THERMODYNAMICS
TUTORIAL 7
IDEAL ENGINE CYCLES

This tutorial is set at QCF Level 5 to 6

On completion you should be able to:

- ***Explain the ideal Carnot cycle with closed system process.***
- ***Explain the air standard cycle for a Hot Air Engine (Stirling Engine)***
- ***Explain the air standard cycles for Spark Ignition and Compression Ignition Engines.***
- ***Explain the air standard cycles for Gas Turbine Engines.***

Contents

1. Introduction to Ideal Engine Cycles

Ideal engine cycles are based on a working substance of pure air and they are also known as ***Air Standard Cycles.*** Only ideal thermodynamic processes are used.

Closed system cycles are intended to represent as closely as possible the actual cycle used in real engines. These can be broken down to spark Ignition cycles, compression ignition cycles and various others that do not usually involve ignition.

Open system cycles represent gas and steam turbine cycles. Steam cycles are covered in a later tutorial.

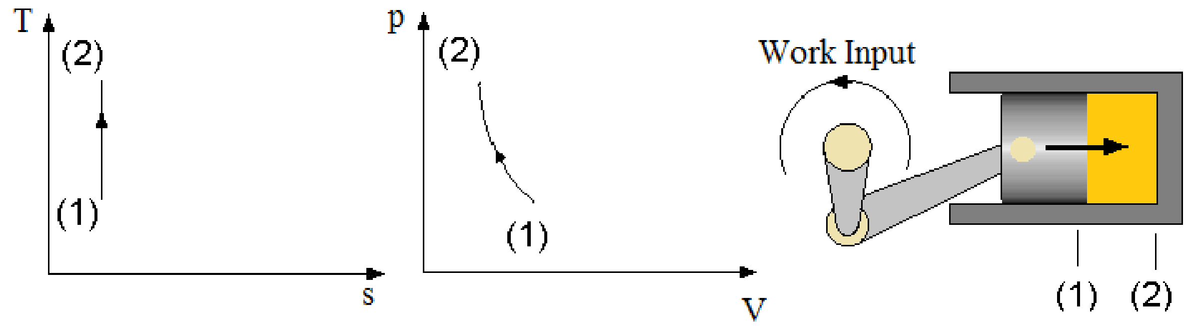
The efficiency predicted by the theory for each cycle is usually compared to that of a Carnot Cycle which represents the most efficient cycle possible. The Carnot principle has been explained earlier. Here we will study the ideal closed system Carnot cycle for a reciprocating engine.

This tutorial is concerned with closed system cycles only.

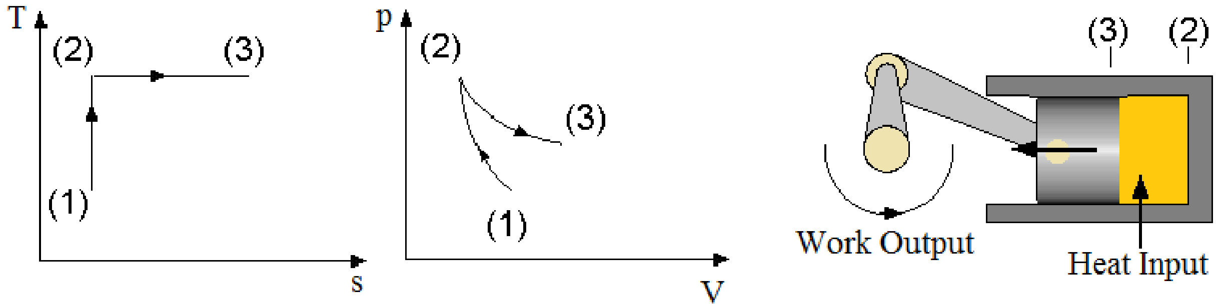
2 Closed System Carnot Cycle

The cycle could be conducted on gas or vapour in a closed or open cycle. The cycle described here is for gas in a cylinder fitted with a piston. It consists of four closed system processes as follows.

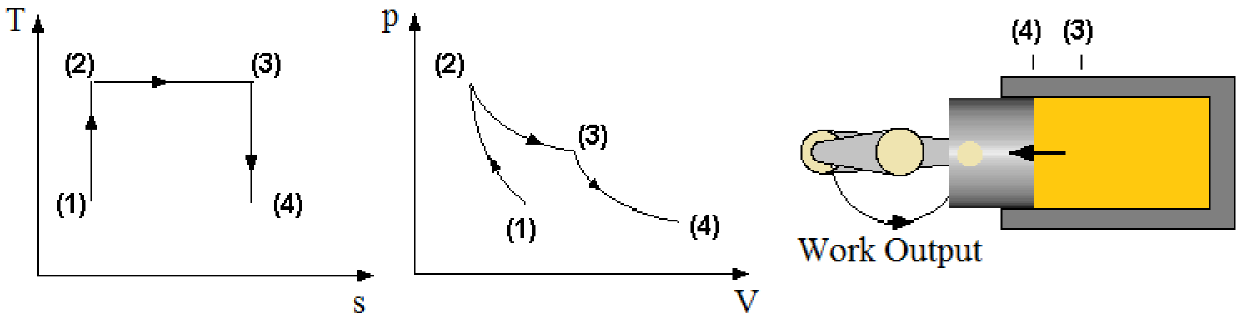
1 to 2 The fluid is compressed isentropically. Work is put in and no heat transfer occurs.



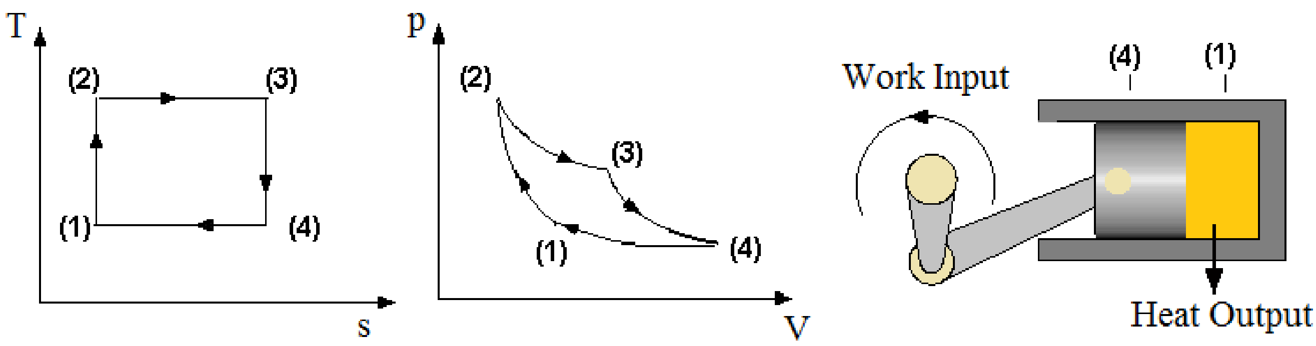
2 to 3 The fluid is heated isothermally. This could only occur if it is heated as it expands so there is work taken out and heat put in.



