THERMODYNAMICS

TUTORIAL 9

TURBINE THEORY

This tutorial is set at NQF Level 5 to 6

On completion of this tutorial you should be able to

- Explain the principles of Impulse and Reaction Turbines.
- Explain and use Vector Diagrams to determine the power produced by flow over the turbine vanes.
- Define the parameters needed to determine the performance of turbines.
- Calculate the performance of turbines.

Content

1. *Introduction*

The following is mainly about steam turbine design and theory but it generally applies to gas turbines as well bearing in mind the methods of determining their properties are different. Surprisingly there seems to be very little information available on the design and construction of gas turbines.

A Brief History

120 B.C. Hero of Alexandria constructs a simple reaction turbine.

This was constructed from a spherical vessel with two spouts as shown. Heat turned the water inside into steam that escaped through the spouts and made the vessel rotate.

1629 Branca, an Italian, created the first impulse turbine.

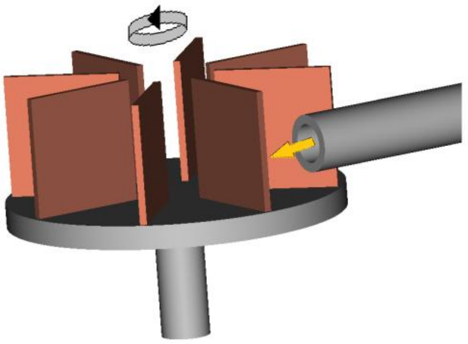
Steam issuing from a nozzle struck the vanes on a wheel and made it revolve.

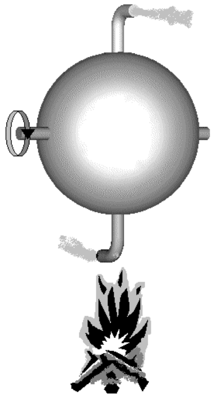
Windmills, developed in medieval times formed the main source of power for centuries.

1884 Charles Parsons developed the first practical reaction turbine. This machine developed around 7 kW of power.

1889 De Laval developed the first practical impulse turbine capable of producing around 2 kW of power.

Others who developed the impulse turbine were ***Rateau*** in France and ***Curtis*** in the U.S.A.





Brancas's Turbine (Impulse) Figure 1 Hero's Turbine (Reaction)

Turbines are generally classified as either ***impulse*** or ***reaction***. This refers to the type of force making it rotate.

2. *Impulse Theory*

2.1 *General Theory*

Impulsive Forces are exerted on an object when it diverts or changes the flow of a fluid passing over it. A very basic impulse turbine is the windmill and this converts the kinetic energy of the wind into mechanical power. Consider a rotor with vanes arranged around the edge. Fluid is directed at the vanes by a set of nozzles.

In the case of steam turbines the symbol used for steam velocity is C (the S.I. symbol specified for vapours). C will be used from this point forward.

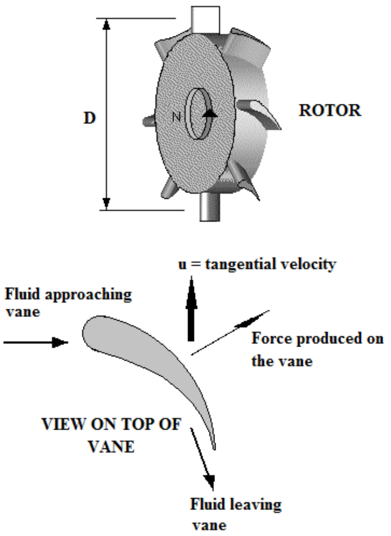


Fig. 2

All the pressure is converted into velocity (kinetic energy) in the nozzles. There is no pressure drop over the row of moving vanes (or blades). The resulting force on the vane is entirely due to the change in the momentum of the fluid and this is entirely due to the change in direction as the steam (or gas or liquid) flows over the blades. It is of interest to note that the name impulsive comes from Newton's second law of motion.

Impulse = change in momentum

Impulsive force = rate of change in momentum.

$$F = \dot{m}\,\Delta C$$

$\dot{m}$ is the mass flow rate in kg/s and ΔC is the change in velocity of the fluid. This is a vector quantity and may be applied to any direction. If we make ΔC the change in velocity in the direction of motion we obtain the force making the rotor turn. This direction is usually called the whirl direction and ΔC_w means the change in velocity in the whirl direction. (It is tangential in the rotor illustrated).

$$F = \dot{m}\Delta C_w$$

Suppose the vanes to be rotating on a mean circle of diameter D at N rev/s. The linear velocity (tangential in the rotor shown) of the vanes is u m/s. This is given by the following equation.

$$u = \pi DN$$

2.1.1 Diagram Power

The power produced by any moving force is the product of force and velocity. The power of the ideal rotor is given by the following equation.

$$P = \dot{m}\,\Delta C_w\,u = \dot{m}\,\Delta C_w\,\pi ND$$

This is the fundamental way of finding the power produced by fluids passing over moving vanes. ΔC_w is a vector quantity and it is found by drawing the vector diagrams for the velocities. For this reason, the power is called ***Diagram Power*** (D.P.).

$$D.P. = \dot{m}\,\Delta C_w \pi ND$$

This formula applies to any type of turbine (steam, gas or water).The main problem is determining ΔC_w

WORKED EXAMPLE No. 1

The vanes on a simple steam turbine are mounted on a rotor with a mean diameter of 0.6 m. The steam flows at a rate of 0.8 kg/s and the velocity in the whirl direction is changed by 80 m/s. The turbine rotates at 600 rev/min. Calculate the diagram power.

SOLUTION

Rotor Speed	$N = 600/60 = 10$ rev/s
Velocity of the vanes	$u = \pi ND = \pi \times 10 \text{ x } 0.6 = 18.85$ m/s
Diagram Power	$DP = \dot{m}\,u\,\Delta C_w = 0.8 \times 18.85 \times 80 = 1\,206.5$ W

SELF ASSESSMENT EXERCISE No. 1

1. A steam turbine has its vanes on a mean diameter of 1.2 m and rotates at 1 500 rev/min. The change in the velocity of whirl is 65 m/s and the change in the axial velocity is 20 m/s. The flow rate is 1 kg/s. Calculate the following.

   i. The diagram power. (6.12 kW)

   ii. The axial force. (20 N)

2. A steam turbine is to be designed to rotate at 3 000 rev/min and produce 5 kW of power when 1 kg/s is used. The vanes will be placed on a mean diameter of 1.4 m. Calculate the change in the velocity of whirl that will have to be produced. (22.7 m/s)

3. A gas turbine has rotor blades on a mean diameter of 0.5 m and the rotor turns at 2000 rev/min. The change in the whirl velocity is 220 m/s and the diagram power is 2 MW. Calculate the mass flow rate of gas. (173.6 kg/s)

2.2 ***Vector Diagram Analysis for Impulse Turbine***

Consider a pure impulse turbine similar to figure 2. Let the blades be symmetrical in shape. The velocity of the steam coming out of the nozzle is an absolute or true velocity and is denoted C_1. The velocity of the blade at the mean diameter is u and given by $u = \pi ND$ where D is the mean diameter of the blades on the rotor and N the rotor speed in rev/s. The angles are measured relative to the direction of u and are designated α for the absolute velocities C and β for the blade surface angle.

Note that in a pure impulse turbine the ***entire expansion of the steam is in the nozzle*** and ***the entire drop in pressure is in the nozzle*** and so the velocity C_1 can be calculated.

The steam leaving the nozzle strikes the blade and is diverted by the blade. For maximum efficiency the angle of the steam jet must be such that the steam arrives on the moving vane so that it is travelling parallel to the surface of the blade. This is called ***shockless entry***. This is essential to avoid energy being wasted.

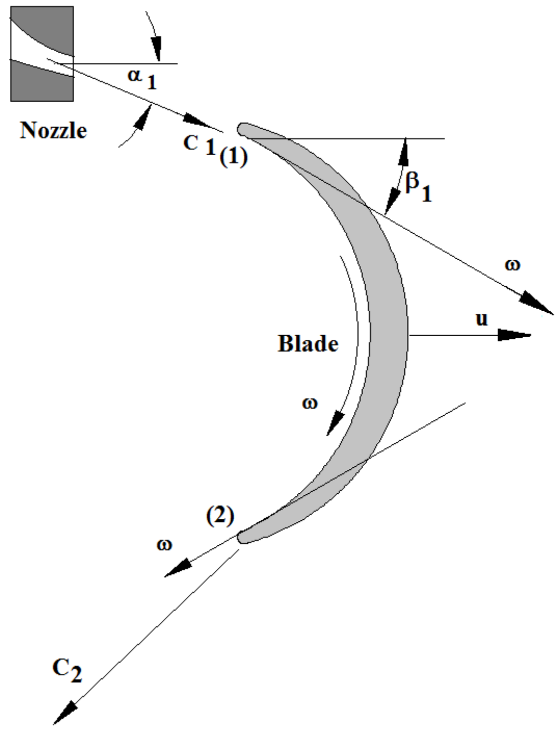


Figure 3

At the inlet to the blade (point 1) the steam on the surface will have two components, the velocity of the blade u plus the velocity of the steam relative to the surface of the blade which is often (but confusingly) designated as ω. The rule for shockless entry is that the ***velocity vectors*** must add up so that:

$$\mathbf{C_1 = u + \omega_1}$$

Figure 4 shows the vector diagram at inlet to the. C_{w1} is the component of C_1 that is in the direction of u. It is normal to draw this to scale and determine the unknown data.

