THERMODYNAMICS
TUTORIAL 8
POSITIVE DISPLACEMENT COMPRESSORS and EXPANDERS

This tutorial is set at QCF Level 4 to 5

On completion of this tutorial you should be able to

- Solve problems involving positive displacement compressors
- Describe the various types of positive displacement compressors
- Define the various parameters used to define the performance of positive displacement compressors
- Describe some types of gas and steam expanders

Contents

1. Compressed Air

1.1 Types

Air is an expansive substance and dangerous when used at high pressures. For this reason, most applications are confined to things requiring low pressures (10 bar or lower) but there are industrial uses for high pressure air up to 100 bar.

The common source of the air is the compressor. There are many types of positive displacement compressors with different working principles and working conditions. These are examples.

1) Reciprocating.
2) Sliding vane compressors.
3) Lobe compressors.
4) Helical screws.
5) Centrifugal.
6) Axial turbine compressors.

This tutorial only applies to reciprocating compressors which is the main positive displacement type. Every revolution of the shaft produces a definite volume change in the chamber fixed by the geometry of the machine. In theory if the machine is stopped the gas or steam cannot pass from inlet to outlet or vice versa because a positive barrier is formed.

The function of all of them is to draw in air from the atmosphere and produce air at pressures substantially higher. Usually a storage vessel or receiver is used with the compressor. The same principles are applied to the compression of other gasses. The reciprocating compressor is probably the most versatile of all the types and is only out performed by rotary types when large volumes at low pressures are required. For high pressures, the reciprocating compressor is almost universal.

1.2 Atmospheric Vapour

Air and vapour mixtures are covered in detail in a later tutorial. We should note, however, the effects it has on the performance of an air compressor. Atmospheric air contains ***Water Vapour*** mixed with the other gases. When the air is cooled to the dew point, the vapour condenses into water and we see rain or fog. The ratio of the mass of water vapour in the air to the mass of the air is called the ***Absolute Humidity.*** The quantity of water that can be absorbed into the air at a given pressure depends upon the temperature. The hotter the air, the more water it can evaporate. When the air contains the maximum possible amount of vapour it is at its dew point and rain or fog will appear. The air is then said to have 100% humidity. When the air contains no water vapour at all (dry air), it has 0% humidity. This refers to ***Relative Humidity***. For example if the air has 40% humidity it means that it contains 40% of the maximum that it could contain. There are various ways to determine the humidity of air and instruments for doing this are called ***Hygrometers***.

The importance of humidity to air compressors is as follows. When air is sucked into the compressor, it brings with it water vapour. When the air is compressed the pressure and the temperature of the air goes up and the result is that the compressed air will have a relative humidity of about 100% and it will be warm. When the air leaves the compressor it will cool down and the water vapour will condense. Water will then clog the compressor, the receiver and the pipes.

Water causes damage to air tools, ruins paint sprays and corrodes pipes and equipment. For this reason the water must be removed and the best way is to use a well designed compressor installation and distribution network.

1.3 Typical Compressor Layout

The diagram below shows the layout of a two stage reciprocating compressor typically for supplying a workshop.

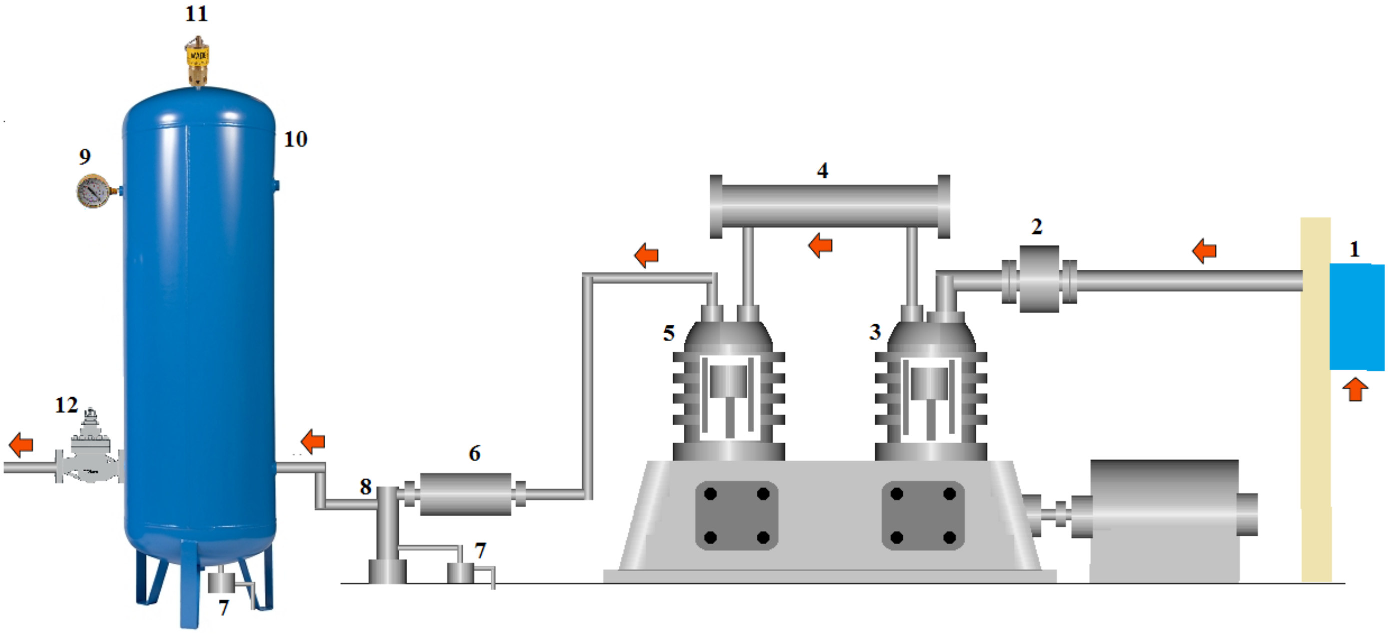

Figure 1

1. Induction box and silencer on outside of building with course screen
2. Induction filter
3. Low pressure stage
4. Intercooler
5. High pressure stage
6. Silencer
7. Drain trap
8. After cooler
9. Pressure gauge
10. Air receiver.
11. Safety pressure relief valve
12. Stop valve

1.4 Free Air Delivery

When a gas such as air flows in a pipe, the mass of the air depends upon the pressure and temperature. It would be meaningless to talk about the volume of the air unless the pressure and temperature are considered. For this reason the volume of air is usually stated as ***Free Air Delivery or FAD.***

FAD refers to the volume the air would have if let out of the pipe and returned to atmospheric pressure at the same temperature.

The FAD is the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at a higher pressure. Suppose atmospheric conditions are $p_a T_a$ and V_a (the FAD) and the compressed conditions are p, V and T.

