THERMODYNAMICS 201

TUTORIAL No.8

COMBUSTION OF FUELS

On completion of this tutorial you should be able to

. write down combustion equations.

. solve the oxygen and air requirements for the combustion of solid, liquid and gaseous fuels.

. determine the products of combustion.

. determine the air/fuel ratio from the products of combustion.

. solve problems involving energy released by combustion.

.solve problems involving dissociation of products.

. solve past paper questions.

Let's start by revising the basics.

1. INTRODUCTION

Combustion is the process of chemical reaction between fuel and oxygen(reactants). The process releases heat and produces products of combustion. The main elements which burn are :

CARBON

HYDROGEN

SULPHUR

The heat released by 1 kg or m^3 of fuel is called the calorific value.

The oxygen used in combustion processes normally comes from the atmosphere and this brings nitrogen in with it which normally does nothing in the process but makes up the bulk of the gases remaining after combustion.

The main elements in combustion are then :

	Symbol	Atomic Mass	Molecular Mass	Product
Carbon	C	12		CO_2
Hydrogen	H_2	1	2	H_2O
Sulphur	S	32		SO_2
Oxygen	O_2	16	32	
Nitrogen	N_2	14	28	

If the water formed during combustion leaves as vapour, it takes with it the latent heat of evaporation and thus reduces the energy available from the process. In this case the calorific value is called the lower Calorific value (LCV). If the products cool down after combustion so that the vapour condenses, the latent heat is given up and the calorific value is then the higher calorific value (HCV).

Solid and liquid fuels are normally analysed by mass to give the content of carbon, hydrogen, sulphur and any other elements present. Often there is silica, moisture and oxygen present in small quantities which have some effect on process. The silica leaves slaggy deposits on the heat transfer surfaces in boilers.

Gaseous fuels are normally analysed by volumetric content and are in the main hydrocarbon fuels.

For purposes of calculation, the content of air is considered to be :

	VOLUMETRIC	GRAVIMETRIC
Oxygen	21%	23%
Nitrogen	79%	77%

The sulphur content of the fuel is considered to be a pollutant and so undesirable. The theoretically correct quantity of air or oxygen required to just exactly burn the fuel expressed as a ratio to the fuel burned, is called the STOICHIOMETRIC RATIO. In practice it is found that not all the oxygen in the reactant reaches the fuel elements and that excess air is required in order to ensure complete combustion. This results in oxygen appearing in the products. If too little air or oxygen is supplied, the result is incomplete combustion resulting in the formation of carbon monoxide CO instead of carbon dioxide CO_2. The resulting products contain water H_2O. Industrial equipment for measuring the contents of the products usually remove the water from the sample and the products are then called the dry products.
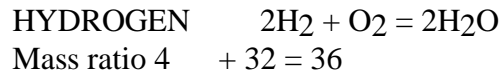
2. COMBUSTION CHEMISTRY

2.1 SOLID AND LIQUID FUELS

In the case of solid and liquid fuels, we do the combustion of each element separately. The important rule is that you must have the same number of atoms of each substance before and after the process. This may be obtained by juggling with the number of molecules.

$$CARBON \qquad C + O_2 = CO_2$$
$$Mass\ ratio\ 12 + 32 = 44$$
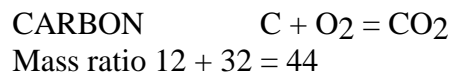
Hence 1kg of C needs 32/12kg of O_2 and makes 44/12kg of CO_2

$$HYDROGEN \qquad 2H_2 + O_2 = 2H_2O$$
$$Mass\ ratio\ 4 \quad + 32 = 36$$

Hence 1kg of H_2 needs 8kg of O_2 and makes 9 kg of H_2O

$$SULPHUR \qquad S + O_2 = SO_2$$
$$32 + 32 = 64$$

Hence 1 kg of S needs 1kg of O_2 and makes 2kg of SO_2.

2.2. GASEOUS FUELS

Typical hydrocarbons are :

Methane	CH_4
Ethane	C_2H_6
Propane	C_3H_8
Butane	C_4H_{10}
Pentane	C_5H_{12}
Hexane	C_6H_{14}
Heptane	C_7H_{16}
Octane	C_8H_{18}
Ethene	C_2H_4 (Ethylene)
Propene	C_3H_6 (Propylene)
Ethyne	C_2H_2 (Acetylene)
Benzenol	C_6H_6 (Benzene)
Cyclohexane	C_6H_{12}

The combustion equation follows the following rule :

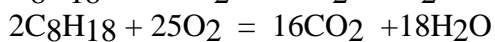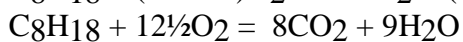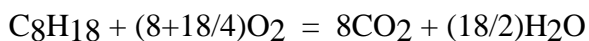$$C_aH_b + (a+b/4)O_2 = (a)CO_2 + (b/2)H_2O$$
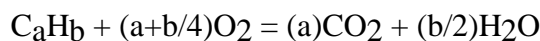
If this results in fractional numbers of molecules, then the whole equation may be multiplied up.