3 to 4 The fluid continues to expand isentropically with no heat transfer. Work output is obtained.



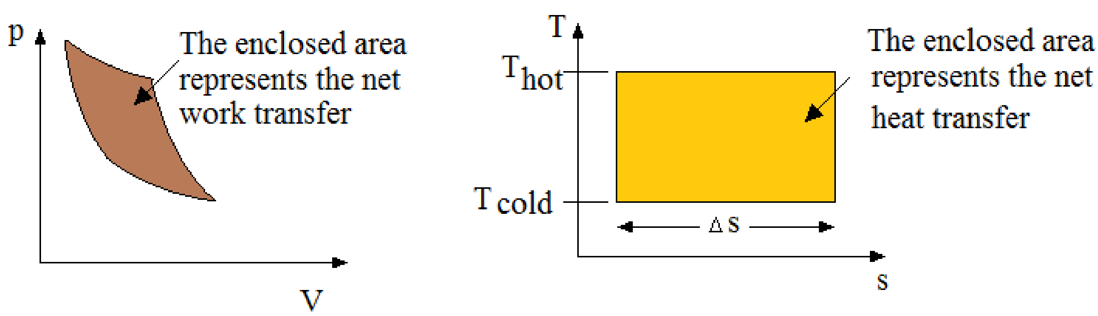
4 to 1 The fluid is cooled isothermally. This can only occur if it cooled as it is compressed, so work is put in and heat is taken out. At the end of this process every thing is returned to the initial condition.



The total work taken out is W_{out} and the total work put in is W_{in}.

To be an engine, W_{out} must be larger than W_{in} and a net amount of work is obtained from the cycle. It also follows that since the area under a p-V graph represents the work done, then the area enclosed by the p-V diagram represents the net work transfer.

It also follows that since the area under the T-s graph is represents the heat transfer, the area enclosed on the T-s diagram represents the net heat transfer. This is true for all cycles and also for real engines.



Applying the first law

$$\mathbf{Q_{nett} = W_{nett}}$$

For isothermal heat transfers

$$Q = \int T ds = T\Delta s \text{ since T is constant}$$

The efficiency would then be given by

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_{cold}\Delta s_{cold}}{T_{hot}\Delta s_{hot}}$$

It is apparent from the T- s diagram that the change in entropy Δs is the same at the hot and cold temperatures. It follows that

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$$

This expression, which is the same as that used for the ideal model, gives the Carnot efficiency and it is used as a target figure that cannot be surpassed (in fact not even attained).

WORKED EXAMPLE No. 1

A heat engine draws heat from a combustion chamber at 300°C and exhausts to atmosphere at 10°C. What is the maximum possible thermal efficiency that could be achieved?

SOLUTION

The maximum efficiency possible is the Carnot efficiency. Remember to use absolute temperatures.

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{273 + 10}{273 + 300} = 1 - \frac{283}{573} = 0.505 \text{ or } 50.5\%$$

SELF ASSESSMENT EXERCISE No. 1

1. A heat engine works between temperatures of 1 100° C and 120°C. It is claimed that it has a thermal efficiency of 75%. Is this possible?
   (Answer the maximum efficiency cannot exceed 71%)

2. Calculate the efficiency of a Carnot Engine working between temperatures of 1 200°C and 200° C.
   (Answer 67.9%)

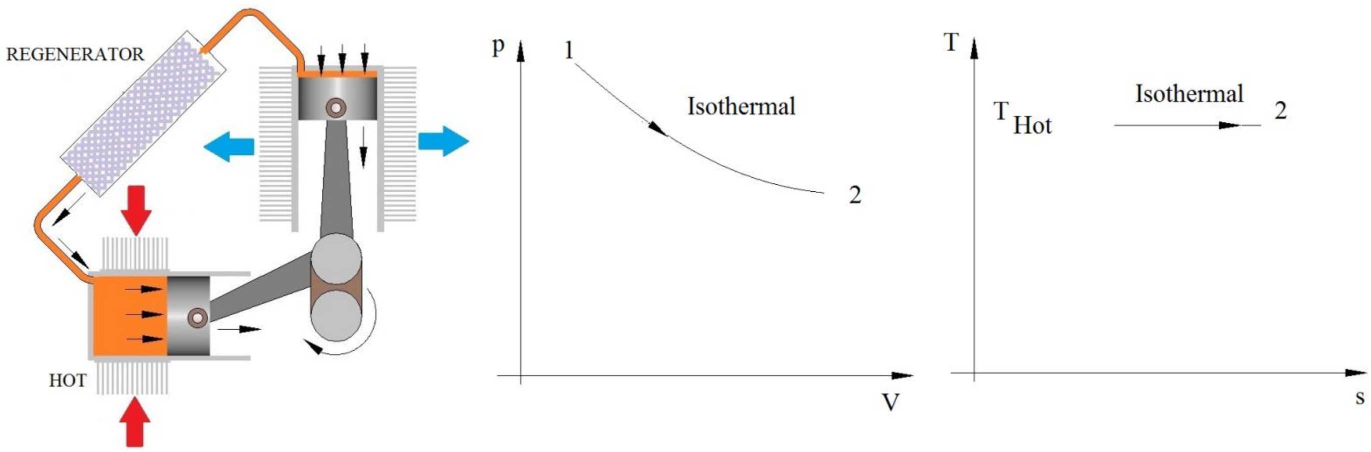
3 The Stirling Cycle

There is a group of engines called ***Hot Air Engines*** that uses an external source of heat rather than internal combustion. The air in the engine undergoes a series of closed system processes. The Stirling hot air engine is probably the best known but there are others not described here. You will find a good description and animation at the web link below. Other links will also be revealed with a web search.

http://www.animatedengines.com/vstirling.html

This particular engine was invented in 1816 by Rev. Robert Stirling of Scotland. It is an example of a hot air engine and there are quite a few ideas and designs for such engines which are quite efficient and can use simple sources of heat. The following design uses two cylinders with 90^o crank displacement between them. They are connected by a ***Regenerator.*** This is a unit full of a material that allows the gas to flow through it easily but quickly absorbs heat when the gas is hot and quickly gives it back when the gas is cold. The gas is usually air but in a sealed unit it can be something else.

The first diagram shows the cycle at a point where the piston in the cold cylinder is about to move away from the head and the piston in the hot cylinder is already moving away from the head and so the gas volume is expanding from (1) to (2). All of the gas is in the hot cylinder so ***external heat*** is added as it expands counteracting the natural drop in temperature. The pressure falls as the volume expands.