Applying the gas law we have

$$\frac{pV}{T} = \frac{p_a V_a}{T_a} \quad V_a = \frac{pVT_a}{Tp_a} = \text{FAD}$$

2. *Cycle for Reciprocating Compressor*

2.1 Theoretical Cycle

The diagram shows the basic design of a reciprocating compressor. The piston reciprocates drawing in gas, compressing it and expelling it when the pressure inside the cylinder reaches the same level as the pressure in the delivery pipe.

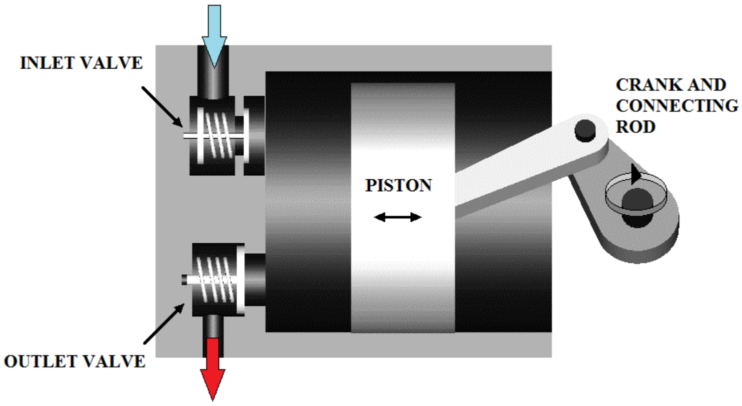

Figure 2

If the piston expels all the air and there is no restriction at the valves, the pressure - volume cycle is as shown below.

Gas is induced from 4 to 1 at the inlet pressure. It is then trapped inside the cylinder and compressed according the law $pV^n = C$. At point 2 the pressure reaches the same level as that in the delivery pipe and the outlet valve pops open. Air is then expelled at the delivery pressure. The delivery pressure might rise very slightly during expulsion if the gas is being compacted into a fixed storage volume. This is how pressure builds up from switch on.

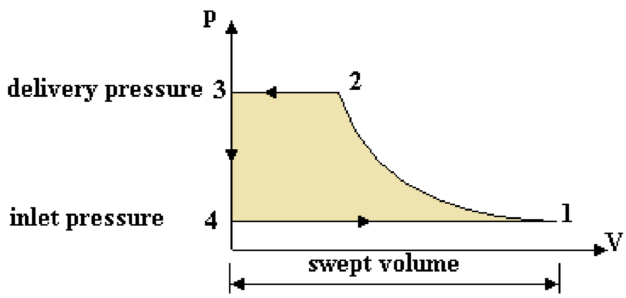

Figure 3

2.2 *Volumetric Efficiency*

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.

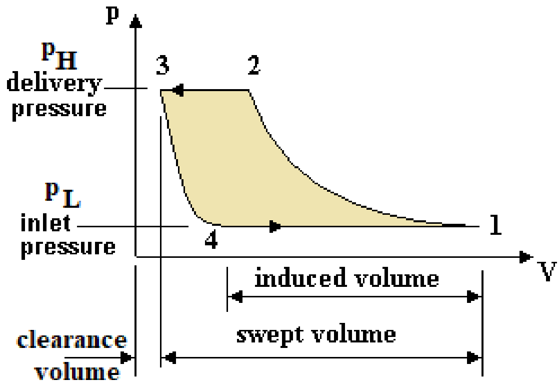

Figure 4

The volumetric efficiency is defined as

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

This efficiency is made worse if leaks occur past the valves or piston.

The clearance ratio is defined as

$$c = \frac{\text{Clearance Volume}}{\text{Swept Volume}} = \frac{V_3}{V_1 - V_3}$$

Ideally the process 2 to 3 and 4 to 1 are isothermal. That is to say, there is no temperature change during induction and expulsion.

WORKED EXAMPLE No. 1

Gas is compressed in a reciprocating compressor from 1 bar to 6 bar. The FAD is 13 dm^3/s. The clearance ratio is 0.05. The expansion part of the cycle follows the law $pV^{1.2} = C$. The crank speed is 360 rev/min. Calculate the swept volume and the volumetric efficiency.

SOLUTION

Swept Volume = V   Clearance volume = 0.05 V

Consider the expansion from 3 to 4 on the p-V diagram.

p_4 = 1 bar p_3 = 6 bar. $p_3V_3^{1.2} = p_4V_4^{1.2}$

$6(0.05V)^{1.2} = 1(V_4^{1.2})$

V_4 = 0.222V or 22.2% of V

F.A.D. = 0.013 m^3/s.

Induced volume = $V_1 - V_4$ = 1.05V - 0.222V = 0.828V

Induced volume = 0.013

V = 0.013/0.828 = 0.0157 m^3/s

Crank speed = 6 rev/s so the swept volume = 0.0157/6 = 2.62 dm^3.

V_1 = V + 0.05V = 1.05V

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}} = \frac{0.828V}{V} = 82.8\ \%$$

WORKED EXAMPLE No. 2

Show that if the clearance ratio of an ideal single stage reciprocating compressor is c that the volumetric efficiency is given by

$$\eta_{vol} = 1 - c\left[\left\{\frac{p_H}{p_L}\right\}^{\frac{1}{n}} - 1\right]$$

p_L is the inlet pressure and p_H the outlet pressure.

SOLUTION

Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$ Clearance volume = V_3

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} \qquad c = \frac{V_3}{V_1 - V_3}$$

$$V_1 - V_3 = \frac{V_3}{c} \qquad \frac{V_1}{V_3} = \frac{1 + c}{c}$$

$$\eta_{vol} = \frac{c\,(V_1 - V_4)}{V_3} = c\left\{\frac{V_1}{V_3} - \frac{V_4}{V_3}\right\}$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} = \left(\frac{p_H}{p_L}\right)^{\frac{1}{n}}$$

$$\eta_{vol} = c\left\{\frac{1 + c}{c} - \left(\frac{p_H}{p_L}\right)^{\frac{1}{n}}\right\}$$

$$\eta_{vol} = 1 + c - c\left(\frac{p_H}{p_L}\right)^{\frac{1}{n}} = 1 - c\left[\left\{\frac{p_H}{p_L}\right\}^{\frac{1}{n}} - 1\right]$$

In real compressors the warm cylinder causes a slight temperature rise over the induction from 4 to 1. The gas is restricted by the valves and p_1 is slightly less than p_4. The valves also tend to move so the real cycle looks more like this.