WORKED EXAMPLE No.1

Write out the combustion equation for C_8H_{18}

SOLUTION

$C_8H_{18} + (8+18/4)O_2 = 8CO_2 + (18/2)H_2O$
$C_8H_{18} + 12\frac{1}{2}O_2 = 8CO_2 + 9H_2O$
$2C_8H_{18} + 25O_2 = 16CO_2 + 18H_2O$

There are other gases which burn and the main one to know about is Carbon Monoxide (CO) which is partially burned carbon. The equation for the combustion of CO is :

$$2CO + O_2 = 2CO_2$$

---

3. COMBUSTION BY MASS

The only rule to be observed in deducing the quantities of each substance is the law of conservation of mass. The proportions of the mass is that of the molecular masses. This is shown in the following example.

WORKED EXAMPLE No.2

A fuel contains by mass 88% C, 8%H_2, 1%S and 3% ash (silica). Calculate the stoichiometric air.

SOLUTION

CARBON $\qquad\qquad$ $C + O_2 = CO_2$
Mass ratio $\qquad$ $12 + 32 = 44$

Hence 0.88kg of C need (32/12)x0.88=2.347kg of oxygen.
It makes (44/12) x 0.88 = 3.227 kg of carbon dioxide.

HYDROGEN $\qquad\qquad$ $2H_2 + O_2 = 2H_2O$
Mass ratio $\qquad\qquad$ $4 \ + \ 32 = 36$

hence 0.08kg of hydrogen needs (32/4) x 0.08 = 0.64kg of oxygen.

SULPHUR $\qquad\qquad$ $S + O_2 = SO_2$
Mass ratio $\qquad\qquad$ $32 + 32 = 64$

Hence 0.01kg of sulphur needs 0.01kh of oxygen and makes 0.02kg of sulphur dioxide.

TOTAL OXYGEN needed is 2.347 + 0.64 + 0.01 = 2.997kg

TOTAL AIR needed is 2.997/23% = 13.03kg

The STOICHIOMETRIC air/fuel ratio is 13.03/1

WORKED EXAMPLE No.3

If the air supplied is 20% more than the stoichiometric value, find the analysis of the dry products by mass.

SOLUTION

If 20% excess air is supplied then the air supplied is:

$$120\% \times 13.03 = 15.637 \text{ kg}$$

Oxygen is also 20% excess so $0.2 \times 2.997 = 0.599$kg is left over.
Nitrogen in the air is 77% x 15.637 = 12.04kg

List of products :

Nitrogen	12.04kg	=75.8%
Carbon dioxide	3.227kg	=20.3%
Sulphur dioxide	0.02kg	=0.1%
Oxygen	0.599kg	=3.8%
Total dry product	15.886kg	=100%

It is of interest to note that for a given fuel, the % of any product is a direct indication of the excess air and in practice the carbon dioxide and/or oxygen is used to indicate this. This is important in obtaining optimal efficiency in a combustion process.

SELF ASSESSMENT EXERCISE No. 1

1. A boiler burns fuel oil with the following analysis by mass :

    80% C 18% H_2 2% S

    30% excess air is supplied to the process. Calculate the stoichiometric ratio by mass and the % Carbon Dioxide present in the dry products.
    (15.62/1   14.9% CO_2)

2. A boiler burns coal with the following analysis by mass :

    75% C 15% H_2 7% S   remainder ash

    Calculate the % Carbon Dioxide present in the dry products if 20% excess air is supplied.
    (16.5% CO_2)

3. Calculate the % of each dry product when coal is burned stoichiometrically in air. The analysis of the coal is:

    80% C   10% H_2 5% S and 5% ash.

    (76.7% N, 22.5% CO_2 0.8% SO_2)

4. COMBUSTION BY VOLUME

First we need to revise gas mixtures and understand the meaning of VOLUMETRIC CONTENT. To do this we must understand Dalton's law of partial pressures and Avagadro's Law.

First let us define the kmol. A kmol of substance is the number of kg numerically equal to the apparent molecular mass. For example 12 kg of Carbon is a kmol, so is 32 kg of O_2 and 2 kg of H_2 and 28 kg of N_2.

The molecular mass of a substance is expressed as kg/kmol so the molecular mass of O_2 , for example, is 32 kg/kmol.

Avagadro's Law states :

$1m^3$ of any gas at the same pressure and temperature contains the same number of molecules. It follows that the volume of a gas at the same p and T is directly proportional to the number of molecules. From this we find that the volume of a kmol of any gas is the same if p and T are the same.

Dalton's law states:

The total pressure of a mixture is the sum of the partial pressures. The partial pressure is the pressure each gas would exert if it alone occupied the same volume at the same temperature.

Consider two gases A and B occupying a volume V at temperature T. Using the Universal gas law for each :

$$p_A V_A = m_A R_o T / \tilde{N}_A \qquad p_B V_B = m_B R_o T / \tilde{N}_B$$

where $\tilde{N}$ is the relative molecular mass.

$p_A/p_B = m_A \tilde{N}_B / m_B \tilde{N}_A$ = ratio of the kmol fractions.
p_A and p_B are the partial pressures.
V_A and V_B are the partial volumes.