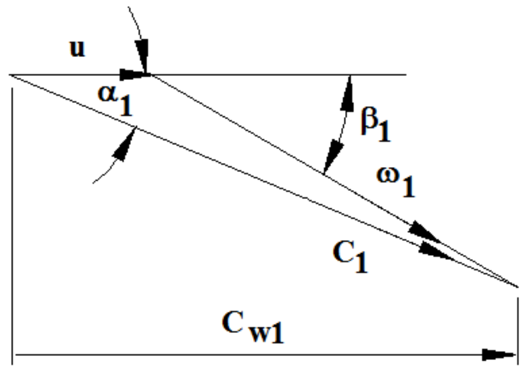


Figure 4

It is most likely that the known values are u and C_1. If we know α_1 the others can be calculated by applying the cosine rule to the triangle:

$$\boldsymbol{\omega_1^2 = C_1^2 + u^2 - 2\,u\,C_1 \cos(\alpha_1)}$$ from which we solve $\boldsymbol{\omega_1}$

If it is β_1 that is known then:

$C_1^2 = \boldsymbol{\omega_1}^2 + u^2 - 2\,u\,\boldsymbol{\omega_1} \cos(180 - \beta_1)$ note that $\cos(180 - \beta_1) = -\cos(\beta_1)$

$C_1^2 = \omega_1^2 + u^2 + 2\,u\,\omega_1 \cos(\beta_1)$

Rearrange into quadratic form $\boldsymbol{\omega_1^2 + \omega_1 \{2\,u \cos(\beta_1)\} + (u^2 - C_1^2) = 0}$

This may be solved for $\boldsymbol{\omega_1}$ using the quadratic equation. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

We can get the nozzle angle α_1 by applying trigonometry and $\alpha_1 = \mathrm{asin}\left\{\frac{\omega_1 \sin(\beta_1)}{C_1}\right\}$

Or if we know the nozzle angle we can get the ideal blade angle $\beta_1 = \mathrm{asin}\left\{\frac{C_1 \sin(\alpha_1)}{\omega_1}\right\}$

Now examine the vector diagram at exit from the blade. The steam is swept around blade and exits (point 2). The steam has velocity ω_2 relative to the blade plus the velocity of the blade u so the absolute velocity of the steam leaving the vane is $\mathbf{C_2 = u + \omega_2}$

The exit vector is shown in figure 5. The magnitude of $\boldsymbol{\omega_2}$ is the same as $\boldsymbol{\omega_1}$ unless friction slows the steam down on the blade. C_{w2} is the component of C_2 in the direction of the blade velocity u and in this case it is swept backwards to the direction.

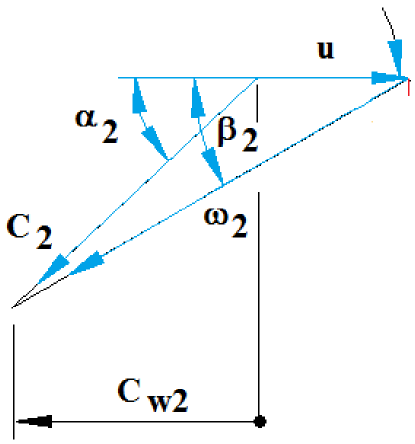


Figure 5

Knowing the value of $\boldsymbol{\omega_2}$, u and β_2 we can construct the diagram and scale off the unknown values. We can also use trigonometry and apply the cosine rule again. $C_2^2 = \omega_2^2 + u^2 - 2\, u\, \omega_2 \cos(\beta_2)$

Rearrange into quadratic form $\omega_2^2 - \omega_2 \{2\, u \cos(\beta_2)\} + (u^2 - C_2^2)$

This may be solved for $\boldsymbol{\omega_2}$ using the quadratic equation. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

We can get the angle of the steam leaving the vane α_2 by applying trigonometry and $\alpha_2 = \mathrm{asin}\left\{\frac{\omega_2 \sin(\beta_1)}{C_2}\right\}$

Since the blade velocity u is common to the inlet and outlet, the two diagrams may be drawn together as shown in figure 6.

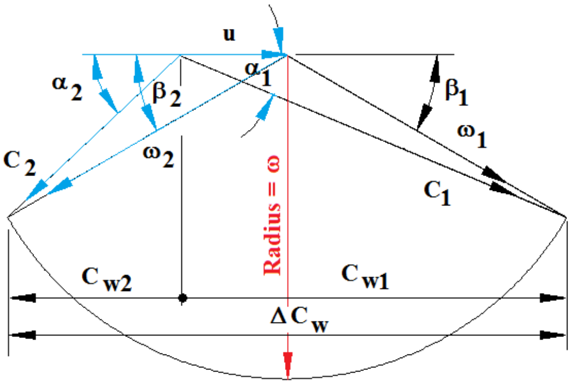


Figure 6

The change in velocity of the steam in the direction of whirl (direction of u) is ΔC_w. In this way we can determine ΔC_w either by scaling it from the diagram or calculating it with trigonometry.

$$\boldsymbol{\Delta C_w} = \omega_1 \cos(\beta_1) + \omega_2 \cos(\beta_2)$$

Put into the formula to calculate the diagram power

$$\text{Diagram Power} = \text{D.P.} = \dot{m}\, u\, \Delta C_w = \dot{m}\, u\, \{\omega_1 \cos(\beta_1) + \omega_2 \cos(\beta_2)\}$$

2.2.1 Friction

If there is no friction $\omega_1 = \omega_2$

If there is friction then the steam is slowed down on the blade so $\omega_1 > \omega_2$

We define the blade friction coefficient as $k = \omega_2/\omega_1$

We can now correct the formulae to $\boldsymbol{\Delta C_w} = \omega_1\{\cos(\beta_1) + k \cos(\beta_2)\}$

2.2.2 Diagram Efficiency

The energy of the steam approaching the blade is the kinetic energy $\frac{1}{2}\, \dot{m}\, C_1^2$

The diagram efficiency is defined as:

$$\eta_d = \frac{\text{D.P.}}{\text{K.E. at Inlet}} = \frac{\dot{m}u\, \Delta C_w}{\frac{1}{2}\, \dot{m} C_1^2} = \frac{2u\, \Delta Cw}{C_1^2}$$

2.2.3 Optimal Efficiency

In order to simplify this, let us look at a symmetrical blade ($\beta_1 = \beta_2$) and note $\omega_2 = k\,\omega_1$.

$\Delta C_w = \omega_1 \cos(\beta_1) + \omega_2 \cos(\beta_2) = \omega_1 \cos(\beta_1) + k\,\omega_1 \cos(\beta_1) = \omega_1 \cos(\beta_1)(1 + k)$

From the vector diagram $\omega_1 \cos(\beta_1) = C_1 \cos(\alpha_1) - u$ substitute into the above

$\Delta C_w = \omega_1 \cos(\beta_1)(1 + k) = \{C_1 \cos(\alpha_1) - u\}\,(1 + k)$

$$\eta_d = \frac{2u\,\Delta Cw}{C_1^2} = \frac{2u\,\{(1+k)\,\{\,C_1\cos(\alpha_1) - u\}\}}{C_1^2} = 2(1+k)\left\{\frac{u}{C_1}\cos(\alpha_1) - \left(\frac{u}{C_1}\right)^2\right\}$$

It is common to express this as

$\eta_d = 2(1+k)\{r\cos(\alpha_1) - r^2\}$ where $r = \frac{u}{C_1}$ and is called the blade speed ratio.

For a given nozzle angle and velocity we can plot η_d against r (figure 7) and we see that there is a value where the efficiency is a maximum.