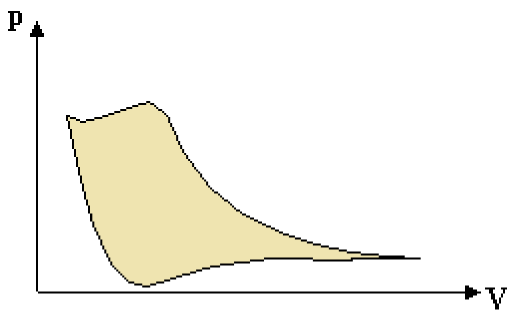

Figure 5

WORKED EXAMPLE No. 3

A single stage reciprocating compressor produces a FAD of 2 d m^3/s at 420 rev/min. The inlet conditions are 1 bar and 10^{o}C. The polytropic index is 1.2 for the compression and expansion. The outlet pressure is 8 bar. The clearance volume is 10 cm^3.

Due to the restriction of the inlet valve and the warming effect of the cylinder walls, the pressure at the start of compression is 0.97 bar and the temperature is 17^{o}C.

Determine the volumetric efficiency.

SOLUTION

Because the induction stroke is neither at constant pressure nor constant temperature, we must solve the swept volume by using the expulsion stroke, which is assumed to be at constant pressure and temperature. The numbers of the cycle points are as before.

$$T_2 = 290\left(\frac{9}{0.97}\right)^{\frac{0.2}{1.2}} = 412.2 \text{ K}$$

$$\text{FAD per stroke} = \frac{2 \times 60}{420} = 0.2857 \text{ dm}^3 \text{ per stroke}$$

$$\text{Compressed Volume Expelled} = \frac{0.2857 \times p_a \times T_H}{T_a \times p_H}$$

$$\text{Expulsion Volume} = V_2 - V_1 = \frac{0.2857 \times 1 \times 412.2}{283 \times 8} = 0.052 \text{ dm}^3$$

$$p_3V_3^n = p_4V_4^n \qquad 8 \times 0.01^{1.2} = 1 \times V_4^{1.2} \qquad V_4 = 0.0566 \text{ dm}^3$$

$$V_2 = 0.01 + 0.052 = 0.062 \text{ dm}^3$$

$$p_1V_1^n = p_2V_2^n \quad 0.96V_1^n = 8 \times 0.062^{1.2} \qquad V_1 = 0.363 \text{ dm}^3$$

$$\text{Induced Volume} = V_1 - V_4 = 0.303 \text{ dm}^3$$

$$\text{Swept Volume} = V_1 - V_3 = 0.353 \text{ dm}^3$$

$$\eta_{vol} = \frac{0.306}{0.353} = 86.7\%$$

2.3 Indicated Power

Ignoring the Clearance Volume

The work done in one cycle is the area enclosed by the p -V diagram. Referring to the ideal cycle (figure 6a) we may obtain this by integrating the elementary strips shown.

$$W = \int_{p_1}^{p_2} V dp$$

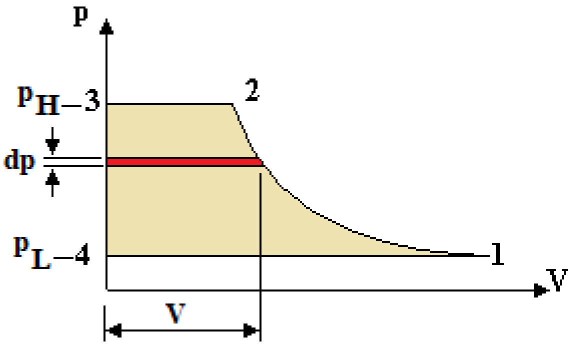

Figure 6a

$$pV^n = C \qquad V = \left(\frac{C}{p}\right)^{1/n} = C^{1/n}p^{-1/n} \text{ and } C^{1/n} = Vp^{1/n}$$

$$W = \int_{p_1}^{p_2} V\,d\,p = \int_{p_1}^{p_2} C^{1/n}p^{-1/n}d\,p = C^{1/n}\int_{p_1}^{p_2} p^{-1/n}d\,p$$

$$W = C^{1/n}\left[\frac{p^{1-1/n}}{1-1/n}\right]_{p_1}^{p_2} = \left(\frac{n}{n-1}\right) C^{1/n}\left[p_2^{1-1/n} - p_1^{1-1/n}\right]$$

$$W = \left(\frac{n}{n-1}\right)\left[C^{1/n}p_2^{1-1/n} - C^{1/n}p_1^{1-1/n}\right]$$

Substitute $C^{1/n} = V_1p_1^{1/n} = V_2p_2^{1/n}$

$$W = \left(\frac{n}{n-1}\right)\left[V_2p_2^{1/n}p_2^{1-1/n} - V_1p_1^{1/n}\,p_1^{1-1/n}\right] = \left(\frac{n}{n-1}\right)[V_2p_2 - V_1p_1]$$

Since pV = mRT and m is the mass induced and delivered.

$$W = \left(\frac{n}{n-1}\right)mR[T_2 - T_1] = \left(\frac{n}{n-1}\right)mRT_1\left[\frac{T_2}{T_1} - 1\right]$$

It has been shown elsewhere that by combing the gas law and the polytropic law that:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-1/n} = (r)^{1-1/n} \qquad \text{where } r = \frac{p_2}{p_1} \text{ the compression ratio}$$

$$W = \left(\frac{n}{n-1}\right)mRT_1\left\{(r)^{1-1/n} - 1\right\}$$

If we multiply by the number of strokes per second (N) we get the indicated Power **IP = W × N**

We have developed three equations for the indicated Work :-

$$\mathbf{W = \left(\frac{n}{n-1}\right)[p_2V_2 - p_1V_1] \;\ldots\ldots (A)}$$

$$\mathbf{W = \left(\frac{n}{n-1}\right)mR[T_2 - T_1] \;\ldots\ldots (B)}$$

$$\mathbf{W = \left(\frac{n}{n-1}\right)mRT_1\left\{(r)^{1-1/n} - 1\right\} \;\ldots\ldots (C)}$$

Isothermal Work

It can be shown that for minimum work the compression process should be isothermal with n = 1. In this case the integration process is different and repeating the process we get the following:

Noting $T_1 = T_2 = T$ for an isothermal process

$$pV = mRT \qquad V = \frac{mRT}{p} = mRTp^{-1}$$

$$W_{iso} = \int_{P_1}^{p_2} V\,dp = mRT_1\int_{P_1}^{p_2} p^{-1}dp = mRT_1\,[\ln(p)]_{P_1}^{p_2} = mRT\ln\left(\frac{p_2}{p_1}\right)$$

$$\mathbf{W_{iso} = mR\,T\,ln\,(r)}$$

Including the Clearance Volume Effect

Refer to figure 6b. The processes 1 to 2 and 3 to 4 are both polytropic $pV^n = C$.

The enclosed area may be found by subtracting the work output (3 to 4) from the work input (1 to 2) found previous.