These are the volumes each gas would occupy if they were separated and kept at the original p and T. This concept is very useful in problems involving the combustion of gases. It also follows that the partial volumes are directly related to the partial pressures so that $V_A/V_B = p_A/p_B$
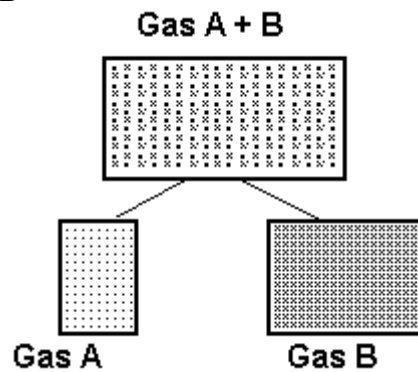

Figure 1

When not mixed the pressure is p and the volumes are V_A and V_B. Hence :

$$pV_A/T = mR_o/\tilde{N}_A \qquad \grave{} \qquad p = m_AR_oT/\tilde{N}_AV_A \ldots\ldots\ldots\ldots(1)$$

$$pV_B/T = mR_o/\tilde{N}_B \qquad p = m_BR_oT/\tilde{N}_BV_B \ldots\ldots\ldots\ldots(2)$$

Since (1) = (2) then :

$m_A/\tilde{N}_AV_A = m_B/\tilde{N}_BV_B$ and so $V_A/V_B = (m_A/\tilde{N}_A)(m_B/\tilde{N}_B)$ which shows that in a mixture, the partial volumes are in the same ratio as the kmol fractions which in turn are in proportion to the number of molecules of each gas.
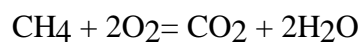
When mixed they both have volume V, hence:

$$p_A = m_AR_oT/\tilde{N}_A \, V \ldots\ldots\ldots(3)$$

$$p_B = m_BR_oT/\tilde{N}_B \, V \ldots\ldots\ldots(4)$$

(3)/(1) gives $p_A/p = V_A/V$ and (4)/(2) gives $p_B/p = V_B/V$
hence $V_A/V_B = p_A/p_B$

Consider the combustion of Methane.

$$CH_4 + 2O_2 = CO_2 + 2H_2O$$

Since the volumetric content of each gas is in the same ratio as the kmol fractions then volumetric content is in the same proportion as the molecules. Hence it needs 2 volumes of oxygen to burn 1 volume of methane.

The volume of air needed is 2/21% = 9.52 volumes. Hence it burn 1 m^3 of methane we need 9.52 m^3 of air for stoichiometric combustion. If the products are at the same p and T as the original reactants, we would obtain 1 m^3 of carbon dioxide and 2 m^3 of water vapour which would probably condense and cause a reduction in volume and/or pressure.

9

WORKED EXAMPLE No.4

Calculate the % CO_2 in the dry products when methane is burned with 15% excess air by volume.

SOLUTION

$$CH_4 + 2O_2 = CO_2 + 2H_2O$$

Volume ratio   1   2   1   2
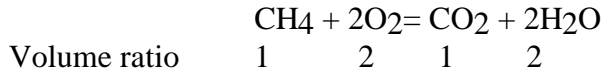
The stoichiometric air is 2/21% = 9.524 m^3
The actual air is 9.524 x 115% = 10.95 m^3

Analysis of dry products :

|		
Nitrogen	79% x 10.95	8.65 m^3
Carbon Dioxide		1.00 m^3
Oxygen	15% x 2	0.30 m^3
Total		9.95 m^3

The % Carbon Dioxide = (1/9.95) x 100 = 10%

When the fuel is a mixture of gases, the procedure outlined must be repeated for each combustible gas and the oxygen deduced for the volume of each in 1 m^3 of total fuel.

WORKED EXAMPLE No. 5

A fuel is a mixture of 60% Methane and 30% carbon monoxide and 10% oxygen by volume. Calculate the stoichiometric oxygen needed.

SOLUTION

As before, the volume of oxygen required to burn 1 m^3 of methane is 2m^3. To burn 0.6m^3 needs 1.2m^3 of oxygen. For carbon monoxide we use the combustion equation :

$$2CO + O_2 = 2CO_2$$
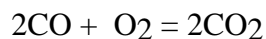
Hence to burn 1 m^3 of CO need 0.5 m^3 of oxygen, so to burn 0.3 m^3 needs 0.15 m^3 of oxygen.

The total oxygen needed is 1.2 + 0.15 = 1.35 m^3 . However there is already 0.1 m^3 in the fuel so the **stoichiometric oxygen needed 1.25m^3**

SELF ASSESSMENT EXERCISE No. 2

1. Find the air fuel ratio for stoichiometric combustion of Ethene by volume. (26.19/1)

2. Find the air fuel ratio for stoichiometric combustion of Butane by volume.(30.95/1). Calculate the % carbon dioxide present in the dry flue gas if 30% excess air is used. (10.6%)

3. Find the air fuel ratio for stoichiometric combustion of Propane by volume.(23.81/1). Calculate the % oxygen present in the dry flue gas if 20% excess air is used. (3.8%)

4. A gaseous fuel contains by volume :

    5% CO_2, 40% H_2 , 40% CH_4, 15% N_2

    Determine the stoichiometric air and the % content of each dry product. (4.76 m^3, 89.7%,N_2 10.3% CO_2).