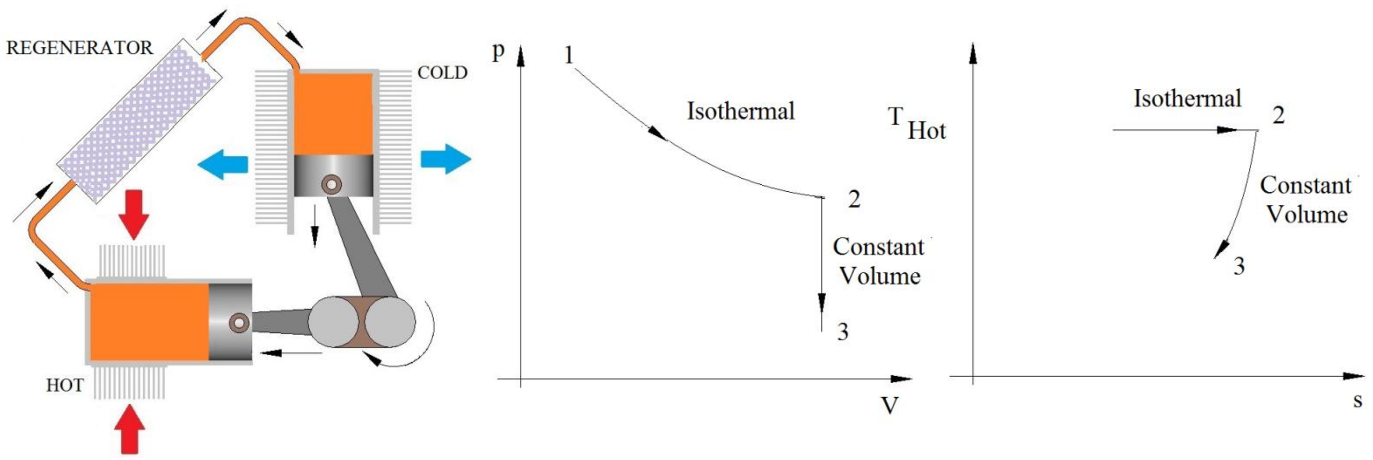
The work done during the expansion process is ideally

$$W = -p_1V_1\ln\left(\frac{V_2}{V_1}\right) = -mRT_{Hot}\ln\left(\frac{V_2}{V_1}\right)$$

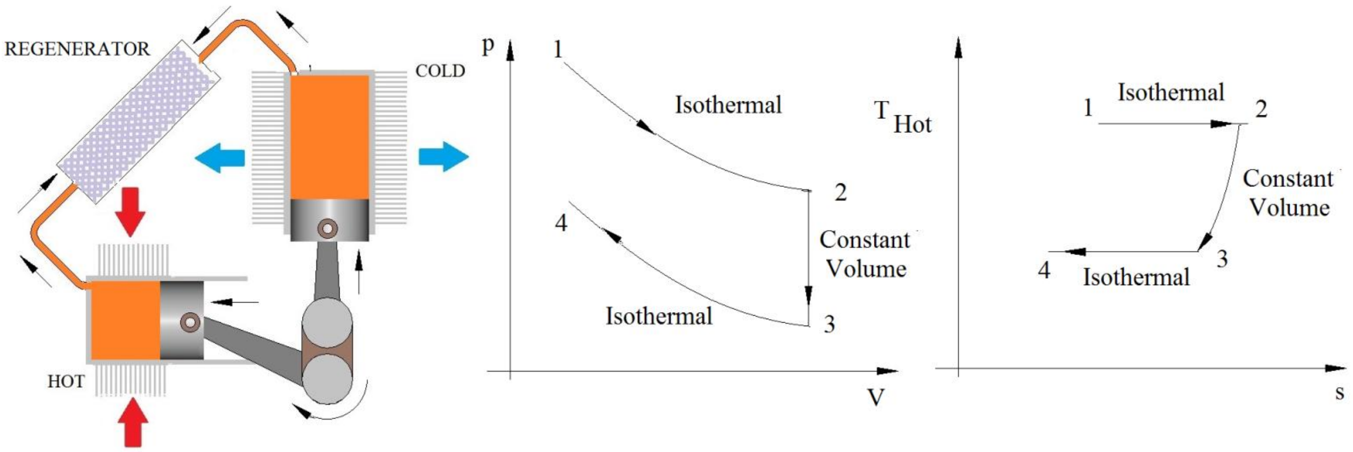
The heat supplied is ideally the equal and opposite to the work transfer since $Q + W = \Delta U = m\ c_v\ \Delta T = 0$

$$Q_{in} = mRT_{hot}\ln\left(\frac{V_2}{V_1}\right)$$

The second diagram shows the gas has finished expanding and the piston in the hot cylinder is starting to move towards the head. The piston in the cold cylinder is still moving down. Gas is pushed through the regenerator and this absorbs some ***internal heat*** from the hot gas which cools so the pressure falls (2) to (3). The volume of the gas is constant as the pistons are moving in opposite directions. W = 0
There is ***no external heat transfer***.



In the third diagram the two pistons are moving towards the head compressing the gas. Most of the gas is in the cold cylinder so it cools and contracts. Heat is removed to the surroundings. The cooling counteracts the natural effect of compression which otherwise raises the temperature so ideally the compression is isothermal.



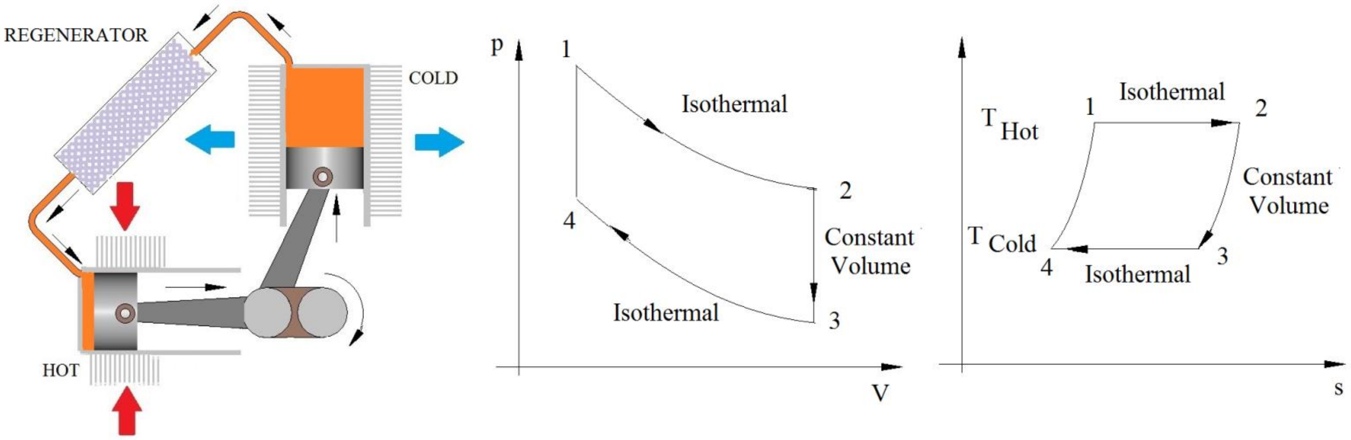
The work done during the compression process is ideally

$$W = mRT_{Cold}\ln\left(\frac{V_3}{V_4}\right) = mRT_{Cold}\ln\left(\frac{V_2}{V_1}\right)$$

$$Q_{out} = mRT_{Cold}\ln\left(\frac{V_4}{V_3}\right) \quad \text{(Not needed for the efficiency)}$$

In the fourth diagram the piston in the hot cylinder is starting to move away from the head and the piston in the cold cylinder is moving towards the head. Most of the air is in the cold cylinder. As it passes through the regenerator it picks up some heat from ***internal heat transfer*** and the pressure rises. ***There is no external heat transfer***.