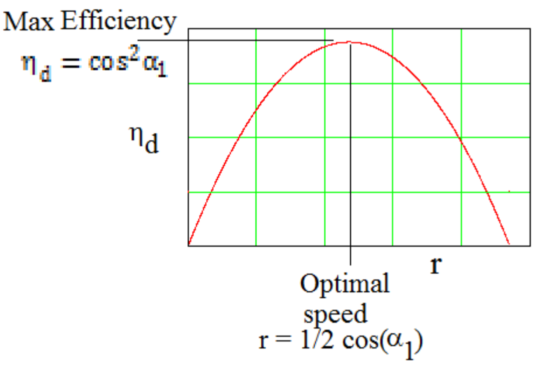


Figure 7

To find the maximum point, differentiate η_d with respect to r and equate to zero

$$\frac{d\eta_d}{dr} = 2(1+k)\{\cos(\alpha_1) - 2r\} = 0$$

$$\cos(\alpha_1) = 2r \quad \text{Hence} \quad u = \frac{C_1\cos(\alpha_1)}{2}$$

The optimal efficiency is

$$\eta_d = 2(1+k)\{r(2r) - r^2\} = 2(1+k)r^2 = 2(1+k)r^2$$

If k = 1

$$\eta_d = 4r^2 = \cos^2\alpha_1$$

WORKED EXAMPLE No. 2

A pure impulse turbine has a single row of symmetrical blades. The steam exits the nozzles with a velocity of 195 m/s at 23° to the tangent. The blades rotate on a mean diameter of 0.318 m.
Determine:

i. the best wheel speed
ii. the optimal diagram efficiency
iii. the blade angles

Check your answers by drawing the vector diagram to scale. Assume k = 1

SOLUTION

$C_1 = 195$ m/s $\quad \alpha_1 = 23^o$

$u = \frac{C_1}{2}\cos(\alpha_1) = \frac{195}{2}\cos(23^o) = 89.75\ m/s$

$u = \pi ND \quad N = \frac{89.75}{\pi\ 0.318} = 89.8\ rev/s$ or 5 390 rev/min

$\omega_1^2 = C_1^2 + u^2 - 2\,u\,C_1\cos(\alpha_1) = 195^2 + 89.75^2 - 2 \times 89.75 \times 195 \times \cos(23^o) = 13\,860 \quad \omega_1 = 117.73$

$C_1 \sin(\alpha_1) = \omega_1 \sin(\beta_1)$

$195 \sin(23^o) = 76.2 = 117.73 \sin(\beta_1) \quad \beta_1 = 40.3^o$

$\eta_d = \cos^2\alpha_1 = 0.85$

Check D.P. = $u\,\Delta C_w = 90 \times 180\ 269 = 16\,200$ W per unit mass

K.E. supplied = $C_1^2/2 = 1\,952/2 = 19\,012.5$ W per unit mass

$\eta_d = 16200/19012.5 = 0.85$

The blade angle at inlet and outlet is 40 °. These could also be calculated. Note that at this optimal condition the absolute velocity C_2 leaves in an axial direction i.e. $\alpha_2 = 90^o$

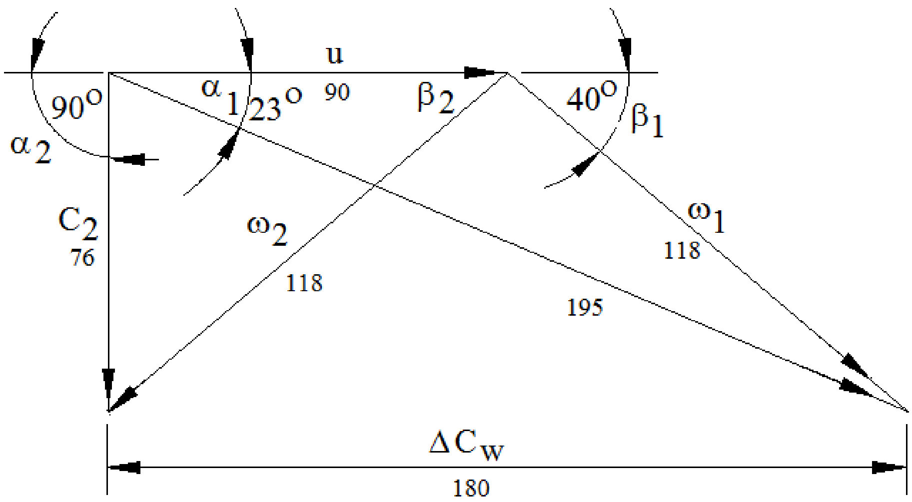


Figure 8

2.3 *Multiple Rows*

A practical impulse turbine needs several sets of moving vanes and fixed nozzles (fixed row) which are also blades or vanes (Figure 9).

2.3.1 *Pressure Compounded*

If the pressure is dropped in stages the design is called ***pressure compounded*** and is also known as a ***Rateau*** turbine after the designer. The steam leaving the moving blades is collected by a barrier and then expanded through the nozzles. In this way the pressure is dropped in stages from inlet to exit. The fixed row is attached to the casing and the moving row is attached to the rotor. The velocity increases only in the nozzles as shown.

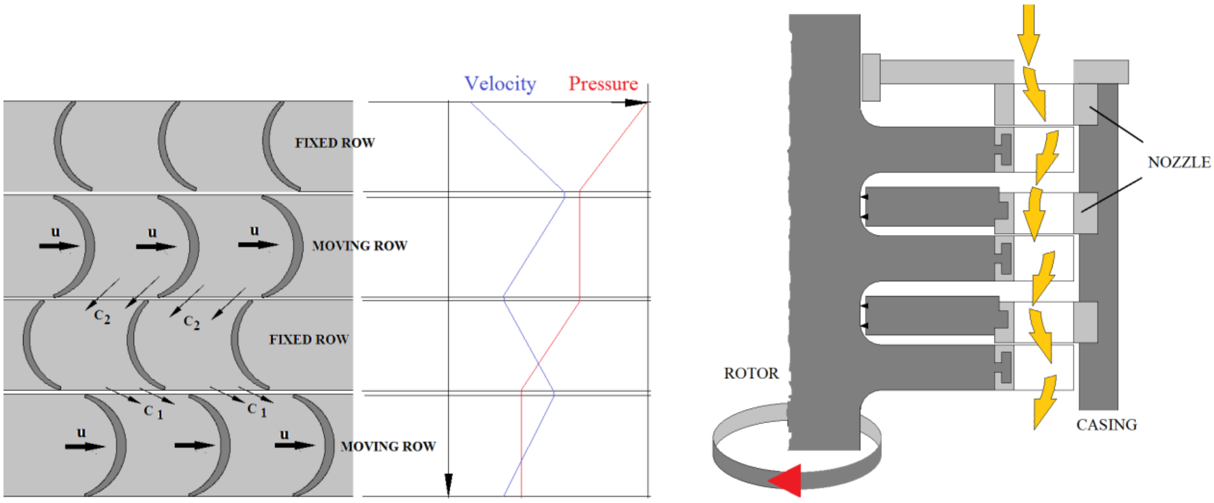


Figure 9

2.3.2 *Velocity Compounded*

It is found in practice that achieving the optimal conditions of steam velocity and blade velocity is not easy with a pressure compounded turbine and a modified design is called ***velocity compounded*** and named after the inventor C. G. Curtis. In this design the entire pressure drop is in the first set of fixed nozzles and subsequent fixed vanes on the casing are there only to deflect the steam to obtain the optimal angle. It is possible to have several similar stages.

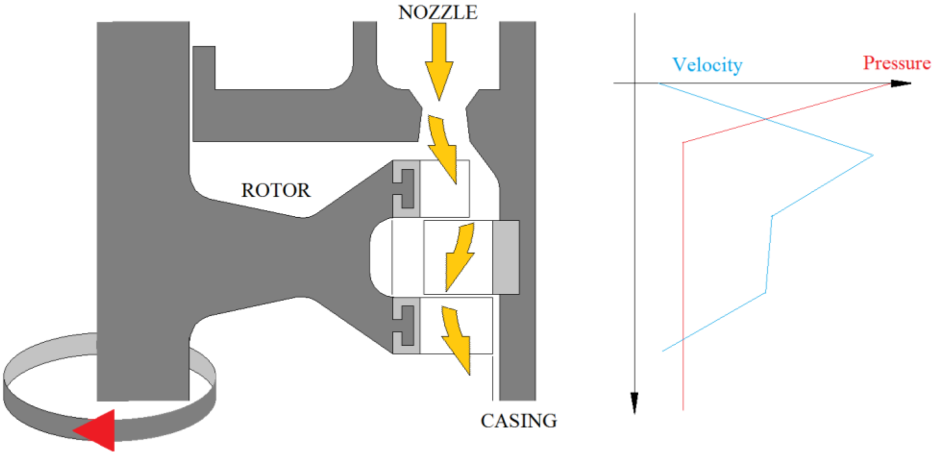


Figure 10

The picture (Figure 11) shows a turbine with 3 sets of rotors (cylinders) and the rotor on the right has been removed to reveal the casing and nozzles.