5. RELATIONSHIP BETWEEN PRODUCT AND EXCESS AIR.

It follows that if we can deduce the % product then given the figure, we can work backwards to determine the air or oxygen that was used.

WORKED EXAMPLE No.6

consider the combustion of methane again.
$CH_4 + 2O_2 \quad = \quad CO_2 + 2H_2O$
1 vol   2 vol   1 vol   2 vols
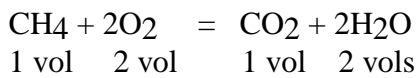
SOLUTION

Let the excess air be x (as a decimal)

The stoichiometric air is 9.52 vols.   Actual air is $9.52(1 + x)$

Dry Products:

Nitrogen	$0.79 \times 9.52(1 + x) =$	$7.524x + 7.524$
Oxygen	$2.000x$	
Carbon Dioxide		1.000
Total		$9.524x + 8.524$

% Carbon monoxide $= 100 \{1/(9.524x + 8.524)\}$

% Oxygen $\quad = 100\{2/(9.524x + 8.524)\}$

For example if the % CO_2 is 10% then the excess air is found as follows :
$\quad 10\% = 100 \{1/(9.524x + 8.524)\}$
$\quad 0.1 = 1/(9.524x + 8.524)$
$\quad (9.524x + 8.524) = 10$
$\quad 9.524x = 1.476$
$\quad \textbf{x=0.155 \ or \ 15.5\%}$

Similarly if the O_2 is 10% then the excess air is 81% (show this for yourself)

If the analysis of the fuel is by mass, then a different approach is needed as follows :

WORKED EXAMPLE No.7

An analysis of the dry exhaust gas from an engine burning Benzole shows 15% Carbon Dioxide present by volume. The Benzole contains 90% C and 10% H_2 by mass. Assuming complete combustion, determine the air/fuel ratio used.

SOLUTION

1 kg of fuel contains 0.9kg of C and 0.1kg of H_2. Converting these into kmol we have 0.9/12 kmol of C and 0.1/2 kmol of H_2. For 1 kmol of dry exhaust gas we have :

$$0.15 \text{ kmol of } CO_2$$
$$Y \text{ kmol of excess } O_2$$

$$1 - 0.15 - Y = 0.85 - Y \text{ kmol of } N_2$$

1 kmol of CO_2 is 44 kg
1 kmol of N_2 is 28 kg
1 kmol of O is 32 kg
0.15 kmol of CO_2 is 0.15 x 44kg
This contains (12/44) carbon so the carbon present is 0.15 x 12 kg
The carbon in the fuel is 0.9 kmol per kmol of fuel. Hence the number of kmols of DEG must be 0.9/(.15 x 12) = 0.5

There are 0.5 kmol of DEG for each kmol of fuel burned.

The Nitrogen present in the DEG is 0.85 - Y kmol per kmol of DEG. This has a mass of
28(0.85 - Y) per kmol of DEG
The oxygen supplied to the process must be :
(23.3/76.7) x 28 x (0.85 - Y) = 7.24 - 8.5Y kg per kmol of DEG.

(using precise proportions of air for accuracy).

The oxygen contained within the carbon dioxide is:

(32/44) x 0.15 x 44 = 4.8 kg per kmol DEG

1 kmol of CO_2 contains 44 kg and 32/44 of this is oxygen. The oxygen in the CO_2 is hence 32 x 0.15 kg per kmol DEG.

The excess oxygen is
$$32Y \text{ kg per kmol DEG}$$

Total oxygen in the products excluding that used to make H_2O is :

$$32 \times 0.15 + 32Y$$

The oxygen used to burn hydrogen is hence :

$$7.24 - 8.5Y - 32 \times 0.15 + 32Y$$

O_2 used to burn H_2 is

$$2.44 - 40.5Y \text{ kg per kmol DEG}$$

For 0.5 kmol this is

$$1.22 - 20.25Y \text{ kg}$$

To burn hydrogen requires oxygen in a ratio of 8/1. There is 0.1 kg of H_2 in each kmol of fuel so 0.8 kg of O_2 is needed. Hence :

$$0.8 = 1.22 - 20.25Y$$
$$Y = 0.0208 \text{kmol per kmol DEG}$$

The nitrogen in the DEG is 0.85 - Y = 0.829 kmol per kmol DEG
The actual Nitrogen = 0.829 x 0.5 x 28 = 11.61 kg
The air supplied must be 11.61/.767 = 15.14kg per kg of fuel. A simple calculation shows the stoichiometric mass of air is 13.73 so there is 10.3% excess air.

