaust gas. Since the mass of fuel added in the combustion chamber is small compared to the air flow we often neglect the difference in mass and the equation becomes

$$
\text { T.R. }=c_{p a}\left(T_{3}-T_{2}\right) / c_{p g}\left(T_{5}-T_{2}\right)
$$

## WORKED EXAMPLE No. 2

A gas turbine uses a pressure ratio of $7.5 / 1$. The inlet temperature and pressure are respectively $100^{\circ} \mathrm{C}$ and 105 kPa . The temperature after heating in the combustion chamber is $1300{ }^{\circ} \mathrm{C}$.
The specific heat capacity $\mathrm{c}_{\mathrm{p}}$ for air is $1.005 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ and for the exhaust gas is 1.15 $\mathrm{kJ} / \mathrm{kg} \mathrm{K}$. The adiabatic index is 1.4 for air and 1.33 for the gas. Assume isentropic compression and expansion. The mass flow rate is $1 \mathrm{~kg} / \mathrm{s}$.

Calculate the air standard efficiency if no heat exchanger is used and compare it to the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.

## SOLUTION

Referring to the numbers used on fig. 6 the solution is as follows.
Air standard efficiency $=1-\mathrm{r}_{\mathrm{p}}(1-1 / \gamma)=1-7.5^{-0.286}=\mathbf{0 . 4 3 8} \mathbf{0 r} \mathbf{4 3 . 8 \%}$
Solution with heat exchanger
$\mathrm{T}_{2}=\mathrm{T}_{1} \mathrm{r}_{\mathrm{p}}(1-1 / \gamma)=283(7.5)^{0.286}=503.6 \mathrm{~K}$
$\mathrm{T}_{5}=\mathrm{T}_{4} / \mathrm{r}_{\mathrm{p}}(1-1 / \gamma)=1573 /(7.5) 0.248=954.1 \mathrm{~K}$
Use the thermal ratio to find T3.
$0.8=1.005\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right) / 1.15\left(\mathrm{~T}_{5}-\mathrm{T}_{2}\right)$
$0.8=1.005(\mathrm{~T} 3-503.6) / 1.15(954.1-503.6)$
$\mathrm{T}_{3}=916 \mathrm{~K}$

In order find the thermal efficiency, it is best to solve the energy transfers.
$\mathrm{P}(\mathrm{in})=\mathrm{mcpa}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)=1 \times 1.005(503.6-283)=221.7 \mathrm{~kW}$
$\mathrm{P}($ out $)=\mathrm{mc}_{\mathrm{pg}}(\mathrm{T} 4-\mathrm{T} 5)=1 \times 1.15(1573-954.1)=711.7 \mathrm{~kW}$
$\mathrm{P}($ net $)=\mathrm{P}($ out $)-\mathrm{P}($ in $)=490 \mathrm{~kW}$
$\Phi($ in $)$ combustion chamber $)=\mathrm{mc}_{\mathrm{pg}}(\mathrm{T} 4-\mathrm{T} 3)$
$\Phi(\mathrm{in})=1.15(1573-916)=755.5 \mathrm{~kW}$
$\eta$ th $=\mathrm{P}($ net $) / \Phi(\mathrm{in})=490 / 755.5=\mathbf{0 . 6 5}$ or $\mathbf{6 5 \%}$

## SELF ASSESSMENT EXERCISE No. 2

1. A gas turbine uses a pressure ratio of $7 / 1$. The inlet temperature and pressure are respectively $10{ }^{\circ} \mathrm{C}$ and 100 kPa . The temperature after heating in the combustion chamber is $1000{ }^{\circ} \mathrm{C}$. The specific heat capacity $\mathrm{C}_{\mathrm{p}}$ is $1.005 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ and the adiabatic index is 1.4 for air and gas. Assume isentropic compression and expansion. The mass flow rate is $0.7 \mathrm{~kg} / \mathrm{s}$.

Calculate the net power output and the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.
( 234 kW and 57\%)
2. A gas turbine draws in air from the atmosphere at 1.02 bar and 300 K . The air is compressed to 6.4 bar isentropically. The air entering the turbine is at 1500 K and it expands isentropically to 1.02 bar. Assume the specific heat Cp is $1.005 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$ and $\gamma$ is 1.4 for both the turbine and compressor. Ignore the addition of mass in the burner. Calculate the following.
i. The air standard efficiency. (40.8\%)
ii. The efficiency when an exhaust heat exchanger with a thermal ratio of 0.75 is added. (70.7\%)