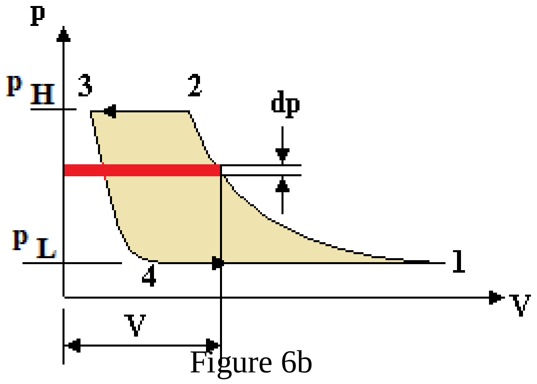

Figure 6b

Using equation (A)

$$W = W_{1-2} - W_{3-4} = \frac{n}{n-1}[p_2V_2 - p_1V_1] - \frac{n}{n-1}[p_4V_4 - p_3V_3]$$

Rearrange as follows

$$W = \left(\frac{n}{n-1}\right)p_1V_1\left\{\left(\frac{p_2V_2}{p_1V_1}\right) - 1\right\} - \left(\frac{n}{n-1}\right)p_4V_4\left\{\left(\frac{p_3V_3}{p_4V_4}\right) - 1\right\}$$

Using the polytropic law $pV^n = C$ eliminate the volumes

$$\frac{p_2V_2}{p_1V_1} = \left(\frac{p_2}{p_1}\right)^{1-1/n} \text{ and } \frac{p_3V_3}{p_4V_4} = \left(\frac{p_3}{p_4}\right)^{1-1/n}$$

Substitute

$$W = \left(\frac{n}{n-1}\right)p_1V_1\left\{\left(\frac{p_2}{p_1}\right)^{1-1/n} - 1\right\} - \left(\frac{n}{n-1}\right)p_4V_4\left\{\left(\frac{p_3}{p_4}\right)^{1-1/n} - 1\right\}$$

$$W = \left(\frac{n}{n-1}\right)p_1V_1\{r^{1-1/n} - 1\} - \left(\frac{n}{n-1}\right)p_4V_4\{r^{1-1/n} - 1\}$$

Where r is the pressure ratio and $p_1 = p_4$ hence

$$\mathbf{W = \left(\frac{n}{n-1}\right)p_1\{r^{1-1/n} - 1\}\{V_1 - V_4\}\ \ldots\ldots\ (D)}$$

$V_1 - V_4$ is the swept volume and the mass drawn in and expelled is $m_1 - m_3 = m_1 - m_4$

$$(m_1 - m_3) = \frac{p_1(V_1 - V_4)}{RT_1} \text{ so } V_1 - V_4 = \frac{(m_1 - m_3)RT_1}{p_1}$$

$$\mathbf{W = (m_1 - m_3)RT_1\left(\frac{n}{n-1}\right)\left\{r^{1-\frac{1}{n}} - 1\right\}}$$

The mass delivered per second is $\dot{m} = m_1 - m_3 \times N$ rev/s

$$\mathbf{I.P. = \dot{m}\ R\ T_1 \left(\frac{n}{n-1}\right)\left[(r)^{1-\frac{1}{n}} - 1\right] \ldots\ldots (E)}$$

Using equation (B)

$m_4 = m_3$ is the mass expanded from (4) and (3) and $m_1 = m_2$ is the mass compressed from (1) and (2) and the difference is the mass expelled. Assume n is the same value for both processes subtract W_{1-2} from W_{3-4} the indicated work per stroke is:

$$W = W_{1-2} - W_{3-4} = \left(\frac{n}{n-1}\right) m_1 R[T_2 - T_1] - \left(\frac{n}{n-1}\right) m_3 R[T_3 - T_4]$$

$(T_2 - T_1) = (T_3 - T_4)$

$$W = \left(\frac{n}{n-1}\right) R\,\{m_1[T_2 - T_1] - m_3[T_2 - T_1]\}$$

$$W = \left(\frac{n}{n-1}\right) (m_1 - m_3) R[T_2 - T_1]$$

$$IP = W \times \text{strokes per second}$$

This is the same as using the mass per second.

$$\mathbf{I.P. = \dot{m}\ R\ T_1 \left(\frac{n}{n-1}\right)\left[(r)^{1-\frac{1}{n}} - 1\right]}$$

If the process was repeated for isothermal compression and expansion we would get the same expression as before for the work done.

$$\mathbf{W_{iso} = mR\ T_1\ ln\ (r)}$$

2.4 Isothermal Efficiency

The minimum indicated power is obtained when the index n is a minimum. The ideal compression is hence isothermal with n = 1.

The isothermal efficiency is defined as:

$$\eta_{iso} = \frac{\textbf{Isothermal Work}}{\textbf{Actual Work}} = \frac{\mathbf{mR\ T_1\ ln\ (r)}}{\mathbf{\left(\frac{n}{n-1}\right) mR[T_2 - T_1])}}$$

$$\eta_{iso} = \frac{\mathbf{(n-1)T_1 ln(r)}}{\mathbf{n(T_2 - T_1)}}$$

Note that in the ideal case, T_1 and T_2 are the inlet and outlet temperatures.

SELF ASSESSMENT EXERCISE No. 1

Show how the volumetric efficiency of an ideal single stage reciprocating air compressor may be represented by the equation

$$\eta_{vol} = 1 - c\left[\left\{\frac{p_H}{p_L}\right\}^{\frac{1}{n}} - 1\right] \text{ note } \frac{p_H}{p_L} = r \text{ (the compression ratio)}$$

Where c is the clearance ratio, p_H the delivery pressure and p_L the induction pressure.
A reciprocating air compressor following the ideal cycle has a free air delivery of 60 dm^3/s. The clearance ratio is 0.05. The inlet is at atmospheric pressure of 1 bar. The delivery pressure is 7 bar and the compression is polytropic with an index of 1.3. Calculate the following.

i. The ideal volumetric efficiency. (82.7%)
ii. The ideal indicated power. (14.7 kW)
iii. Sketch curves of η_{vol} against r for typical values of n and c.

3. *Multiple Compressor Stages*

3.1 *The Effect of Intercooling*

The advantage of compressing the fluid in stages is that intercoolers may be used and the overall compression is nearer to being isothermal. Consider the p - V diagram for a two stage compressor.

The cycle 1 to 4 is a normal cycle conducted between p_L and p_M. The air is expelled during process 3 to 4 at p_M and constant temperature. The air is then cooled at the intermediate pressure and this causes a contraction in the volume so that the volume entering the high pressure stage is V_5 and not V_2. The high pressure cycle is then a normal cycle conducted between p_M and p_H.