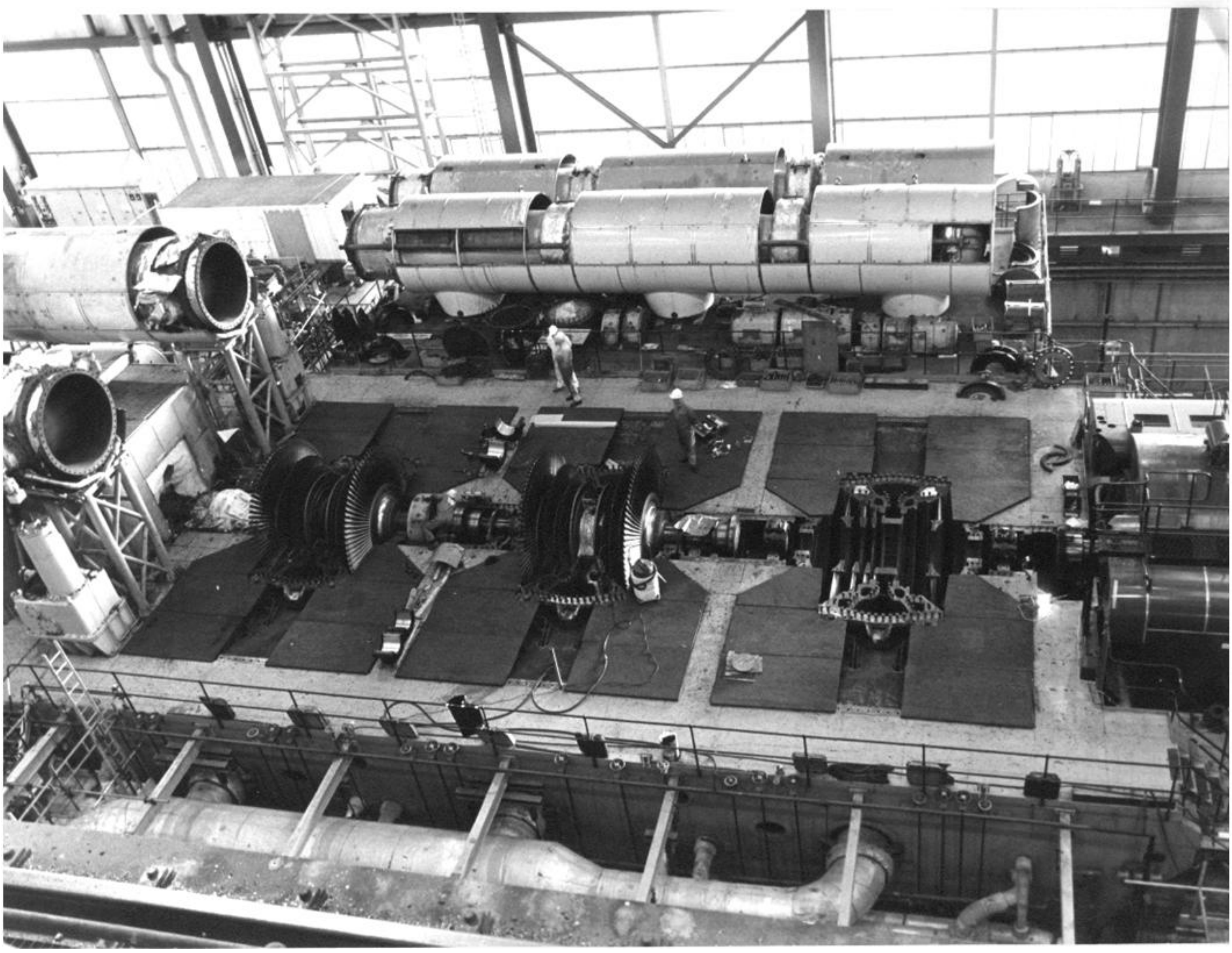


Figure 11

WORKED EXAMPLE No. 3

A pure impulse turbine has a single row of symmetrical blades with an inlet and outlet angle of 30^o to the tangent. The steam exits the nozzles with a velocity of 195 m/s. The blades rotate on a mean diameter of 0.318 m at 3000 rev/min. Draw the blade vector diagram to scale. Assuming shockless entry calculate:

i. the diagram power for unit mass flow
ii the angle of the nozzles for shockless entry
iii. the Diagram Efficiency
iv. the kinetic energy in the steam leaving the row

SOLUTION

$u = \pi ND = \pi \times (3000/60) \text{ x } 0.318 = 50$ m/s

$C_1 = 195$ m/s $\quad \beta_1 = 30^o$ and $\beta_2 = 30^o$ swept back $\quad \dot{m} = 1$ kg/s

Constructing the vector diagram as shown produces $\mathbf{\Delta C_w} = 260$ m/s

D.P. = $\dot{m}\, u\, \Delta C_w = 1 \times 50 \text{ x } 260 = 13\ 000$ W or 13 kW

The angle of the nozzles is $\alpha_1 = 23^o$

$$\eta_d = \frac{2u\,\Delta Cw}{C_1^2} = \frac{2 \text{ x } 50 \times 260}{195^2} = 0.684 \text{ or } 68.4\%$$

Or we could calculate the K.E. at inlet = $\frac{1}{2}\dot{m}\, C_1^2 = \frac{1}{2} \times 1 \times 195^2 = 19\ 012$ W or 19.012 kW

$\eta_d = 13/19 = 0.684$

The exit velocity of the steam is $C_2 = 110$ m/s

The kinetic energy is $\frac{1}{2}\dot{m}\, C_2^2 = \frac{1}{2} \times 1 \times 110^2 = 6\ 050$ W or 6.05 kW

Check Energy Balance D.P. + K.E. at exit = 13 + 6 = 19 kW = K.E. at inlet

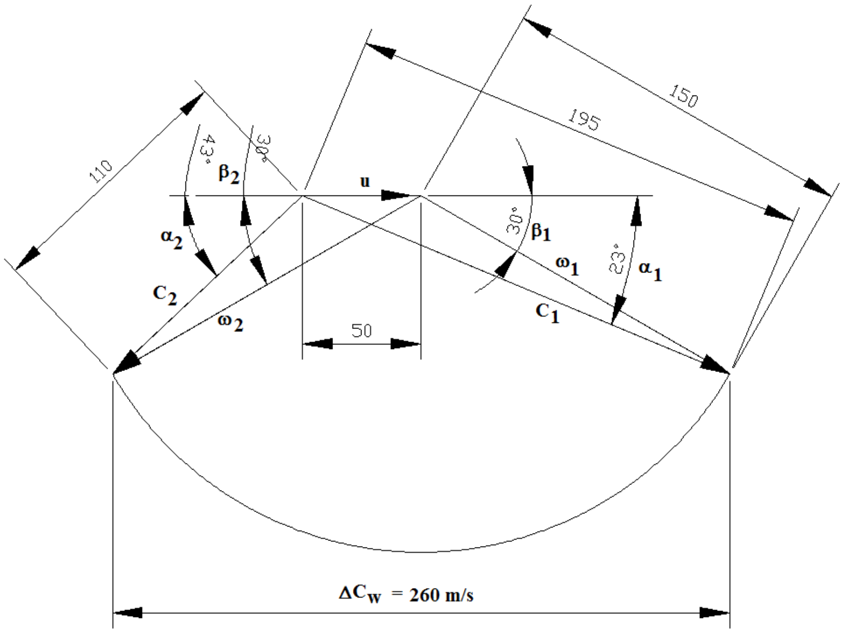


Figure 12

Students should try calculating the answers with the formulae

2.3.4 Axial Force

If the moving blades are not symmetrical and/or there is friction on the blades there is a change in the velocity in the direction of the rotor axis. This produces a change in momentum in that direction and so an axial force is produced. This would require a large thrust bearing in the turbine design.

The vector diagram for such a set up is shown in figure 13. The axial component of the absolute steam velocity is C_A and the change in the axial direction is ΔC_A. The axial force is the rate of change in momentum in the axial direction $\mathbf{F_A = \dot{m}\, \Delta C_A}$ and $C_{A1} = \omega_1 \sin \alpha_1$ and $C_{A2} = \omega_2 \sin \alpha_2$

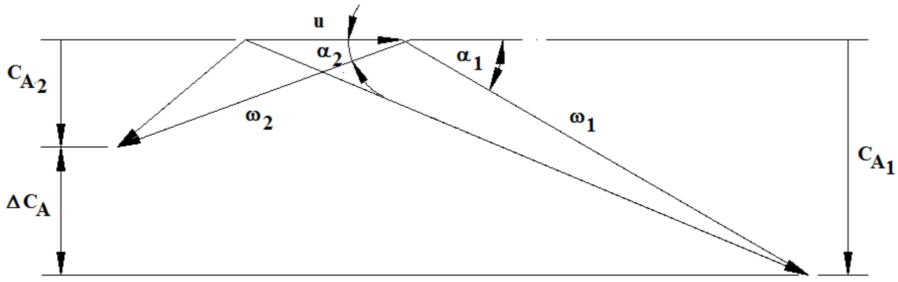


Figure 13

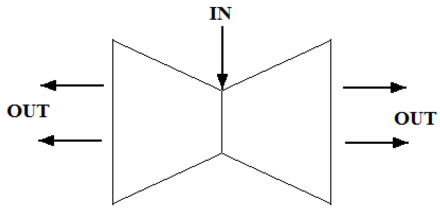


Figure 14

The force can be eliminated by placing two identical rotors back to back so the axial thrust cancels out. Figure 14 shows the schematic for such an arrangement and you can see this in figure 15. The steam enters at the middle and half flows one way and half the other.