SELF ASSESSMENT EXERCISE No. 3

1. C_2H_6 is burned in a boiler and the dry products are found to contain 8% CO_2 by volume. Determine the excess air supplied. (59%)

2. The analysis of the dry exhaust gas from a boiler shows 10% carbon dioxide. Assuming the rest is oxygen and nitrogen determine the excess air supplied to the process and the % excess air. The fuel contains 85% C and 15% H_2

    (21.5 kg , 44.5%)

Now we will look at a complete example involving all the principles so far covered.

6. ENERGY RELEASED BY THE REACTION

The contents of the fuel and air or oxygen prior to combustion are called the reactants. The resulting material is called the products. The process releases energy but the amount of energy depends upon the temperature before and after the reaction.

Consider a mixture of reactants at condition (1) which is burned and the resulting products are at condition (2). In order to solve problems we consider that the reactants are first cooled to a referance condition (0) by removing energy Q_1. The reaction then takes place and energy is released. The products are then brought back to the same referance conditions (0) by removing energy Q_2. The energy Q_1 and Q_2 are then returned so that the final condition of the products is reached (2).
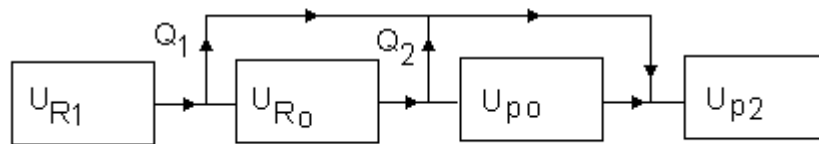

Figure 2

For constant volume combustion (closed system), we use Internal Energy. Balancing we have

$$U_{p2} - U_{R1} = (U_{Ro} - U_{R1}) + (U_{po} - U_{Ro}) + (U_{p2} - U_{po})$$

The energy released by combustion is in this case the Internal Energy of combustion and this occurs at standard conditions of 1 bar and 25$^\text{o}$C. This pressure is designated p^θ and the internal energy of combustion is designated ΔU^θ. When this is based on 1 kmol it is designated Δu^θ

$$U_{p2} - U_{R1} = (U_{Ro} - U_{R1}) + \Delta U_o^\theta + (U_{p2} - U_{po})$$

The standard conditions chosen for the combustion are 1 bar and 25$^\text{o}$C. At this temperature the internal energy of all gases is the same (-2 479 kJ/kmol). The figure is negative because the zero value of internal energy arbitrarily occurs at a higher temperature.

If the process is conducted in a steady flow system, enthalpy is used instead of internal energy. The reasoning is the same but U is replaced by H.

$$H_{p2} - H_{R1} = (H_{Ro} - H_{R1}) + \Delta H_o^\theta + (H_{p2} - H_{po})$$

Δh_o^θ may be found in the thermodynamic tables for some fuels. The figures are quoted in kJ per kmol of substance.

For the products In terms of kmol fractions $\qquad h_{po} = u_{po} + n_p R_o T_o$

15

For the reactants In terms of kmol fractions $\qquad$ $h_{Ro} = u_{Ro} + n_R R_o T_O$

where n is the kmols.

$$\Delta h_o{}^\theta = (u_{po} + n_p R_o T_O) - (u_{Ro} + n_R R_o T_O)$$
$$\Delta h_o{}^\theta = (u_{po} - u_{Ro}) - n_R R_o T_O + n_p R_o T_O$$
$$\Delta h_o{}^\theta = (u_{po} - u_{Ro}) + (n_p - n_R) R_o T_O$$
$$\Delta u_o{}^\theta = \Delta h_o{}^\theta + (n_p - n_R) R_o T_O$$

If the combustion produces equal numbers of kmols before and after, the pressure would be constant (assuming constant volume and no condensation). $n_p = n_R$ so $\Delta h_o{}^\theta$ is the same as the internal energy of reaction $\Delta v_o{}^\theta$. For example consider the combustion of ethylene.

$$C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$$
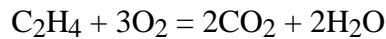
In this case there are 4 kmols before and after . When this occurs, we may use the specific heat c_p to solve the problems. $\Delta u_o{}^\theta = \Delta h_o{}^\theta = c_p \Delta T$

The specific heats are listed in the thermodynamic tables.

Note that in order to make the method of solution conform to standard data, the combustion equations should always be based on 1 kmol of fuel. The heat transfer Q_1 is found either by use of the mean specific heat or by looking up the enthalpy of the gas at the required temperatures (enthalpy of formation) and deducing the change.

In general for a constant volume we should use $\Delta u_o{}^\theta$ and C_v to solve problems.

For constant pressure with no work being done (e.g. a combustion chamber) we should use $\Delta h_o{}^\theta$ and C_p.

Since tables only list $\Delta h_o{}^\theta$ and C_p we may find

$\Delta u_o{}^\theta = \Delta h_o{}^\theta + (n_p - n_R) R_o T_O$ where is 298.1 K and R_o is 8.314 kJ/kmol K and n is the number of kmols of product.