The shaded area of the diagram represents the work saved by using the intercooler. The optimal saving is obtained by choosing the correct intermediate pressure. This may be found as follows.

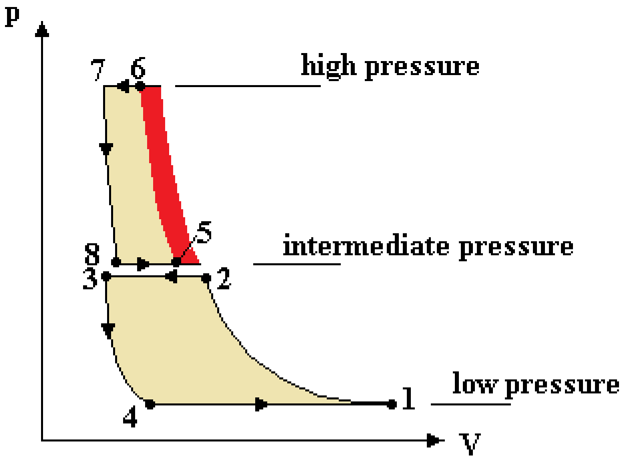

Figure 7

3.2 *Optimal Interstage Pressure*

$W = W_1 + W_2$ where W_1 is the work done in the low pressure stage and W_2 is the work done in the high pressure stage.

$$W = \frac{m\,R\,n(T_2 - T_1)}{(n-1)} + \frac{m\,R\,n(T_6 - T_3)}{(n-1)}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/n} \quad \text{and} \quad T_6 = T_5\left(\frac{p_6}{p_5}\right)^{1-1/n}$$

Assuming the value of n is the same for each stage:

$$W = m\,R\left[\left\{\frac{nT_1}{(n-1)}\right\}\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} - 1\right] + m\,R\left[\left\{\frac{nT_6}{(n-1)}\right\}\left(\frac{p_6}{p_5}\right)^{1-\frac{1}{n}} - 1\right]$$

Since $p_2 = p_5 = p_m$ and $p_6 = p_H$ and $p_1 = p_L$

$$W = m\,R\left[\left\{\frac{nT_1}{(n-1)}\right\}\left(\frac{p_M}{p_L}\right)^{1-\frac{1}{n}} - 1\right] + m\,R\left[\left\{\frac{nT_6}{(n-1)}\right\}\left(\frac{p_H}{p_M}\right)^{1-\frac{1}{n}} - 1\right]$$

In order to find the minimum value of W we differentiate with respect tp P_M and equate to zero.

$$\frac{dW}{dP_M} = m\,RT_1(p_L)^{\frac{1-n}{n}}(p_M)^{-\frac{1}{n}} - m\,RT_5(p_H)^{\frac{n-1}{n}}(p_M)^{\frac{1-2n}{n}}$$

If the intercooler returns the air to the original inlet temperature so that $T_1 = T_5$, then equating to zero reveals that for minimum work:

$$\mathbf{p_m = \sqrt{p_L \times p_H}}$$

It can further be shown that when this is the case, the work done by both stages is equal.

When K stages are used, the same process reveals that the minimum work is done when the pressure ratio for each stage is:

$$\mathbf{p_m = \sqrt[K]{\frac{p_L}{p_H}} \quad \textit{or} \quad \left(\frac{p_L}{p_H}\right)^{1/k}}$$

WORKED EXAMPLE No. 5

A single acting reciprocating compressor runs at 360 rev/min and takes in air at 1 bar and 15^{o}C and compresses it in 3 stages to 64 bar. The free air delivery is 0.0566 m^3/s. There is an intercooler between each stage, which returns the air to 15^{o}C. Each stage has one piston with a stroke of 100 mm. Calculate the following.

The ideal interstage pressure
The ideal indicated power per stage
The heat rejected from each cylinder
The heat rejected from each intercooler
The isothermal efficiency
The swept volume of each stage
The bore of each cylinder

Ignore leakage and the effect of the clearance volume. The index of compression is 1.3 for all stages.

SOLUTION

$$\text{Pressure Ratio for each stage} = \left(\frac{64}{1}\right)^{\frac{1}{3}} = 4$$

Hence the pressure after stage 1 is 1 × 4 = 4 bar.

The pressure after the second stage is 4 × 4 = 16 bar

The final pressure is 16 × 4 = 64 bar.

$$T_1 = 288 \text{ K} \qquad T_2 = 288 \times 4^{\frac{0.3}{1.3}} = 396.5 \text{ K}$$

$$\dot{m} = \frac{p_1 V}{RT_1} = \frac{1 \times 10^5 \times 0.0566}{287 \times 288} = 0.06847 \text{ kg/s}$$

The indicated power for each stage is the same so it will be calculated for the 1st. stage. Using equation (E)

$$\text{I.P.} = \dot{m}\, RT_1 \left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} - 1\right] = 0.06847 \times 287 \times 288 \left(\frac{1.3}{1.3-1}\right)\left[(4)^{1-\frac{1}{1.3}} - 1\right] = 9\,246 \text{ Watts}$$

CYLINDER COOLING

Consider the energy balance over the first stage.

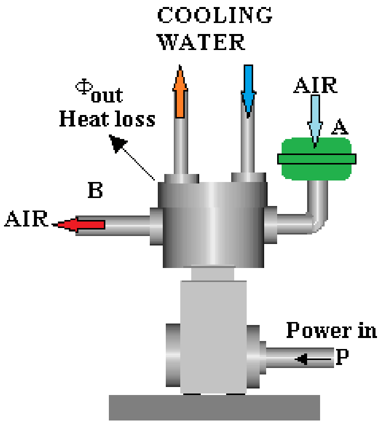

Figure 8

Balancing the energy we have

$$H_A + P(in) = H_B + \Phi\ (out)$$

$$\Phi\ (out) = P(in) - \dot{m}C_p(T_B - T_A)$$

$$\Phi\ (out) = 9.246 - 0.06847 \times 1.005\ (396.5 - 288)$$

$$\Phi\ (out) = 1.78 \text{ kW (rejected from each cylinder)}$$

INTERCOOLER

Now consider the Intercooler. No work is done and the temperature is cooled from T_2 to T_5.

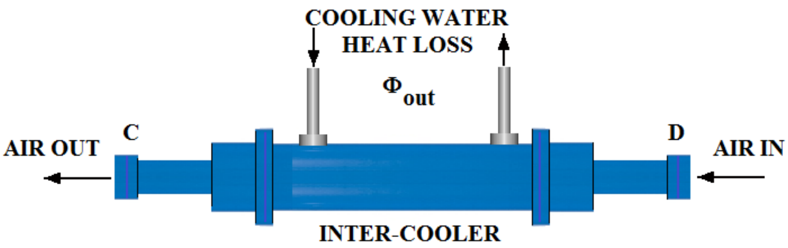

Figure 9

$\Phi\ (out) = \dot{m}C_p(T_2 - T_5) = 0.0687 \times 1.005\ (396.5 - 288) = 7.49$ kW

ISOTHERMAL EFFICIENCY

The ideal isothermal power = $\dot{m}RT_1 \ln(p_1/p_2)$ per stage.