The volume of the gas is constant. W = 0   Q = 0



Thermal Efficiency

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{mRT_{Hot}Ln\left(\frac{V_2}{V_1}\right) - mRT_{Cold}Ln\left(\frac{V_2}{V_1}\right)}{mRT_{Hot}Ln\left(\frac{V_2}{V_1}\right)} = \frac{T_{Hot} - T_{Cold}}{T_{Hot}} = 1 - \frac{T_{Cold}}{T_{Hot}}$$

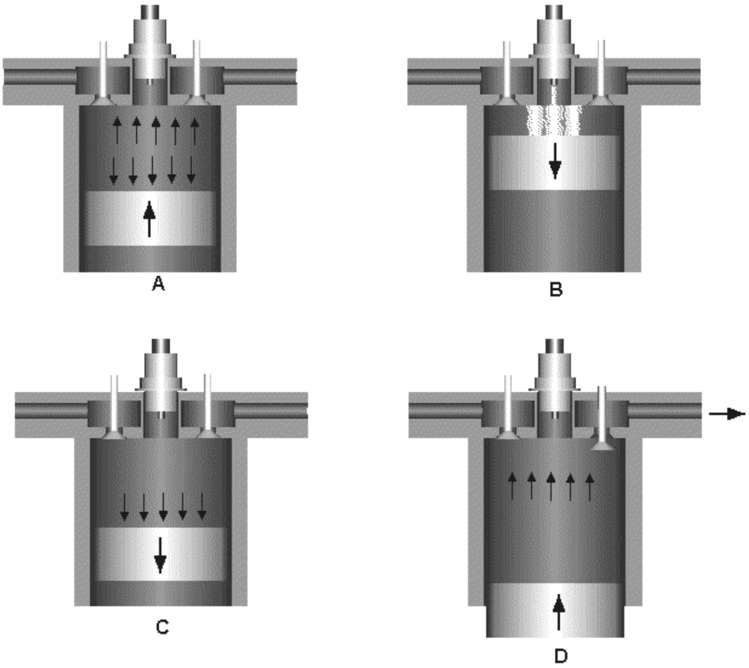
The supposition that external heat transfer only occurs during isothermal processes is in reality far from the actual case.

This ideal case gives the same efficiency as the Carnot Cycle which is the most efficient possible for a cycle operating between two temperatures. In practice the cycle has little resemblance to the ideal one described but these engines continue to attract attention as they have the potential for efficient fuel economy.

4. Spark Ignition Engines

4.1 The Otto Cycle

The Otto cycle represents the ideal cycle for a spark ignition engine. In an ideal spark ignition engine, there are four processes as follows.



Compression Stroke

Air and fuel are mixed and compressed so rapidly that there is no time for heat to be lost. (Figure A) In other words the compression is adiabatic. Work must be done to compress the gas.

Ignition

Just before the point of maximum compression, the air is hot and a spark ignites the mixture causing an explosion (Figure B). This produces a rapid rise in the pressure and temperature. The process is idealised as a constant volume process in the Otto cycle.

Expansion or Working Stroke

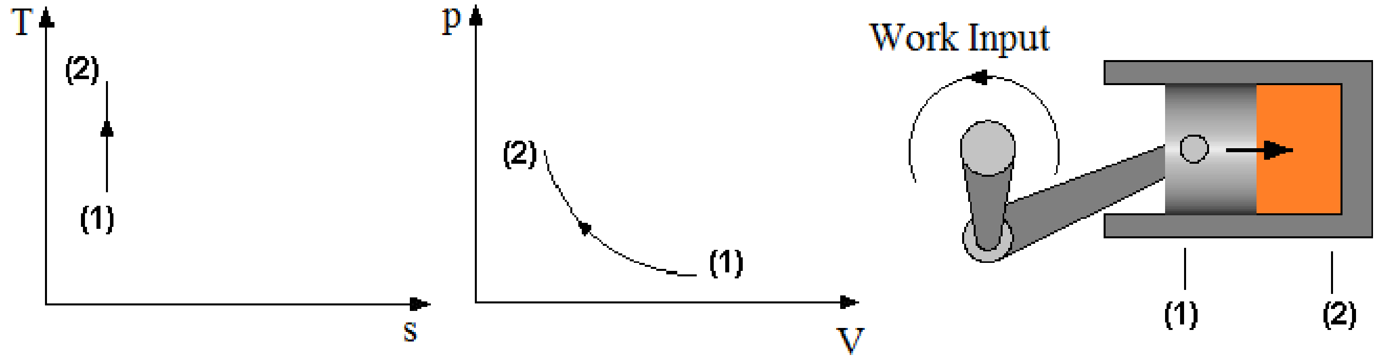
The explosion is followed by an adiabatic expansion pushing the piston and giving out work. (Figure C)

Exhaust

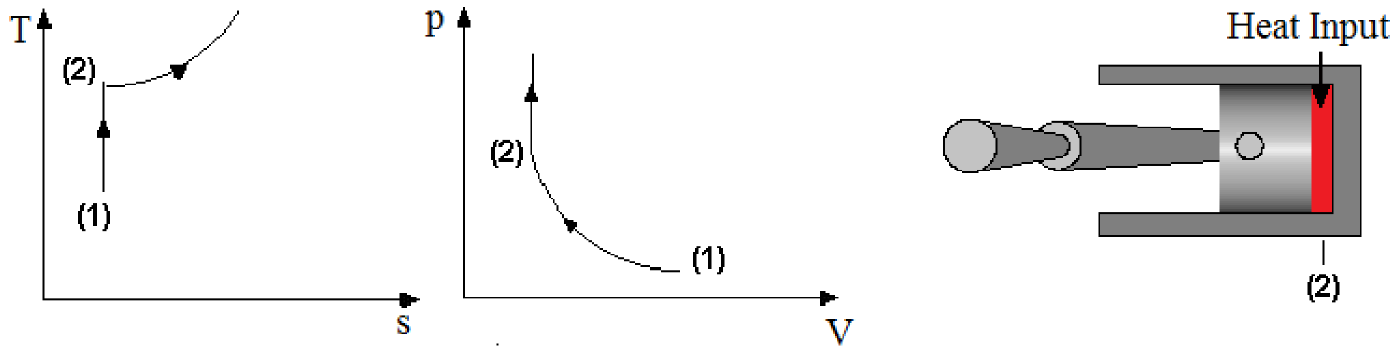
At the end of the working stroke, there is still some pressure in the cylinder. This is released suddenly by the opening of an exhaust valve. (Figure D) This is idealised by a constant volume drop in pressure in the Otto cycle. In 4 stroke engines a second cycle is performed to push out the products of combustion and draw in fresh air and fuel. It is only the power cycle that we are concerned with.