Because the volume of the steam or gas increases greatly as it progresses along the axis, the height of the blades increases in order to accommodate it. Figure 15 shows this. The exhaust steam has such a large volume that entry to the condenser is through the large passages underneath. The condenser occupies the space below the turbine hall.

Figure 15

The mean diameter D of the blades changes for each row so solving the diagram power is more difficult. If the blades are long then the tangential velocity u is different at the root to the tip. For maximum efficiency the blade angle would have vary with radius.

WORKED EXAMPLE No. 4

The velocity of steam leaving the nozzles of an impulse turbine is 1000 m/s and the nozzle angle is 20° to the tangential direction. The blade velocity is 350 m/s and the blade friction coefficient is 0.7. The blades are symmetrical. For a unit mass flow rate determine the following:

i. the blade angle at inlet
ii. the diagram power
iii the diagram efficiency
iv the axial force

SOLUTION

From the data given C_1 = 1 000 m/s u = 350 m/s $\alpha_1 = 20^o$ k = 0.7 $\dot{m}$ = 1 kg/s

First solve using trigonometry

$\omega_1^2 = C_1^2 + u^2 - 2\,u\,C_1 \cos(\alpha_1) = 1\,000^2 + 350^2 - 2 \times 350 \times 1000 \cos(20^o) = 464.7 \times 10^3$

ω_1 = 681.7 m/s

$$\beta_1 = \text{asin}\left\{\frac{C_1 \sin(\alpha_1)}{\omega_1}\right\} = \text{asin}\left\{\frac{1000 \sin(20)}{681.7}\right\} = 30.1^o$$

Since the blades are symmetrical $\beta_2 = 30.1^o$

$\omega_2 = k\,\omega_1 = 0.7 \times 681.7 = 477.2$ m/s

$\Delta C_w = \omega_1 \cos(\beta_1) + \omega_2 \cos(\beta_2) = 681.7 \cos(30.1) + 477.2 \cos(30.1) = 1002.6$ m/s

D.P. = $\dot{m}\,u\,\Delta C_w = 1 \times 350 \times 1002.6 = 350.9 \times 10^3$ W or 350.9 kW

$$\eta_d = \frac{2u\,\Delta Cw}{C_1^2} = \frac{2 \times 350 \times 1002.6}{1000^2} = 0.7 \text{ or } 70\%$$

$C_{A1} = \omega_1 \sin(\beta_1) = 681.7 \times \sin(30.1) = 341.9$ m/s

$C_{A1} = \omega_2 \sin(\beta_2) = 477.2 \times \sin(30.1) = 239.3$ m/s

$F_A = \dot{m}\,\Delta C_A = 1 \times (341.9 - 239.3) = 102.6$ N

If the vector diagram drawn to scale is shown in figure 16

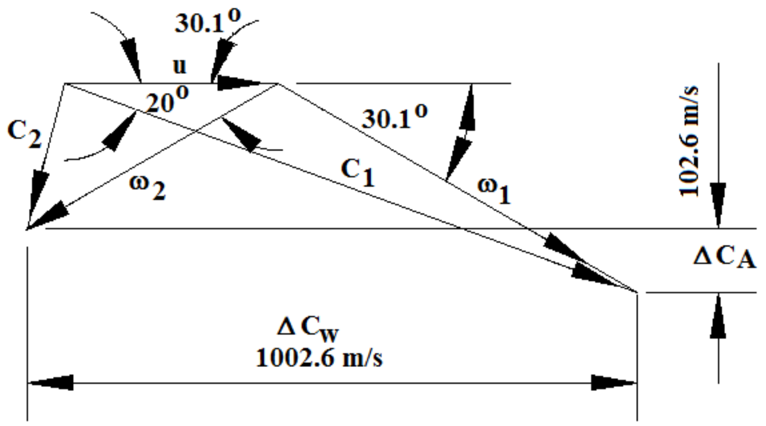


Figure 16

SELF ASSESSMENT EXERCISE No. 2

1. The velocity of steam leaving the nozzles of an impulse turbine is 375 m/s and the nozzle angle is 20^o to the tangential direction. The blade velocity is 165 m/s and the blade friction coefficient is 0.85. The axial velocity is to be constant. For a unit mass flow rate determine the following:

   i. the blade angle at inlet and outlet (34^o and 42^o)
   ii. the diagram power (54.8 kW per unit mass)
   iii the diagram efficiency (78 %)
   iv. the optimal efficiency (81 %)

2. The first stage of a steam turbine is a two row velocity compounded impulse wheel. The velocity of the steam leaving the fixed nozzles is 610 m/s and leaves at 16^o to the tangential direction. The mean velocity of the moving blades is 122 m/s. The exit angle of the moving blades is 18^o. The exit angle of the second row of fixed blades is 21^o. The exit angle of the second moving row is 35^o. The blade friction coefficient is 0.9 for all the blades.

   Determine the inlet angles for all the blades assuming shock free entry throughout.
   Answers 20^o, 24.5^o and 34.1^o

   Assuming a unit mass flow, calculate the diagram power and diagram efficiency for the wheel.
   Answers 145 kW and 78%

   What would be the maximum possible diagram efficiency? Answer 92.4%
   Calculate the axial force. Answer 39.4 N

3. *Reaction Theory*

Reaction Forces are exerted on an object when it causes the velocity of the fluid to change. This could be a change in magnitude or a change in direction or both. When a fluid accelerates in a nozzle, the kinetic energy of the fluid increases and since energy is conserved, the pressure of the fluid drops. In other words, the pressure behind the fluid forces it through the nozzle causing it to speed up. The force required to accelerate the fluid is in the direction of the acceleration. Every force has an equal and opposite reaction so an equal and opposite force is exerted on the nozzle. This is the principle used in rockets.

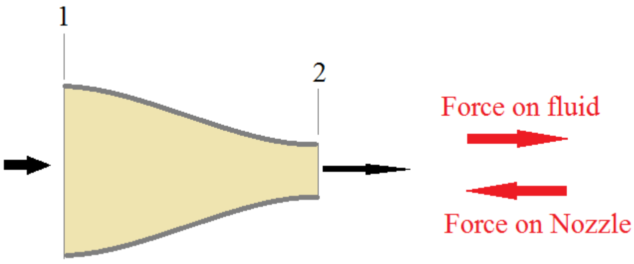


Figure 17

Applying the law of energy conservation between (1) and (2) we find

$$\frac{C_1^2}{2} + h_1 = \frac{C_2^2}{2} + h_2 \qquad \frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2$$

Some may regard the following as trivial but for those who like to see the evidence here is the proof that the above equation holds true even when the nozzle moves. Otherwise move on to the next page.

The force on the fluid is: $F = \dot{m}\,\Delta C = \dot{m}\,\{C_2 - C_1\}$ and the force on the nozzle is equal and opposite.
If this force makes the nozzle move at u to the left, the power developed is $F\,u = u\{C_2 - C_1\}$per unit mass.

The absolute velocities of the steam are now $C_2 - u$ and $C_1 - u$

The energy equation becomes

$$\frac{(C_1 - u)^2}{2} + h_1 = \frac{(C_2 - u)^2}{2} + h_2 - \text{ specific power}$$

$$\frac{(C_1 - u)^2}{2} + h_1 = \frac{(C_2 - u)^2}{2} + h_2 - u(C_2 - C_1)$$

Clear the brackets and simplify

$$\frac{C_1^2}{2} + \frac{u^2}{2} + \frac{2uC_1}{2} + h_1 = \frac{C_2^2}{2} + \frac{u^2}{2} + \frac{2uC_2}{2} + h_2 - uC_2 - uC_1$$

$$\frac{C_1^2}{2} + uC_1 + h_1 = \frac{C_2^2}{2} + uC_2 + h_2 - uC_2 - uC_1$$

$$\frac{C_1^2}{2}\; h_1 = \frac{C_2^2}{2} + h_2 \text{ hence } \frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2$$

This shows the energy equation can be applied to any nozzle using the relative velocities.

A pure reaction turbine would be a set of nozzles on a wheel with the reaction force pushing the vanes in a tangential direction. In practice the steam is expanded in several stages. Each stage contains one set of fixed vanes that serve to accelerate the steam and guide it on to the next set of moving vanes. The steam is not only deflected by the moving blades but also undergoes a further drop in pressure producing an increase in the relative velocity.

It is also normal to keep the axial velocity constant throughout the turbine so $C_{A1} = C_{A2} = C_A$ and so there is no axial force. Consider one pair of blades or stage and make it the first stage. The steam enters the first fixed row at point (0) and has shockless entry to the first moving row at (1) and exits at (2) into the next fixed row. This is illustrated in figure 18. The steam accelerates from a low velocity at (0) (this is often taken as zero for the first stage) to C_1. The pressure and enthalpy is reduced in consequence. The steam enters the moving row with absolute velocity C_1 at an absolute angle of α_1. The moving blade must have angle β_1 at entry to make the relative velocity ω_1 correct for shockless entry. The steam expands through the moving blades driving the rotor. The absolute velocity and enthalpy drops further. If there is a following row of fixed blades the angles at (2) must be correct for shockless entry (α_2 and β_2) but we are not considering the next stage at the moment.