$C_v = C_p - R = C_p - R_o/\text{molecular mass}$

WORKED EXAMPLE No.8

1. A vessel contains 0.2 m^3 of C_2H_4O and oxygen in its stoichiometric ratio. The mixture is at 1 bar pressure and 25°C. The mixture is ignited and allowed to cool back to 25°C. Determine

    i. the final pressure.

    ii. the amount of condensate formed.

    iii. The heat transfer.

    iv. the enthalpy of reaction per kmol of C_2H_4O.

    The enthalpy of formation ($\Delta h_o{}^\theta$) for the gases involved is shown below for a temperature of 298 K.

	Molecular mass. kg/kmol	Enthalpy of reaction (kJ/kmol)
CO_2(gas)	44	- 393 520
H_2O(gas)	18	- 241 830
H_2O(liquid)	18	-285 820
O_2(gas)	32	0
C_2H_4O	44	-52 630

SOLUTION

$$C_2H_4O + 2\tfrac{1}{2}O_2 = 2CO_2 + 2H_2O$$

	C_2H_4O	$2\tfrac{1}{2}O_2$	$2CO_2$	$2H_2O$
mass ratio	44	80	88	36
kmol ratio	1	2½	2	1

p= 1bar   V = 0.2 m^3 T = 298 K

$V = 0.2 m^3 = V_f + V_{ox} = 3\tfrac{1}{2}$ Volumes

$p = 1$ bar $= p_f + p_{ox}$

$p_f = 1 \times (1/3.5) = 0.2857$ bar

$p_{ox} = 1 \times (2.5/3.5) = 0.7143$ bar

mass of fuel $= pV\tilde{N}/R_oT = 0.2857 \times 10^5 \times 0.2 \times 44/(8314.4 \times 298)$

mass of fuel = 0.1015 kg

mass of oxygen $= pV\tilde{N}/R_oT = 0.7143 \times 10^5 \times 0.2 \times 32/(8314.4 \times 298)$

mass of oxygen = 0.1845 kg

total mass = 0.286 kg

Mass of $CO_2 = (88/124) \times 0.286 = 0.203$ kg

Mass of $H_2O = (36/124) \times 0.286 = 0.083$ kg

total mass = 0.286 kg

$p(CO_2) = mR_oT/\tilde{N}V = 0.203 \times 8314.4 \times 298/(44 \times 0.2)$
$\qquad\qquad = 0.5716 \times 10^5 = 0.5716$ bar

Since condensate forms, the gas is saturated with water vapour so $p(vapour) = p_g$ @ 28ºC from the steam tables.

$p(vapour) = 0.03166$ bar

Total pressure = 0.5716 + 0.03166 = 0.603 bar (answer (i)

Mass of vapour $= V/v_g$

where $v_g = 43.4$ m^3/kg @ 28ºC from tables.

Mass of vapour = 0.2/43.4 = 0.004608 kg

Mass of condensate formed = 0.083 - 0.0046 = 0.0784 kg

Answer (ii)

Now consider the reaction. Since it starts and finishes at 25ºC there is no initial cooling required ($Q_1 = 0$).

18

REACTANT

Fuel   Mass = 0.1015 kg   kmol = 0.1015/44 = 0.00231 kmol
Oxygen Mass = 0.1845 kg   kmol = 0.1845/32 = 0.00576 kmol

Enthalpy of formation for oxygen = 0
Enthalpy of formation for C_2H_4O = -52630 kJ/kmol
H_f = 0.00231(-52630) = -121.4 kJ (Minus relative to higher point of reference)

PRODUCTS

CO_2 kmol = 0.203/44 = 0.00461 kmol
H_f = 0.00461 x (-393 520) = - 1815.6 kJ

H_2O (gas)   kmol = 0.0408/18 = 0.00227 kmol
H_f = 0.00227 x (-241830) = - 548.1 kJ

H_2O (water)   kmol = 0.0784/18 = 0.00436 kmol
H_f = 0.00436 x (-285820) = - 1244.9 kJ
Total = -3608 kJ

The change in enthalpy = -3608 - (-121.4)

= **-3486.6 kJ = ΔH_o**   **Answer (iv)**

$\Delta h_o{}^\theta$ = -3486.6/0.00231 = **1.509 GJ/kmol**   **Answer (v)**

19

WORKED EXAMPLE No.9

Air and Ethane (C_2H_6) are mixed with twice the stoichiometric ratio at 600K in a vessel at 12 bar pressure. Determine the temperature and pressure after combustion assuming no energy losses. The enthalpy of combustion at 25°C is $\Delta H_o = -1\,427\,860$ kJ/kmol

SOLUTION

$$C_2H_6 + 3.5\,O_2 = 3H_2O + 2CO_2$$

kmol   1   3.5   3   2
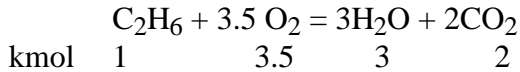
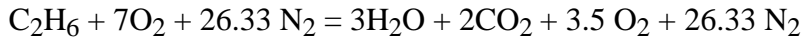The air required = 3.5/0.21 = 16.67 kmol
Actual air = 33.33 kmol
Nitrogen = 0.79 x 33.33 = 26.33 kmol
Excess oxygen = 3.5 kmol
The equation may be rewritten as
$C_2H_6 + 7O_2 + 26.33\,N_2 = 3H_2O + 2CO_2 + 3.5\,O_2 + 26.33\,N_2$