The four ideal processes that make up the Otto cycle are as follows.

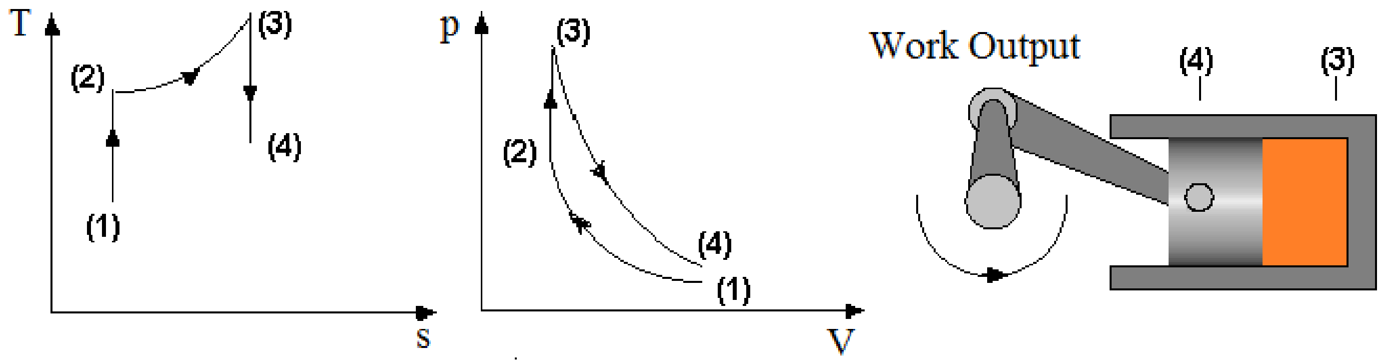
1 to 2. The air is compressed reversibly and adiabatically. Work is put in and no heat transfer occurs.



2 to 3. The air is heated at constant volume. No work is done. $Q_{in} = m\,c_V\,(T_3 - T_2)$

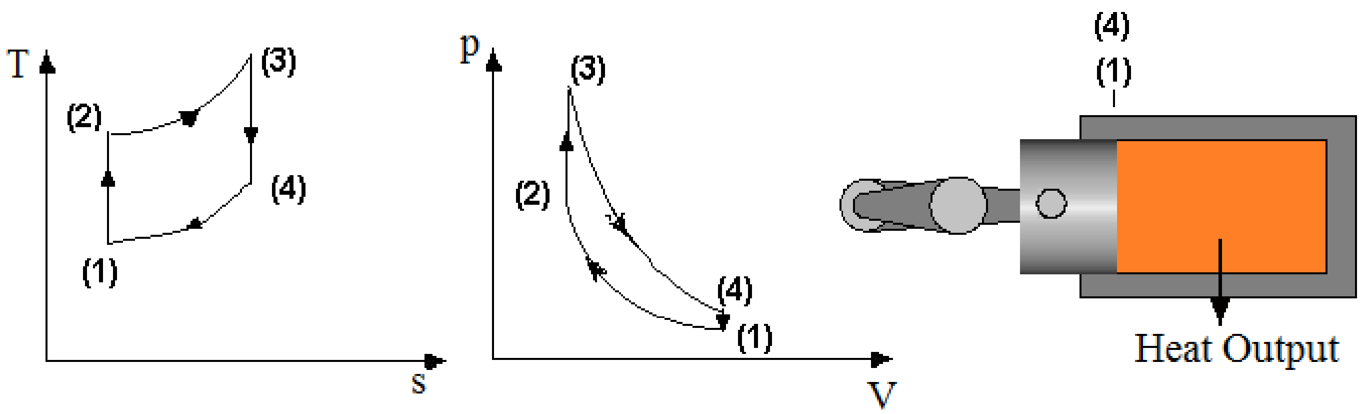


3 to 4. The air expands reversibly and adiabatically with no heat transfer back to its original volume. Work output is obtained.

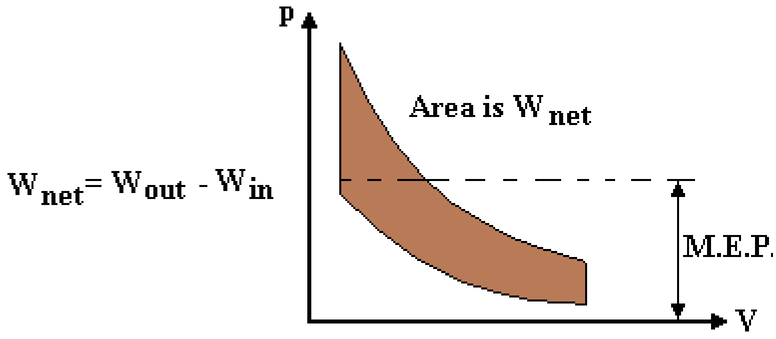


4 to 1. The air is cooled at constant volume back to its original pressure and temperature. No work is done

$Q_{out} = m\,c_V\,(T_4 - T_1)$



If the engine is successful, then W_{out} The area under the top curve) is larger than W_{in} (the area under the lower curve). The enclosed area represents the net work obtained from the cycle.

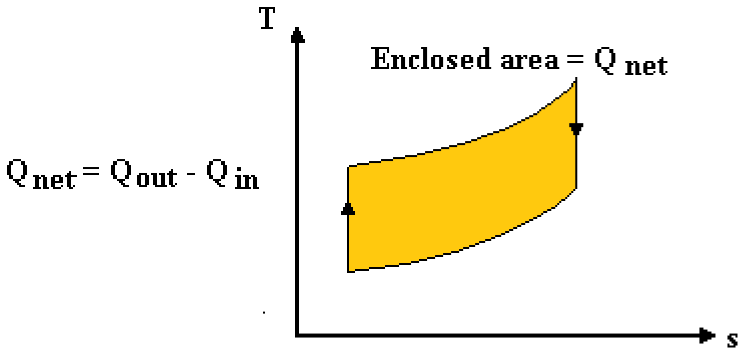


The Mean Effective Pressure (MEP) is the average pressure such that

W(net) = Enclosed Area = MEP × A × L
W(net) = MEP × Swept Volume

This is true for all cycles and for real engines.

A corresponding net amount of heat must have been transferred into the cycle of:



Applying the first law, it follows $\mathbf{Q_{net} = W_{net}}$

It also follows that since the heat transfer is equal to the area under a T - S graph, then the area enclosed by the cycle on the T - S diagram is equal to the Qnet and this is true for all cycles.