P(isothermal) = $0.06847 \times 287 \times 288 \ln 4 = 7.846$ kW

$\eta_{(iso)} = 7.846/9.246 = 84.9$ %

SWEPT VOLUMES

Consider the first stage.

The F.A.D. is 0.0566 m^3/s.

In the ideal case where the air is drawn in at constant temperature and pressure from the atmosphere, the FAD is given by:

FAD = Swept Volume × Speed and the speed is 6 rev/s

Hence S.V. (1st. Stage) = 0.0566/6 = 0.00943 m^3

S.V. = Bore Area Stroke

$0.00943 = \pi D^2/4 \times 0.1 \quad D_1 = 0.347$ m.

Now consider the second stage.

The air at inlet has a pressure of 4 bar.

The volume drawn is hence 1/4 of the original FAD.

The swept volume of the second stage is hence 0.00943/4 = 0.00236 m^3.

$0.00236 = \pi D^2/4 \times 0.1$ hence $D_2 = 0.173$ m

By the same reasoning the swept volume of the third stage is

SV(3rd stage) = 0.00943/16 = 0.000589 m^3.

$0.000589 = \pi D^2/4 \times 0.1 \quad D_3 = 0.0866$ m

SELF ASSESSMENT EXERCISE No. 2

1. A single acting 2 stage compressor draws in 8.5 m^3/min of free air and compresses it to 40 bar. The compressor runs at 300 rev/min. The atmospheric conditions are 1.013 bar and 15°C. There is an intercooler between stages which cools the air back to 15°C. The polytropic index for all compressions is 1.3. The volumetric efficiency is 90% for the low pressure stage and 85% for the high pressure stage. Calculate the following.

a. The intermediate pressure for minimum indicated work. (6.365 bar)
b. The clearance ratio (0.032)
c. The mass flow rate delivered (0.1734 kg/s)
d. The theoretical indicated power for each stage. (34.05 kW)
e. Show that ignoring the clearance volume makes only a small difference to the answer.
f. The heat rejected in each cylinder. (7.513 kW)
g. The heat rejected by the intercooler. (26.54 kW)
h. The swept volumes of both stages. (31.4 dm^3 and 5.3 dm^3)

What advantage is there in using an after-cooler?

2. A single acting 2 stage compressor draws in free air and compresses it to 8.5 bar. The compressor runs at 600 rev/min. The atmospheric conditions are 1.013 bar and 15°C.

The interstage pressure is 3 bar and the intercooler cools the air back to 30°C. The polytropic index for all compressions is 1.28.

Due to the effect of warming from the cylinder walls and the pressure loss in the inlet valve, the pressure and temperature at the start of the low pressure compression stroke is 0.96 bar and 25°C. The high pressure cycle may be taken as ideal.

The clearance volume for each stages is 4% of the swept volume of that stage. The low pressure cylinder is 300 mm diameter and the stroke for both stages is 160 mm.

Calculate the following.

The free air delivery. (5.858m^3/min)
The volumetric efficiency of the low pressure stage. (86.3 %)
The diameter of the high pressure cylinder. (171 mm)
The indicated power for each stage. (14.6 kW and 13.4 kW)

3. A 2 stage reciprocating air compressor has an intercooler between stages. The induction and expulsion for both stages are at constant pressure and temperature. All the compressions and expansions are polytropic.

(a) Neglecting the effect of the clearance volume show that the intermediate pressure, which gives minimum, indicated work is

$$p_m = \sqrt{p_L \times p_H}$$

(b) Explain with the aid of a sketch how the delivery temperature from both cylinders varies with the intermediate pressure as it changes from p_L to p_H.

4. A two stage reciprocating air compressor works between pressure limits of 1 and 20 bar. The inlet temperature is 15^{o}C and the polytropic index is 1.3. Intercooling between stages reduces the air temperature back to 15^{o}C.

Find the free air delivery and mass of air that can be compressed per kW h of work input.
(10.06 m^3/kW h   12.17 kg/kW h)

Find the ratio of the cylinder diameters if the piston has the same stroke. Neglect the effect of the clearance volume.
(d/D = 0.473)

4. *Polytropic or Small Stage Efficiency*

This is an alternative way of approaching isentropic efficiency. In this method, the compression is supposed to be made up of many stages, each raising the pressure a small amount. The theory applies to any type of compressor.

For an adiabatic gas compression the law of compression $pV^{\gamma} = C$ and the gas law $pV/T = C$ may be combined to give:

$$\frac{T}{p^{1-1/\gamma}} = C \qquad T = C\,p^{1-1/\gamma}$$

This can be expressed in differential form

$$dT = C\left(\frac{\gamma-1}{\gamma}\right)p^{-\frac{1}{\gamma}}\,dp \quad \text{divide by p} \quad \frac{dT}{p} = C\left(\frac{\gamma-1}{\gamma}\right)p^{-\frac{1}{\gamma}}\frac{dp}{p}$$

$$\frac{dT}{p^{1-\frac{1}{\gamma}}} = C\left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} \qquad \frac{T}{C} = p^{1-\frac{1}{\gamma}} \text{ substitute } C = \frac{T}{p^{1-\frac{1}{\gamma}}}$$

$$\frac{dT}{p^{1-\frac{1}{\gamma}}} = \frac{T}{p^{1-\frac{1}{\gamma}}}\left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} \qquad \frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p}$$

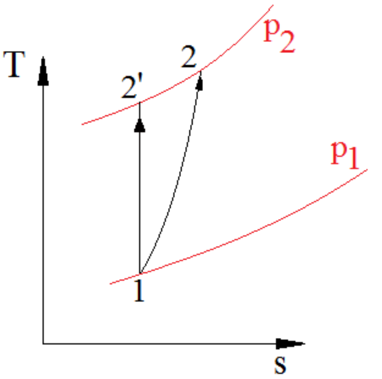

Figure 6

For a compression (1) to (2′) this becomes:

$$\frac{dT'}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} \;\ldots (1)$$

T_1 is the starting temperature and T_2 is the final temperature.