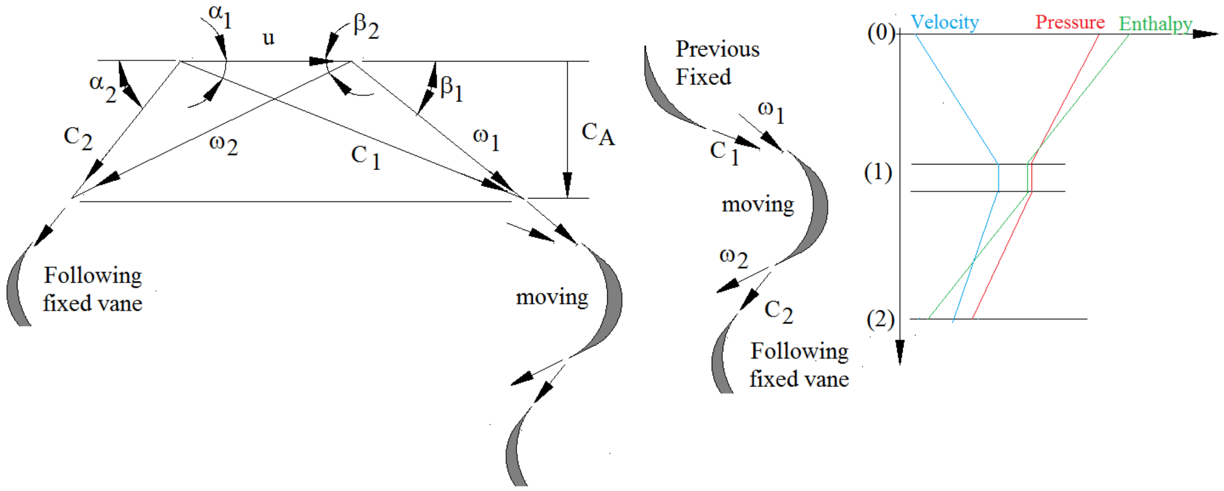


Figure 18

3.1 Degree of Reaction

A reaction turbine designed along these lines must be partly impulse and partly reaction and the degree of reaction in a stage is defined as

$$R = \frac{\text{Enthalpy drop in moving row}}{\text{Enthalpy drop over the stage}} = \frac{h_1 - h_2}{h_o - h_2}$$

Remember the change in enthalpy over the rotor is equal to the change in relative kinetic energy.

$$h_1 - h_2 = \frac{\omega_2^2}{2} - \frac{\omega_1^2}{2} = \frac{1}{2}(\omega_2^2 - \omega_1^2)$$

Applying energy conservation to the rotor we have:

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} + P$$

where P is the diagram power per unit mass flow (energy removed by the rotor)

$$P = (h_1 - h_2) + \left(\frac{C_1^2}{2} - \frac{C_2^2}{2}\right) = \frac{1}{2}(\omega_2^2 - \omega_1^2) + \frac{1}{2}\ (C_1^2 - C_2^2)$$

Applying the energy equation over the stage we have:

$$h_0 + \frac{C_0^2}{2} = h_2 + \frac{C_2^2}{2} + P \qquad P = (h_0 - h_2) + \frac{1}{2}(C_0^2 - C_2^2)$$

The problem with a turbine designed along these lines is that the angles and shape of each row of blades is different and it is much more common to make the velocity the same at the entry to all fixed rows (except perhaps the first row). If this is done as well as keeping the axial velocity constant, then all the fixed blades and moving blades are identical as shown (except perhaps the first row).

This design is named after ***Charles Parsons.*** It follows that the pressure drop over each stage is the same. Figure 19 shows the layout of such a turbine and the vector diagram.

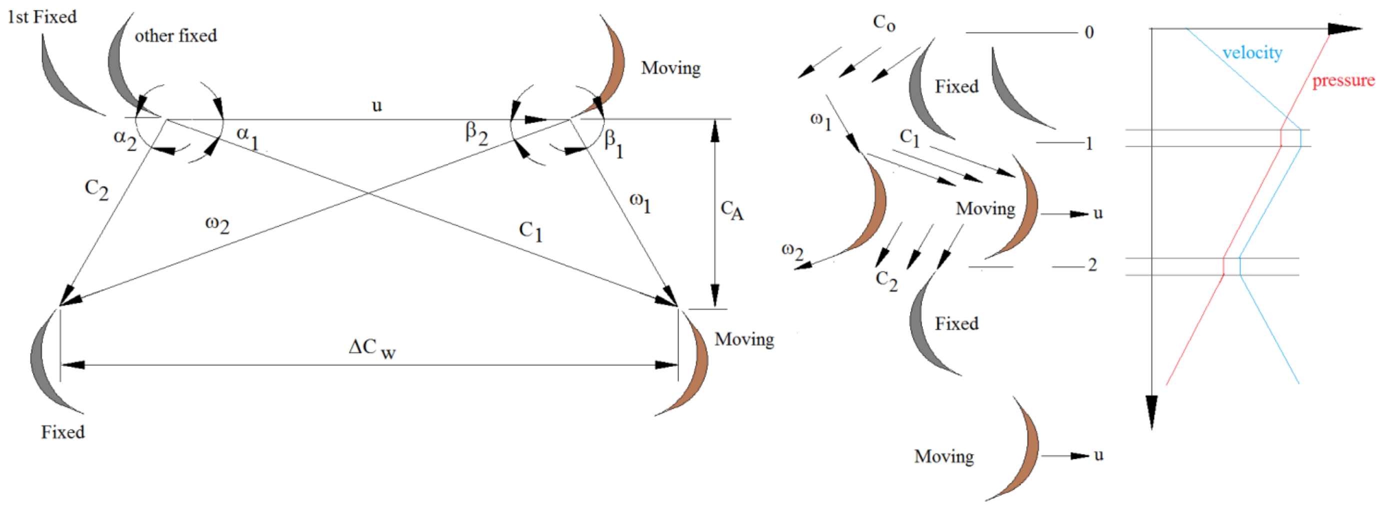


Figure 19

It can be seen from the vector diagram (figure 19) that $\alpha_1 = \beta_2$, $\alpha_2 = \beta_1$, $C_1 = \omega_2$ and $C_2 = \omega_1$ and it follows that C_2 is the same as the velocity exiting the previous fixed stage C_0. This means that the overall change in kinetic energy for the two rows from point (0) to point (2) is zero.

If the velocity is returned to the same value after each stage then $C_0 = C_2$ so

$$P = (h_0 - h_2) = \frac{1}{2}(\omega_2^2 - \omega_1^2) + \frac{1}{2}(C_1^2 - C_2^2)$$

$$R = \frac{h_1 - h_2}{h_0 - h_2} = \frac{(\omega_2^2 - \omega_1^2)}{2P} = \frac{(\omega_2^2 - \omega_1^2)}{(\omega_2^2 - \omega_1^2) + (C_1^2 - C_2^2)}$$

If $\alpha_2 = \beta_1$ and $\alpha_1 = \beta_2$ then $\omega_1 = C_2$ and $\omega_2 = C_1$

$$R = \frac{(\omega_2^2 - \omega_1^2)}{(\omega_2^2 - \omega_1^2) + (\omega_2^2 - \omega_1^2)} = \frac{(\omega_2^2 - \omega_1^2)}{2(\omega_2^2 - \omega_1^2)} = \frac{1}{2}$$

The degree of reaction is hence 50% for a Parson's turbine.

3.2 Diagram Efficiency for Parsons Turbine

The definition of diagram efficiency (also called blade efficiency) is:

$$\eta_d = \frac{\text{D.P.}}{\text{Energy supplied to the moving wheel}}$$

D.P. = $u\,\Delta C_w = u(2\,C_1\cos\alpha_1 - u)$ from the vector diagram

E = Kinetic energy in the absolute velocity entering plus the enthalpy drop over the rotor.

$E = \frac{C_1^2}{2} + (h_1 - h_2)$ this means the kinetic energy of the steam leaving the rotor $\frac{C_2^2}{2}$ is excluded. This is the same as the impulse turbine with the enthalpy drop term added for the reaction part.