4.2 Efficiency

$$\eta = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For the process (1) to (2) we may use the rule:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = {r_v}^{\gamma-1}$$

This is derived from combing the gas law pV = C with the adiabatic expansion law $pV^{\gamma} = C$

For the process (3) to (4) we may similarly write:

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = {r_v}^{\gamma-1}$$

r_v is the volume compression ratio

$$r_v = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

It follows that:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \text{ and } \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Hence:

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\frac{T_3 T_1}{T_2} - T_1}{\frac{T_2 T_4}{T_1} - T_2} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \text{ so } \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{{r_v}^{\gamma-1}} = 1 - {r_v}^{1-\gamma}$$

Since this theoretical cycle is carried out on air for which $\gamma = 1.4$ then the efficiency of an Otto Cycle is given by:

$$\boldsymbol{\eta_{otto} = 1 - {r_v}^{0.4}}$$

This shows that the thermal efficiency depends only on the compression ratio. If the compression ratio is increased, the efficiency is improved. This in turn increases the temperature ratios between the two adiabatic processes and explains why the efficiency is improved.

WORKED EXAMPLE No. 2

An Otto cycle is conducted as follows. Air at 100 kPa and 20^oC is compressed reversibly and adiabatically. The air is then heated at constant volume to 1500^oC. The air then expands reversibly and adiabatically back to the original volume and is cooled at constant volume back to the original pressure and temperature. The volume compression ratio is 8. Calculate the following.

i. The thermal efficiency.

ii. The heat input per kg of air.

iii. The net work output per kg of air.

iv. The maximum cycle pressure.

c_V = 718 kJ/kg γ= 1.4. R = 287 J/kg K

SOLUTION

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

T_1 = 20 +273 = 293K T_3 =1 500 + 273 =1 773 K r_v=8

$$\eta = 1 - r_v{}^{1-\gamma} = 1 - 8^{0.4} = 0.565 \text{ or } 56.5\%$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293(8)^{0.4} = 673.1 \text{ K}$$

$$Q_{in} = mc_v(T_3 - T_2) = 1 \times 718(1\,773 - 673.1) = 789\ \ 700 \text{ J/kg or } 789.7 \text{ kJ/kg}$$

$$W_{net} = \eta Q_{in} = 0.56 \times 789.7 = 446.2 \text{ kJ/kg}$$

From the gas law we have:

$$p_3 = \frac{p_1V_1T_3}{T_1V_3} = \frac{100\,000 \times V_1 \times 1\,773}{293 \times V_3}$$

$$\frac{V_1}{V_3} = 8 \text{ so } p_3 = \frac{100\,000 \ \text{x}\, 1\,773}{293} \times 8 = 4.84 \times 10^6 \text{ Pa or } 4.84 \text{ MPa}$$

If you have followed the principles used here you should be able to solve any cycle.

SELF ASSESSMENT EXERCISE No. 2

Take c_v = 0.718 kJ/kg K, R = 287 J/kg K and γ = 1.4 throughout.

1. In an Otto cycle air is drawn in at 20^{o}C. The maximum cycle temperature is 1 500^{o}C. The volume compression ratio is 8/1. Calculate the following.

   i. The thermal efficiency. (56.5%)
   ii. The heat input per kg of air. (789 kJ/kg).
   iii. The net work output per kg of air. (446 kJ/kg).

2. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40^{o}C respectively. Calculate the following.

   i. The thermal efficiency. (58.5%)
   ii. The maximum cycle temperature. (1450 K).
   iii. The maximum pressure. (4.17 MPa).
   iv. The net work output per kg of air. (293 kJ/kg).

3. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

   The pressure and temperature before compression are 105 kPa and 25^{o}C respectively. The net work output is 500 kJ/kg). Calculate the following.

   i. The heat input. (833 kJ/kg).
   ii. The maximum temperature. (1 906 K)
   iii. The maximum pressure. (6.64 MPa).

4. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40^{o}C respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

   i. The thermal efficiency. (59.4%).
   ii. The net work output. (4.1 kJ/cycle)
   iii. The net power output. (205 kW).

5. An Ericsson Cycle for hot air engines consists of an Isothermal expansion at temperature T_{hot} and expands from pressure p_{high} to pressure p_{low}. This is followed by a constant pressure contraction process at p_{low}. This is followed by an isothermal compression at temperature T_{cold} back to the higher pressure and the cycle is completed by a constant pressure expansion raising the temperature to T_{hot}.

   Sketch the cycle on a T - s and p - V diagram. Show that the theoretical cycle thermal efficiency is the same as for a Carnot Cycle (and Stirling Cycle).

6. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1 300^oC and 20^oC respectively.
(Answer 1.235 kg)

7. The air standard cycle appropriate to the reciprocating spark ignition engine internal-combustion engine is the Otto. Using this, find the efficiency and output of a 2 litre (dm^3), 4 stroke engine with a compression ratio of 9 running at 3 000 rev/min. The fuel is supplied with a gross calorific value of 46.8 MJ/kg and an air fuel ratio of 12.8.

Calculate the answers for two cases.

a. The engine running at full throttle with the air entering the cylinder at atmospheric conditions of 1.01 bar and 10^oC with an efficiency ratio of 0.49.
(Answers 58.5% and 65 kW)

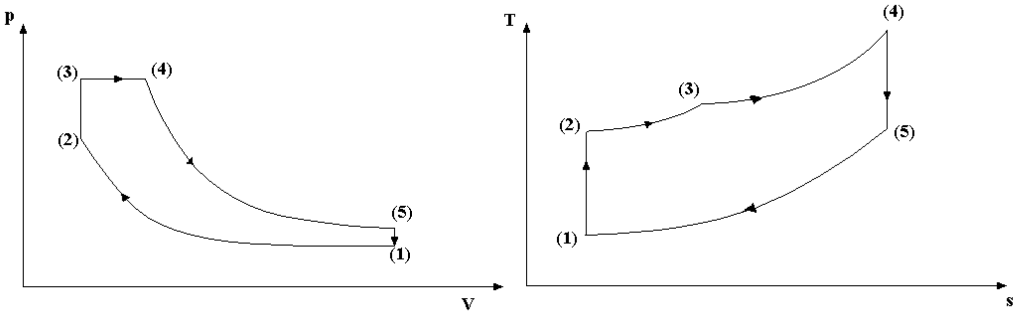
b. The engine running at part throttle with the air entering the cylinder at 0.48 bar and efficiency ratio 0.38.
(Answers 58.5% and 24 kW)

8. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air.
(Answers 871 kJ/kg and 1 129 kJ/kg)

5. Compression Ignition Engines

5.1 The Dual Combustion Cycle

This is the air standard cycle for a modern fast running diesel engine. First the air is compressed isentropically making it hot. Fuel injection starts before the point of maximum compression. After a short delay in which fuel accumulates in the cylinder, the fuel warms up to the air temperature and detonates causing a sudden rise in pressure. This is ideally a constant volume heating process. Further injection keeps the fuel burning as the volume increases and produces a constant pressure heating process. After cut off, the hot air expands isentropically and then at the end of the stroke, the exhaust valve opens producing a sudden drop in pressure. This is ideally a constant volume cooling process. The ideal cycle is shown below.