The process may be idealised as follows
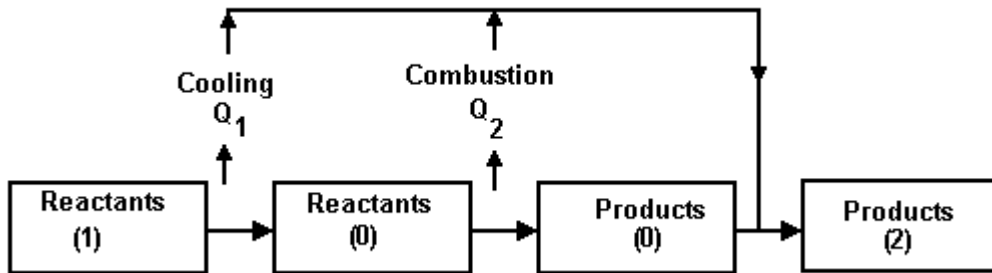
First find the enthalpy of the reactants. The mean temperature of the reactants relative to 25°C is {(25+273) + 600}/2 = 449K near enough 450K for the tables. We look up specific heats in the thermodynamic tables at 450 K. The temperature change from 25°C to 600 K is 302 K. We proceed to work out the heat transfer based on 1 kmol of fuel, Q_1 as follows.

Table of values

C_2H_6 $C_p = 2.402$ kJ/kg K   mass = 1 kmol x 30 = 30 kg
                              $Q_1 = 30$ x 2.402 x 302 = 21 762 kJ

O_2 $C_p = 0.956$ kJ/kg K   mass = 7 kmol x 32 = 224 kg
                              $Q_1 = 224$ x 0.956 x 302 = 64671.5 kJ

N_2 $C_p = 1.049$ kJ/kg K   mass = 26.33 kmol x 28 = 737.24 kg
                              $Q_1 = 737.24$ x 1.049 x 302 = 233556 kJ

Total $Q_1 = -319\,989.7$ kJ per kmol of fuel (negative leaving system)

Next we repeat the process for the products to find $Q_1 + Q_2$

In order to use a mean specific heat we must guess the approximate final temperature of the products. A good guess is always 2000 K so the mean of 25°C and 2000 K is near enough 1150 K. Using this we work out the heat transfer to the products with an unknown temperature change from 25°C to T_2 of ΔT.

H_2O $C_p = 2.392$ kJ/kg K   mass = 3 kmol x 16 = 48 kg
$$Q = 48 \times 2.392 \times \Delta T = 114.8 \, \Delta T$$

O_2 $C_p = 1.109$ kJ/kg K   mass = 3.5 kmol x 32 = 112 kg
$$Q_1 = 112 \times 1.109 \times \Delta T = 124.2 \, \Delta T$$

N_2 $C_p = 1.196$ kJ/kg K   mass = 26.33 kmol x 28 = 737.24 kg
$$Q = 737.24 \times 1.196 \times \Delta T = 881.7 \, \Delta T$$

CO_2 $C_p = 1.270$ kJ/kg K   mass = 2 kmol x 44 = 88 kg
$$Q = 88 \times 1.270 \times \Delta T = 111.76 \, \Delta T$$

Total $Q_1 + Q_2 = 1232.5 \, \Delta T$ kJ per kmol of fuel (positive entering system)

$Q_2 = -1\,427\,860$ kJ/kmol of fuel (from question).

Conserving energy we have $1232.5 \, \Delta T = 1\,427\,860 + 319\,989.7$
hence $\Delta T = 1232.5$ K and $T_2 = 1716$ K which is different from our original guess of 2000 K but more accurate.

Next we must repeat the last stage with a more accurate mean temperature.
Mean temperature = (298 + 1716)/2 = 1007 K. Say 1000 K.

H_2O $C_p = 2.288$ kJ/kg K $Q = 48 \times 2.288 \times \Delta T = 109.8 \Delta T$

O_2 $C_p = 1.090$ kJ/kg K $Q_1 = 112 \times 1.09 \times \Delta T = 122.1 \Delta T$

N_2 $C_p = 1.167$ kJ/kg K $Q = 737.24 \times 1.167 \times \Delta T = 860.4 \Delta T$

CO_2 $C_p = 1.234$ kJ/kg K $Q = 88 \times 1.234 \times \Delta T = 108.6 \Delta T$

Total $Q_1 + Q_2 = 1201 \Delta T$ kJ per kmol of fuel (positive entering system)

Conserving energy we have $1201 \Delta T = 1\,427\,860 + 319\,989.7$

hence $\Delta T = 1455$ K and $T_2 = 1753$ K which is different from our original guess of 1716K but more accurate. the true answer is between 1716 and 1753 K and may be narrowed down by making more steps but two is usually sufficient in the exam.

Finally the pressure. $p_1V_1 / N_1T_1 = R_o = p_2V_2 / N_2T_2$ and the volumes are equal.