It was shown earlier that

$$h_1 - h_2 = \frac{\omega_2^2}{2} - \frac{\omega_1^2}{2} \qquad E = \frac{C_1^2}{2} + \left(\frac{\omega_2^2}{2} - \frac{\omega_1^2}{2}\right)$$

For the Parsons turbine $\omega_2 = C_1$

$$E = \frac{C_1^2}{2} + \left(\frac{C_1^2}{2} - \frac{\omega_1^2}{2}\right) = C_1^2 - \frac{\omega_1^2}{2}$$

$$\eta_d = \frac{u\,(2C_1\cos\alpha_1 - u)}{C_1^2 - \frac{\omega_1^2}{2}}$$

Applying the cosine rule to the input vector triangle

$$\omega_1^2 = C_1^2 + u^2 - 2uC_1\cos\alpha_1$$

$$\eta_d = \frac{u\,(2C_1\cos\alpha_1 - u)}{C_1^2 - \frac{(C_1^2 + u^2 - 2uC_1\cos\alpha_1)}{2}} = \frac{2u\,(2C_1\cos\alpha_1 - u)}{C_1^2 - u^2 + 2uC_1\cos\alpha_1}$$

Often this is changed to use the ratio $r = u/C_1$

$$\eta_d = \frac{2r\,(2\cos\alpha_1 - r)}{1 - r^2 + 2r\cos\alpha_1}$$

3.3 Optimal Efficiency

If we plot the diagram efficiency against r using typical values we see there is a clear maximum. It can be shown that the maximum occurs when $r = \cos(\alpha_1)$ (you can prove this by differentiating if you wish). The optimal efficiency is then

$$\eta_d = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1} \text{ and } r = \cos(\alpha_1)$$

Compare this to the impulse turbine.

$\eta_d = \cos^2\alpha_1 \qquad r = ½\cos(\alpha_1)$

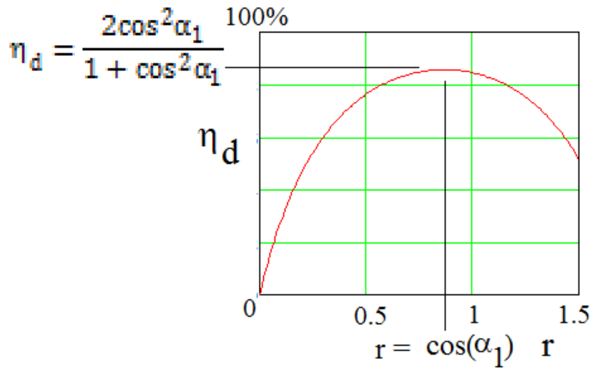


Figure 20

3.4 Axial Flow

It should be borne in mind that steam and gas, unlike liquids, undergoes a volume increase when the pressure falls. This would produce steam velocities that are much too big so the vanes on the rotor and the casing (figure 21) increase in height as the pressure falls. For a given pair of rows it is assumed that the average height is h and the mean diameter of rotation is D. The volume of steam passing through the annular ring is the product of area and axial velocity C_A.

$$V = \pi D h C_A$$

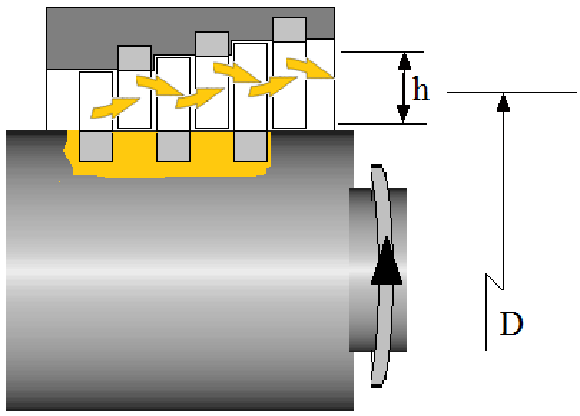


Figure 21

This ignores the effect of the blade thickness.
If the blades are short the area is $A = \pi D h$
If the blades are tall $A = \pi (D_o^2 - D_i^2)$

D_o and D_i are the outer and inner diameters

For a mass flow $\dot{m}$

$$V = \dot{m}\, v \quad \text{where v is the specific volume}$$

WORKED EXAMPLE No. 5

The fixed row of a 50% reaction turbine (Parson's) stage produces dry saturated steam at 2.7 bar and an absolute velocity of 100 m/s. The mean height of the blades is 40 mm. The moving blades are swept back at 20^o to the tangent. The axial velocity of the steam is ¾ of the blade velocity at the mean diameter. Steam is supplied to the stage at 9080 kg/hour. Ignore the blade thickness when calculating the annulus area.

Determine:

i. the rotor speed in rev/min
ii. the diagram power
iii. the diagram efficiency
iv. the enthalpy drop of the steam over the stage

SOLUTION

You need to start sketching the blade vector diagram to work out how to draw it. We know the following:
$C_2 = C_1 = 100$ m/s $\beta_2 = \alpha_1 = 20^o$ $C_A = ¾ u$ $C_A = C_1 \sin \alpha_1 = 100 \sin 20^o = 34.2$ m/s h = 40 mm
u = 4 x 34.2/3 = 45.6 m/s
This is sufficient to draw the vector diagram. The figures on the diagram are rounded off but accurate figures may be found using trigonometry.

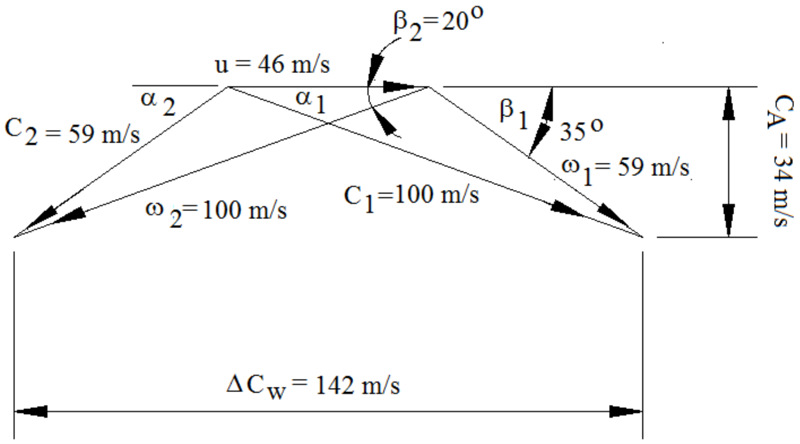


Figure 22

First we need to find the annular area from the volume flow rate so we need the specific volume of the steam leaving the fixed row.

$v = v_g$ @ 2.7 bar which from tables or other sources is 0.6057 m^3/kg
The volume flow rate is hence $V = 9080 \times 0.6057 = 5\,500$ m^3/hour or 91.7 m^3/min or 1.528 m^3/s
$V = 1.528$ m^3/s

The annular area is $A = V/C_A = 1.528/34.2 = 0.0447$ m^2
$A = \pi D h$ hence $D = 0.0447/(\pi \times 0.04) = 0.356$ m

$$N = \frac{u}{\pi \times D} \times 60 = \frac{45.6}{\pi x\, 0.356} \times 60 = 2\,446 \text{ rev/min}$$

$\Delta C_w = 100 \cos(20^o) \times 2 - 46 = 142$ m/s (or scale from diagram)

D.P. = $\dot{m}$ u ΔC_w = (9 080/3 600) × 46 × 142 = 16 475 W or 16.475 kW

The energy input to the moving blades was earlier shown to be

$$E = C_1^2 + \frac{\omega_1^2}{2}$$

Putting in the numbers from the vector diagram
E = 100^2 - ½ × 59^2 = 8 259.5 J per unit mass so E = 8 259.5 × (9 080/3 600) = 2 0832.3 W
η_d = 16.475/20.832 = 0.79 or 79%

$$\eta_d = \frac{u\,(2C_1\cos\alpha_1 - u)}{C_1^2 - \frac{\omega_1^2}{2}} = \frac{46(2\ \times\ 100\ \times \cos(20^o) - 46)}{100^2 - \frac{59^2}{2}} = \frac{6\,529.1}{8\,259.5} = 0.79$$

Specific enthalpy drop over the moving row = ½ (ω_2^2 - ω_1^2) = ½ (100^2 - 59^2) = 3 259.5 J/kg

Specific enthalpy drop over the stage = 2 × 3 259.5 = 65 19 J/kg

SELF ASSESSMENT EXERCISE No. 3

1. A stage of a 50% reaction turbine has a row of blades on a mean diameter of 1 m that rotates at 3000 rev/min. The exit angles of the blades are 30^o and the inlet angles 50^o.The mass flow rate is 10000 kg/min. The stage efficiency is 85%.
   Determine
   i. the diagram power (11.83 MW)
   ii. the diagram efficiency (72.9 %)

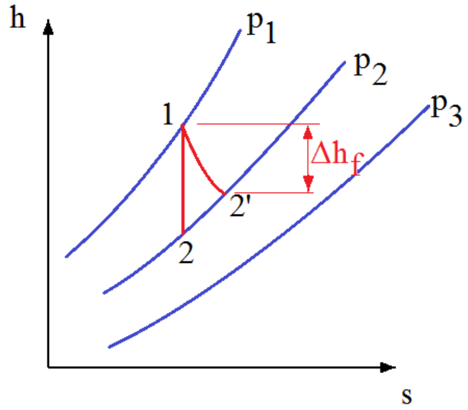
2. The rotor in a stage of a reaction turbine spins at 3000 rev/min and the blades have an exit angle of 20^o, a mean tangential velocity of 91.44 m/s and mean length of 25.4 mm. The blade speed ratio is 0.56. The specific volume of the steam at entry to the rotor is 0.655 m^3/kg. The axial velocity is constant.