The processes are as follows.

(1) to (2) is the compression process idealised as an adiabatic process obeying the law $pV^{1.4}$ = constant. This process requires work to be done.

(2) to (3) is a vertical line with the pressure rising with no change in volume. This represents the detonation and heat is released into the air.

The law is simply V = constant

(3) to (4) is a horizontal line in which it is idealised that the pressure is constant as the fuel is burned and more heat is released into the air. Work is also obtained as the pressure pushes the piston.

The law is simply p = constant

Point (4) is the cut off point.

(4) to (5) is an expansion with the pressure falling and the volume increasing. This is idealised as an adiabatic process obeying the law $pV^{1.4}$ = constant. More work is obtained during this process.

(5) to (1) is a vertical line representing the drop in pressure with no change in volume following the exhaust valve opening. Heat is removed from the system during this process (this is the heat loss in the exhaust)

The analysis of the cycle is as follows.

The heat is supplied in two stages hence $Q_{in} = m\, c_p\, (T_4 - T_3) + m\, c_v\, (T_3 - T_2)$

The heat rejected is $Q_{out} = m\, c_v\, (T_5 - T_1)$

The thermal efficiency may be found as follows.

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

The formula can be further developed to show that

$$\eta_{th} = 1 - \frac{k\beta^{1.4} - 1}{[(k-1) + 1.4k(\beta - 1)]r_v^{0.4}}$$

r_v is the *Volume Compression Ratio*. $r_v = V_1/V_2$

β is the *Cut Off Ratio*. $\beta = V_4/V_3$

k is the ratio p_3/p_2 and this is the process produced by the detonation of the fuel.

This enables us to make theoretical studies to see how the efficiency is affected by the compression ratio and cut off ratio. Examining this formula shows that a higher compression ratio will produce a higher efficiency. Diesel engines have higher compression ratios than petrol engines which partly explain their superior efficiency.

Increasing the cut off ratio increases the work output and also increases the efficiency up to a point.

Slow running large engines have a small detonation so in this case $k \approx 1$

Most students will find this adequate to solve problems concerning the dual combustion cycle. Generally, the method of solution involves finding all the temperatures by application of the gas laws. Those requiring a detailed analysis of the cycle should study the following derivation.

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Obtain all the temperatures in terms of T_2 starting with the isentropic compression (1) to (2)

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_2}{r_v^{\gamma-1}}$$

Constant Volume heating (2) to (3) noting $V_2 = V_3$

$$T_3 = T_2\frac{p_3V_3}{p_2V_2} = T_2\frac{p_3}{p_2} = kT_2$$

Constant Pressure heating (3) to (4) noting $p_3 = p_4$

$$T_4 = T_3\frac{p_4V_4}{p_3V_3} = T_3\frac{V_4}{V_3} = \beta T_3 = \beta kT_2$$

Isentropic expansion (4) to (5)

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{V_4V_2}{V_5V_2}\right)^{\gamma-1} = T_4 = \beta kT_2\left(\frac{\beta}{r_v}\right)^{\gamma-1} = T_2\frac{k\beta^{\gamma}}{r_v^{\gamma-1}}$$

Substitute for all the temperatures in the efficiency formula.

$$\eta_{th} = 1 - \frac{\left(T_2\frac{k\beta^{\gamma}}{r_v^{\gamma-1}} - \frac{T_2}{r_v^{\gamma-1}}\right)}{(kT_2 - T_2) + \gamma(\beta kT_2 - kT_2)} = 1 - \frac{\left(\frac{k\beta^{\gamma}}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}}\right)}{(k-1) + \gamma(\beta k - k)}$$

$$\eta_{th} = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}} = 1 - \frac{k\beta^{1.4} - 1}{[(k-1) + 1.4k(\beta - 1)]r_v^{0.4}}$$

Note that if β =1, the cycle becomes an Otto cycle and the efficiency formulae becomes the same as for an Otto cycle.

WORKED EXAMPLE No. 3

In a theoretical diesel engine cycle the compression starts from 1 bar and 20°C. The compression ratio is 18/1 and the cut off ratio is 1.15. The maximum cycle temperature is 1 360 K. The total heat input is 1 kJ per cycle. Calculate the following.

i. The thermal efficiency of the cycle.
ii. The net work output per cycle.

Compare the result to the Carnot efficiency

SOLUTION

Known data T_1 = 20 +273 = 293 K The hottest temperature is T_4 = 1 360K.
β =1.15 r_v = 18 γ =1.4

$$T_2 = T_1 r_v^{\gamma-1} = 931\text{K}$$

$$T_3 = \frac{V_3 T_4}{V_4} = \frac{T_4}{\beta} = \frac{1360}{1.15} = 1183\text{K}$$

$$\frac{p_3}{p_2} = k = \frac{T_3}{T_2} = 1.27$$

$$\eta_{th} = 1 - \frac{k\beta^{1.4} - 1}{[(k-1) + 1.4k(\beta - 1)]r_v^{0.4}} = 1 - \frac{1.27 \times 1.15^{1.4} - 1}{[(1.27 - 1) + 1.4 \times 1.27(1.15 - 1)]18^{0.4}}$$

η = 0.68 or 68%

This is theoretical and values like this arenot obtained in reality for a variety of reasons.

$W_{net} = \eta \times Q_{in}$ = 0.68 × 1 = 0.68 kJ per cycle.

The Carnot efficiency should be higher.

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{293}{1\ 360} = 0.785$$

The figure of 0.68 is lower so the Carnot principle has not been contravened.

WORKED EXAMPLE No. 4

A dual combustion cycle has a compression ratio of 18/1. The maximum pressure in the cycle is 9 MPa and the maximum temperature is 2 000°C. The pressure and temperature before compression is 115 kPa and 25°C respectively. Calculate the following.

i. The cut off ratio.
ii. The cycle efficiency.
iii. The nett work output per kg of air.

Assume γ =1.4 c_p = 1.005 kJ/kgK c_v = 0.718 kJ/kg K.