The Isentropic Efficiency is

$$\eta_{is} = \frac{T_{2'} - T_1}{T_2 - T_1}$$

Adiabatic Process

$$\frac{T_{2'}}{T_1} = r^{\frac{\gamma-1}{\gamma}} \qquad T_{2'} = T_1\,r^{\frac{\gamma-1}{\gamma}}$$

Polytropic Process

$$\frac{T_2}{T_1} = r^{\frac{n-1}{n}} = r^{\frac{\gamma-1}{\gamma\eta_\infty}} \quad T_2 = T_1\,r^{\frac{\gamma-1}{\gamma\eta_\infty}}$$

Substitute

$$\eta_{is} = \frac{T_1\,r^{\frac{\gamma-1}{\gamma}} - T_1}{T_1\,r^{\frac{\gamma-1}{\gamma\eta_\infty}} - T_1} = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1}$$

Compare

$\frac{T_2}{T_1} = r^{\frac{n-1}{n}}$ and $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$ and it follows that for an isentropic process $\eta_\infty = 1$

WORKED EXAMPLE No. 6

A compressor draws in air at 15°C and 0.3 bar. The air is compressed to 1.6 bar with a polytropic efficiency of 0.86. Determine the temperature and the isentropic efficiency. Take γ =1.4

SOLUTION

$$T_2 = T_1(r)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 288\left(\frac{1.6}{0.3}\right)^{\frac{1.4-1}{1.4\times0.86}} = 288 \times 5.33^{0.332} = 502\ \text{K}$$

$$T_{2'} = T_1\, r^{\frac{\gamma-1}{\gamma}} = 288(5.33)^{\frac{1.4-1}{1.4}} = 288(5.33)^{0.286} = 464.5\ \text{K}$$

$$\eta_{is} = \frac{464.5 - 288}{502 \quad - 288} = 0.825$$

SELF ASSESSMENT EXERCISE No. 3

1. Show that for any compression process the overall efficiency is given by:

$$\eta_{is} == \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} \quad - 1}$$

η_∞ is the polytropic efficiency.

Determine the index of compression for a gas with an adiabatic index of 1.4 and a polytropic efficiency of 0.9. (1.465)

Determine the overall efficiency when the pressure compression ratio is 4/1 and 8/1.
(0.879 and 0.866)

2. A compressor draws in air at 223.3 K temperature and 0.265 bar pressure. The compression ratio is 6. The polytropic efficiency is 0.86. Determine the temperature after compression. Take γ = 1.4
(405 K)

5. *Other Positive Displacement Designs*

This section is added for information only.

5.1 *Screw Types*

Two rotors have helical lobes cut on them in such a way that when they mesh and rotate in opposite directions, air is drawn along the face of the lobes from input to output. Oil is used liberally to seal the air. The oil also acts as a coolant and the diagram shows how the oil and air are separated and then cooled in a radiator. The oil is re-circulated.

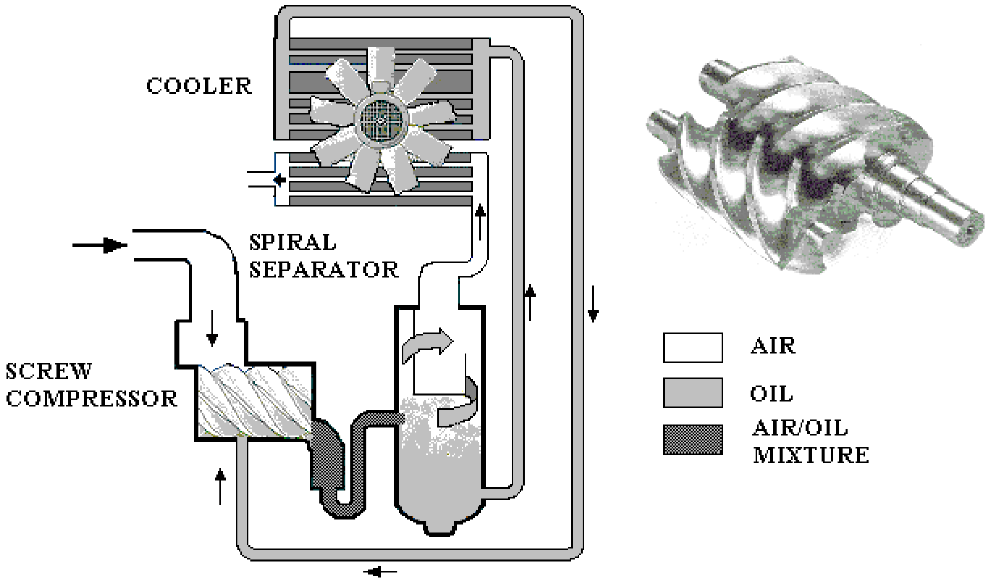

Fig. 11

5.2 *Lobe Types*

Lobe compressors are commonly used as superchargers on large engines. Figure 11 shows the basic design. Air is carried around between the lobes and the outer wall and is expelled when the lobes come together. These compressors are not suitable for high pressures but flow rates around 10 000 m^3/hr are achievable.

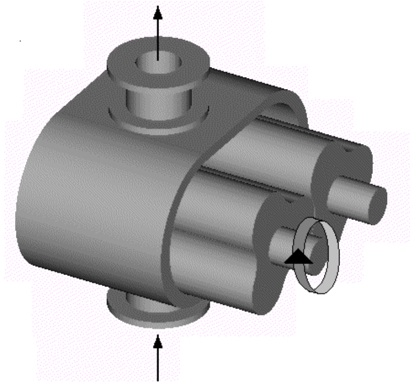

Figure 12

5.3 *Vane Type*

The vanes fit in slots in the rotor. The rotor is eccentric to the bore of the cylinder. When the rotor is turned, centrifugal force throws the vanes out against the wall of the cylinder. The space between the vanes grows and shrinks as the rotor turns so if inlet and outlet passages are cut in the cylinder at the appropriate point, air is drawn in, squeezed and expelled. This type of compressor is suitable for small portable applications and is relatively cheap. Vane compressors often use oil to lubricate and cool the air and a system similar to that shown for the screw compressor is used.

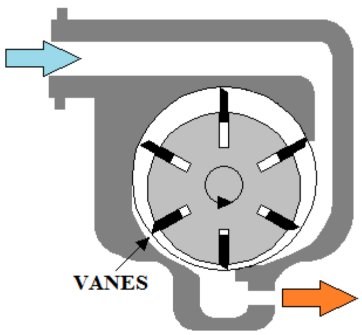

Figure 13

6. Positive Displacement Expanders

With modifications to the valves in particular and maybe to other parts, air compressors can be made to work as a motor and in fact in industrial uses many types of air motors are used. This area of study normally comes under pneumatics and it probably is not the intention for you to study this. Briefly here are some examples of air motors.

The Vane Motor (Figure 14) is basically a vane compressor running backwards. The rotary actuator (Figure 15) can only revolve less than one revolution in either direction.