   Neglecting the blade thickness calculate the mass flow rate of steam. (3.956 kg/s)
   Calculate the diagram power, diagram efficiency and optimum diagram efficiency.
   (Answers 77.92 kW, 85% and 94%)

3. A reaction turbine uses 0.5 kg/s of steam to produce 22.38 kW. The blade velocity ratio is 0.8. The exit angle of the blades is 20^o. The axial velocity is constant.
   Determine:
   i. the enthalpy drop in each moving row.(11.19 kJ/s)
   ii. the blade mean velocity (182.1 m/s)
   iii. the diagram efficiency (93%)
   iv. the optimal efficiency (94%)

4. *The Affect of Friction*

When a vapour or gas expands in a turbine, there are various sources of frictional losses that reduce the power output and results in an increase in the entropy of the fluid. On an h - s chart the expansion from pressures p_1 to p_2 is as shown. Expansion from 1 to 2 is the ideal isentropic expansion and from 1 to 2′ is the actual expansion with friction. The drop in enthalpy is Δh_f. The isentropic or overall efficiency is defined as



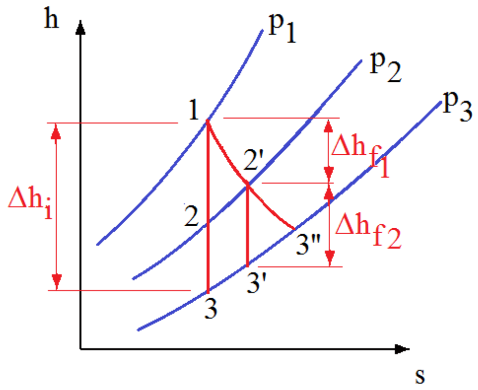
$$\eta_o = \frac{h_1 - h_{2'}}{h_1 - h_2} = \frac{\Delta h_f}{h_1 - h_2}$$

If the expansion shown is just one stage of a multistage turbine then this is called the stage efficiency and defined as:

$$\eta_{s1} = \frac{\Delta h_{f1}}{h_1 - h_2}$$

From which $\Delta h_{f1} = \eta_{s1}\{h_1 - h_2\}$

Now consider a 2 stage turbine expanding fluid from pressure p_1 to p_3. The overall expansion is 1 to 3 but the stage efficiency of the second stage is:



$$\eta_{s2} = \frac{h_{2'} - h_{3''}}{h_{2'} - h_{3'}} = \frac{\Delta h_{f2}}{h_{2'} - h_{3'}}$$

From which $\Delta h_{f2} = \eta_{s2}\{h_{2'} - h_{3'}\}$

For the two stages the overall isentropic efficiency is:

$$\eta_o = \frac{h_1 - h_{3''}}{h_1 - h_3} = \frac{\{h_1 - h_{2'}\} + \{h_{2'} - h_{3''}\}}{h_1 - h_3}$$

$$\eta_o = \frac{\eta_{s1}\Delta h_{f1} + \eta_{s2}\Delta h_{f2}}{\{h_1 - h_3\}}$$

If the stage efficiency is the same for both stages and denoted η_s then:

$$\eta_o = \frac{\eta_s(\Delta h_{f1} + \Delta h_{f2})}{\{h_1 - h_3\}}$$

This could be applied to any number of stages and written as:

$$\eta_o = \frac{\eta_s \sum \Delta h_f}{\Delta h_i}$$

Where Δh_i is ideal or isentropic overall change in enthalpy.

The ***Reheat Factor*** is given by the term

$$\text{R.F.} = \frac{\sum \Delta h_f}{\Delta h_i}$$

$$\boldsymbol{\eta_o = \eta_s \times \text{R.F.}}$$

The same theory can be applied to gas or steam turbines. If any expansion (or compression) is deemed to be made up of an infinite number of small stages, the reheat factor becomes the ***Polytropic Efficiency*** and this is covered in detail in the next tutorial.

WORKED EXAMPLE No. 6

Steam at 15 bar and 300°C is expanded through a reaction turbine to 0.14 bar. All the stages have a stage efficiency of 75%. The reheat factor is 1.04. The turbine is required to produce 12 MW. Calculate the flow of steam required.

SOLUTION

At inlet the steam properties are: p_1 = 15 bar θ_1 = 300°C h_1 = 3 039 kJ/kg s_1 = 6.919 kJ/kg K
At exit the steam properties are : p_2 = 0.14 bar and probably wet steam of unknown dryness fraction.

The ideal entropy value is $s_2 = 6.919$ kJ/kg K $= s_f + x\, s_{fg} = 0.737 + 7.294\, x_2$

$x_2 = (6.919 - 0.737)/7.294 = 0.8475$ (the ideal dryness fraction at exit for an isentropic expansion).

The ideal enthalpy value is $h_2 = h_f + x_2\, h_{fg}$ at 0.14 bar

$h_2 = 220 + (0.8475)(2\,376) = 2\,233.7$ kJ/kg

The ideal power output is $P = \dot{m}(h_1 - h_2) = \dot{m}(3\,039 – 2\,233.7) = 805.3\,\dot{m}$ kW

$\eta_o = \eta_s \times$ R.F. $= 75\% \times 1.04 = 75\%$

The actual power output is $P = (805.3\,\dot{m}) \times \eta_o = (805.3\,\dot{m})\,0.75 = 604\,\dot{m}$ kW

Equating to the required power
$604\,\dot{m} = 12\,000$ kW hence $\dot{m} = 19.9$ kg/s

SELF ASSESSMENT EXERCISE No. 4

Steam at 60 bar and 500°C is expanded through a reaction turbine to 0.07 bar. All the stages have a stage efficiency of 80%. The reheat factor is 1.05. The turbine is required to produce 20 MW. Calculate the flow of steam required.
Answer 20.46 kg/s

5. Extra Tutorial Sheet (For Steam/Gas Turbines) no solutions

Q1. The adiabatic heat drop in a given stage of a multi-stage impulse turbine is 22.1 kJ/kg of steam. The nozzle outlet angle is 16^o the efficiency of the nozzle, defined as the ratio of the actual gain of kinetic energy in the nozzle to adiabatic heat drop, is 92%. The mean diameter of the blades is 1473.2 mm and the revolution per minutes is 1500. Given that the carry over factor ϕ is 0.88, and that the blades are equiangular (the blade velocity coefficient is 0.87). Calculate the steam velocity at the outlet from nozzles, blade angles, and gross stage efficiency.

Q2. The following particulars related to a two row velocity compounded impulse wheel which forms a first stage of a combination turbine.

Steam velocity at nozzle outlet = 579.12 m/s
Mean blade velocity = 115.82 m/s
Nozzle outlet angle = 16^o
Outlet angle first row of moving blades = 18^o
Outlet angle fixed guide blades = 22^o
Outlet angle, second row of moving blades = 36^o
Steam flow rate = 2.4 kg/s

The ratio of the relative velocity at outlet to that at inlet is 0.84 for all blades. Determine for each row of moving blades the following

(a) The velocity of whirl
(b) The tangential thrust on blades
(c) The axial thrust on the blades
(d) The power developed

What is the efficiency of the wheel as a whole?

Q3. A velocity compounded impulse wheel has two rows of moving blades with a mean diameter of 711.2 mm. The speed of rotation is 3 000 rpm, the nozzle angle is 16^o and the estimated steam velocity at the nozzle outlet is 554.73 m/s. The mass flow rate of the steam passing through the blades is 5.07 kg/s.

Assuming that the energy loss in each row of blades (moving and fixed) is 24% of the kinetic energy of the steam entering the blades and referred to as the relative velocity, and that the outlet angles of the blades are: (1) first row of moving blades 18^o, (2) intermediate guide blade 22^o, (3) second row of moving blades is 36^o, draw the diagram of relative velocities and derive the following.

(a) Blade inlet angles
(b) Power developed in each row of blades
(c) Efficiency of the wheel as a whole

Q4. The following particulars refer to a stage of an impulse-reaction turbine.

Outlet angle of fixed blades = 20^o
Outlet angle of moving blades = 30^o
Radial height of fixed blades =100 mm
Radial height of moving blades =100 mm
Mean blade velocity = 138 m/s
Ratio of blade speed to steam speed = 0.625
Specific volume of steam at fixed blade outlet =1.235 m^3/kg
Specific volume of steam at moving blade outlet =1.305 m^3/kg

Calculation the degree of reaction, the adiabatic heat drop in pair of blade rings, and the gross stage efficiency, given the following coefficients which may be assumed to be the same in both fixed and moving blades : $\eta_m = 0.9, \phi = 0.86$.

Q5. Steam flows into the nozzles of a turbine stage from the blades of preceding stage with a velocity of 100 m/s and issues from the nozzles with a velocity of 325 m/s at angle of 20^o to the wheel plane. Calculate the gross stage efficiency for the following data:

Mean blade velocity =180 m/s
Expansion efficiency for nozzles and blades = 0.9
Carry over factor for nozzles and blades = 0.9
Degree of reaction = 0.26
Blade outlet angle = 28^o