$p_1 = 12$ bar $\quad T_1 = 600$ K $\quad N_1 = 4.5$ kmol
$T_2 = 1753$ K $\quad N_2 = 5$ kmol
$p_2 = 12 \times 5 \times 1753/(4.5 \times 600) = 38.9$ bar

SELF ASSESSMENT EXERCISE No. 4

1. The gravimetric analysis of a fuel is Carbon 78%, hydrogen 12%, oxygen 5% and ash 5%. The fuel is burned with 20% excess air. Assuming complete combustion, determine

    i. the composition of the products.
    (72.6% N_2, 17.3% CO_2, 6.5% H_2O and 3.6% O_2)

    ii. the dew point of the products. 47°C)

    iii. the mass of water condensed when the products are cooled to 30°C. (0.67 kg)

2. Carbon monoxide is burned with 25% excess oxygen in a fixed volume of 0.2 m^3. The initial and final temperature is 25°C. The initial pressure is 1 bar. Calculate

    i. the final pressure. (0.874 bar)

    ii. the heat transfer. (574.5 kJ)

    Use your thermodynamic tables for enthalpies of reaction.

3. Similar to Q9 1984

Prove that the enthalpy and the internal energy of reaction are related by

$$\Delta H_o = \Delta U_o + R_o T_o (n_P - n_R)$$

where n_P and n_R are the kmols of products and reactants respectively.

Ethylene (C_2H_4) and 20% excess air at 77°C are mixed in a vessel and burned at constant volume. Determine the temperature of the products. You should use your thermodynamic tables to find ΔU_o or ΔH_o and the table below. (Answer 2263 K)

U (kJ/kmol)	C_2H_4	O_2	N_2	CO_2	H_2O
T(K)					
298.15	- 2479	-2479	-2479	-2479	-2479
300	- 2415	-2440	-2440	-2427	-2432
400	-1557	-297	-355	683	126
2400		54537	50696	95833	73650
2600		60657	50696	95833	73650
2800		66864	62065	117160	92014
3000		73155	67795	127920	101420

Similar to Q8 1993

An engine burns hexane (C_6H_{14}) in air. At a particular running condition the volumetric analysis of the dry products are

CO2   8.7%
CO   7.8 %
N2   83.5%

Calculate the air-fuel ratio by mass and find the stoichiometric ratio.

(Answer 12.9 and 15.17)

7. DISSOCIATION

At the high temperatures and pressures experienced in combustion, dissociation occurs. This results in some of the fuel not burning. CO is produced and in the case of hydrogen, some of it remains as hydrogen after the process even though oxygen is present. The reasons for this will not be covered here other than to say it is predicted by the 2nd law of thermodynamics and involves equilibrium in the chemical process.

When dissociation occurs, the energy released is reduced accordingly and if the amount of unburned fuel is known the previous examples may easily be modified to take account of it.

When hydrogen is burned, it can be shown that the partial pressures of the hydrogen, oxygen and water vapour present in the products are related by

$$k^\theta = \frac{p_{H_2O}\, p^\theta}{p_{H_2}\left(p_{O_2}\right)^{\frac{1}{2}}}$$

The properties tables list values of $\ln K^\theta$.

Similarly when dissociation occurs in the formation of carbon dioxide, the relationship between the partial pressures of CO_2, CO and O_2 is given by

$$k^\theta = \frac{p_{CO_2}\, p^\theta}{p_C\, p_{CO_2}}$$

Other similar equations for other combinations of products may be found in the tables.

SELF ASSESSMENT EXERCISE No. 5

1. Hydrogen is mixed with stoichiometric air at 25°C and burned adiabatically at constant volume. After combustion 6% of the hydrogen remains unburned. Determine the temperature and pressure of the products.

   (Answer the temperature is 2344K after two approximations)

   You need to find K^θ in the tables. Also find $\Delta H_o = 241800$ kJ/kmol. Deduce the partial pressures of the products as a fraction of p and then use K^θ to solve p.

2. A mixture of air and CO is burned adiabatically at constant volume. The air is 90% of the stoichiometric requirement. The mixture is initially at 5 bar and 400K. The only dissociation that occurs is $CO_2 \rightarrow CO + \frac{1}{2}O_2$.

   Show that the equilibrium constant at the final temperature T_2 is

   $$K_p = 1.1212a/(\{(1-a)(0.9-a)^{\frac{1}{2}}(T_2/T_1)^{\frac{1}{2}}\}$$

   where a is the amount of CO_2 kmol in the products per kmol of CO in the reactants.

   If it assumed that initially $T_2 = 2900$ K for which log $K_p = 0.649$, the solution of the above equation gives a=0.784. Check the assumed value of T_2 given that the internal energy of reaction at $T_0 = 298.15$ K is -281750 kJ/kmol.

T (K)	U kJ/kmol			
	CO	O_2	N_2	CO_2
298.15	-2479	-2479	-2479	-2479
400	- 351	- 297	- 355	+ 683
2900	+65715	+69999	+64924	+122530