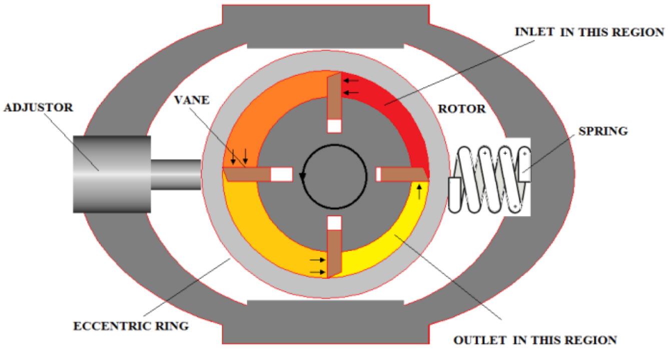

Figure 14 Vane Motor

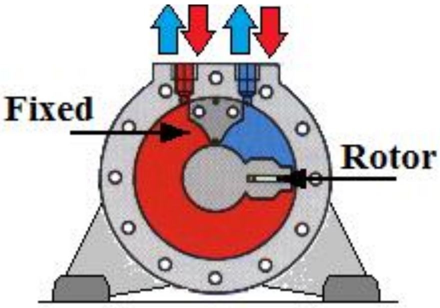

Figure 15 Rotary Actuator

The Piston Type (Figure 16) is designed to rotate a shaft about half a revolution in either direction but this could form the basis of a motor.

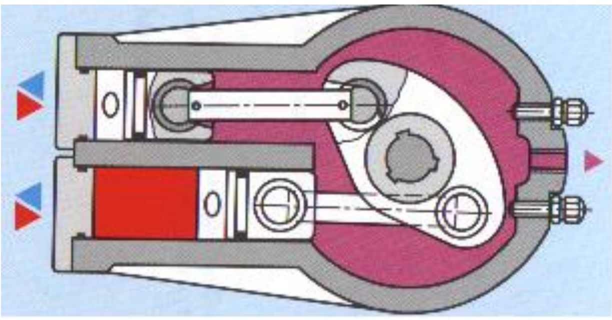

Figure 16

There are many other designs but this could form the basis of a reciprocating engine if the mechanical design allows continuous rotation and a valve system to allow the air in and out at the correct moment.

6.1 *Reciprocating Steam Engines*

Once this topic would have occupied a whole text book as reciprocating steam engines was the basis of the industrial revolution. They were widely used on ships, locomotives and in factories.

In modern times most steam power is produced by turbines but modern technology may produce a revival of smaller engines running on cheap fuel. It is unlikely that you need to study steam engine theory. Reciprocating steam engines need a valve system to admit steam to push the piston and then to allow it to be exhausted. Old designs used mechanical slide valves linked to the crank shaft. A modern design might use electronically controlled valves (similar to the technology used on some modern internal combustion engines).

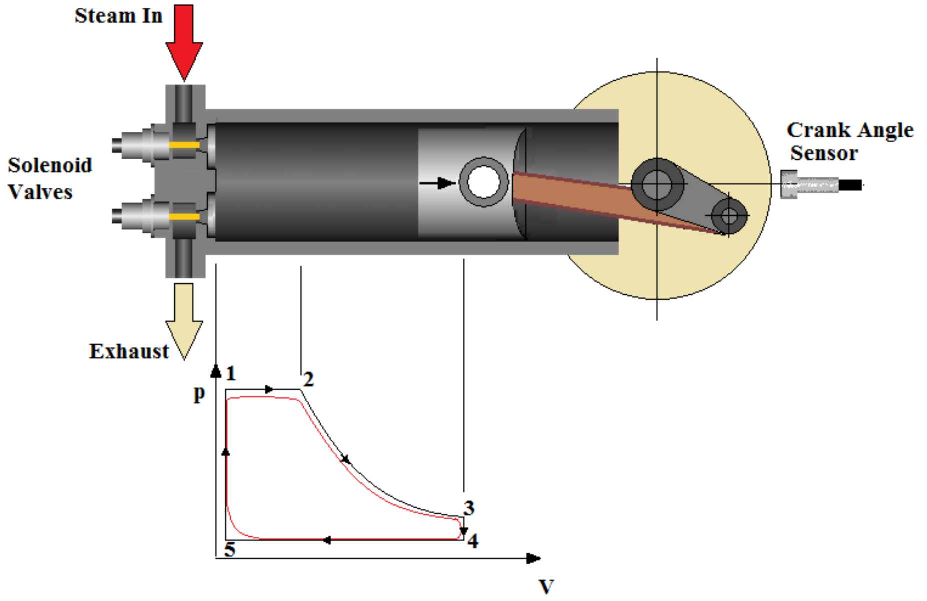

Figure 17

(1) The inlet valve is open and admits high pressure steam to push the piston.
(2) The inlet valve closes (the cut off point) and the steam expands continuing to push the piston.
(3) The exhaust valve opens and the pressure drops.
(4) The piston commences the return stroke pushing out the low pressure steam.
(5) The exhaust valve closes and the inlet valve opens causing a rise in pressure.

This is the same as the ideal cycle for the reciprocating compressor but in reverse. Due to restrictions in the valve and other effects, the corners of the p V diagram are rounded as shown in red.

The theoretical work output and efficiencies are derived in a similar way to the compressor. The steam expansion from 2 to 3 is usually regarded as hyperbolic (pV = C) rather than polytropic ($pV^n = C$). For the ideal cycle the work output per cycle is:

$$W = p_1(V_2 - V_1) + p_2V_2 \ln\left(\frac{V_3}{V_2}\right) - p_5(V_4 - V_5)$$

If the clearance volume is neglected ($V_1 = V_5 = 0$) and if we use the terms p_H and p_L for the high pressure and low pressure then:

$$W = p_HV_2 + p_HV_2 \ln\left(\frac{V_3}{V_2}\right) - p_LV_4$$

The ratio $\left(\frac{V_3}{V_2}\right)$ is called the CUT OFF RATIO r

$$W = p_HV_2 + p_HV_2 \ln(r) - p_L$$
$$W = p_HV_2\{1 + \ln(r)\} - p_LV_4$$

The swept volume is $V_4 - V_3$ and V_3 or V_1 is the clearance volume.

SELF ASSESSMENT EXERCISE No. 4

Assuming the expansion of the steam is hyperbolic (pV = C) show the derivation in full of the formula for the indicated work

$$W = p_H V_2\{1 + \ln(r)\} - p_L V_4$$

Assume an ideal cycle.

A steam engine based on the ideal cycle works between a high pressure of 10 bar and exhaust pressure of 0.5 bar (absolute pressures). The swept volume is 1 litre (0.001 dm^3) and the clearance volume is 20 cm^3. The cut off ratio is 10. Calculate the indicated work.
(Answer 331.8 Joules)

If the engine was run on compressed air, what would be the difference in the work formula?