SOLUTION

Known data T_1 = 298 K T_4 = 2273 Kp_3 = p4 = 9 MPa p_1 = 115 kPa
$V_1/V_2 = V_1/V_3 = 18$ $V_2 = V_3$

$$T_2 = 298 \text{ x } 18^{(\gamma-1)} = 947 \text{ K}$$

$$T_3 = T_1 \frac{p_3 V_3}{p_1 V_1} = 298 \times \frac{9 \times 10^6}{115 \times 10^3} \times \frac{V_3}{V_1} = 298 \times \frac{9 \times 10^6}{115 \times 10^3} \times \frac{1}{18} = 1\ 296 \text{ K}$$

$$\beta = \frac{V_4}{V_3} = \frac{p_3 T_4}{p_4 T_3} = \text{but } p_4 = p_3 \quad \beta = \frac{T_4}{T_3} = \frac{2\ 273}{1\ 296} = 1.75$$

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma-1} \text{ but } \frac{V_4}{V_5} = \frac{V_4}{V_3} \times \frac{V_3}{V_5} = \frac{1.75}{18} = 0.0974$$

$$T_5 = 2\ 273(0.0974)^{0.4} = 895.6 \text{ K}$$

$Q_{in} = mc_p(T_4 - T_3) + mc_v(T_3 - T_2)$ m = 1 kg

Q_{in} =1.005(2 274 -1 296) + 0.718(1 296-947) = 1 232.5 kJ/kg

$Q_{out} = mc_v(T_5 - T_1)$

Q_{out} = 0.718(895.6 - 298) = 429 kJ/g

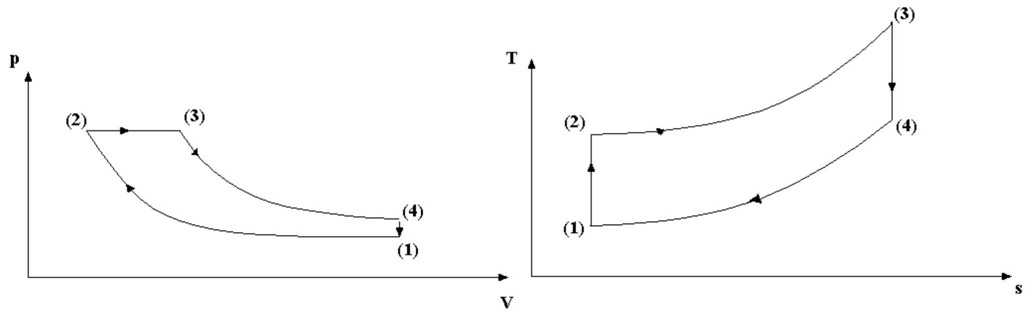
$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{429}{1\ 232} = 0.65 \text{ or } 65\%$$

$$W_{net} = Q_{in} - Q_{out} = 1\ 232 - 429 = 803 \text{ kJ/kg}$$

5.2**. *The Diesel Cycle***

The Diesel Cycle proceeded the dual combustion cycle. The Diesel cycle is a reasonable approximation of what happens in slow running engines such as large marine diesels. The initial accumulation of fuel and sharp detonation does not occur and the heat input is idealised as a constant pressure process only.

Again consider this cycle as being carried out inside a cylinder fitted with a piston. The p-V and T-s cycles diagrams are shown below.



1 - 2 reversible adiabatic (isentropic) compression

2 - 3 constant pressure heating

3 - 4 reversible adiabatic (isentropic) expansion

4 - 1 constant volume cooling

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

The cycle is the same as the dual combustion cycle without the constant volume heating process. In this case since k=1 the efficiency is given by the following formula.

$$\eta_{th} = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

WORKED EXAMPLE No. 5

An engine using the Diesel Cycle has a compression ratio of 20/1 and a cut off ratio of 2. At the start of the compression stroke the air is at 1 bar and 15^oC. Calculate the following.

i. The air standard efficiency of the cycle.
ii. The maximum temperature in the cycle.
iii. The heat input.
iv. The net work output.

SOLUTION

Initial data.

$\beta = 2$ $r_v = 20$ $\gamma = 1.4$ $c_v = 718$ J/kg K for air $T_1 = 288$ K $p_1 = 1$ bar.

The maximum temperature is T_3 and the maximum pressure is p_3 and p_2.

$$\eta_{th} = 1 - \frac{\beta^\gamma - 1}{(\beta - 1)\gamma r_v^{\gamma-1}} = 1 - \frac{2^{1.4-1} - 1}{(2-1) \times 1.4 \times 20^{1.4-1}}$$

$$\eta_{th} = 1 - \frac{1.6391}{4.64} = 0.647 \text{ or } 64.7\ \%$$

$$T_2 = T_1 r_v^{\gamma-1} = 288 \times 20^{0.4} = 954.5 \text{ K}$$

$$T_3 = \frac{V_2 T_2}{V_3} = \beta T_2 = 954.5 \times 2 = 1\,909 \text{ K}$$

$$Q_{in} = m\,c_p(T_3 - T_2) = 1.005(1\,909 - 954.5) = 959.3 \text{ kJ}$$

$$\eta_{th} = \frac{W_{net}}{Q_{in}}$$

$$W_{net} = \eta_{th} \times Q_{in} = 0.647 \times 959.3 = 620.6 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No. 3

1. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30°C and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

   i. The maximum cycle temperature. (2 424 K).
   ii. The net work output per cycle. (864 kJ/kg).
   iii. The thermal efficiency. (67.5 %).

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

   i. The net work output per cycle. (680 kJ/kg).
   ii. The thermal efficiency. (57.6 %).

3. Draw a p - V and T - s diagram for the Dual Combustion Cycle.

   A reciprocating engine operates on the Dual Combustion Cycle. The pressure and temperature at the beginning of compression are 1 bar and 15°C respectively. The compression ratio is 16. The heat input is 1800 kJ/kg and the maximum pressure is 80 bar. Calculate the following.

   i. The pressure, volume and specific volume at all points in the cycle.
   ii. The cycle efficiency. (62.8 %).
   iii. The mean effective pressure. (14.52 bar).

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6. The Basic Gas Turbine Cycle

6.1 Revision of Ideal Steady Flow Gas Processes

Before looking at the ideal cycle let's revise the expansion of a gas in a turbine. When a gas is expanded from pressure p_1 to pressure p_2 adiabatically, the temperature ratio is given by the formula

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$$

The same formula may be applied to a compression process. Always remember that when a gas is expanded it gets colder and when it is compressed it gets hotter. The temperature change is $T_2 - T_1$

Now let's revise the energy transfer for an ideal gas in a steady flow process.

If the velocity of the gas in and out is nearly the same, the energy transfer is equal to the change in enthalpy which is the case here.

The enthalpy change for an ideal gas is

$$\Delta H = \dot{m}\, c_p\, \Delta T$$

The power output of a turbine is

$$P_{in} = \Delta H = \dot{m}\, c_p\, \Delta T$$

The power input to a compressor is

$$P_{out} = \Delta H = \dot{m}\, c_p\, \Delta T$$

The heat Input to a heater is

$$\Phi_{in} = \Delta H = \dot{m}\, c_p\, \Delta T$$

The heat output for a steady flow cooler is

$$\Phi_{out} = \Delta H = \dot{m}\, c_p\, \Delta T$$

WORKED EXAMPLE No. 6

A gas turbine expands 4 kg/s of air from 12 bar and 900°C to 1 bar adiabatically. Calculate the exhaust temperature and the power output. $\gamma = 1.4$ $c_p = 1.005$ kJ/kg K

SOLUTION

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = 1\,173\left(\frac{1}{12}\right)^{1-\frac{1}{1.4}} = 1\,173\left(\frac{1}{12}\right)^{0.2958} = 562.48 \text{ K}$$

The steady flow energy equation states

$$\Phi + P = \text{change in enthalpy/s}$$

Since it is an adiabatic process

$$\Phi = 0$$

so

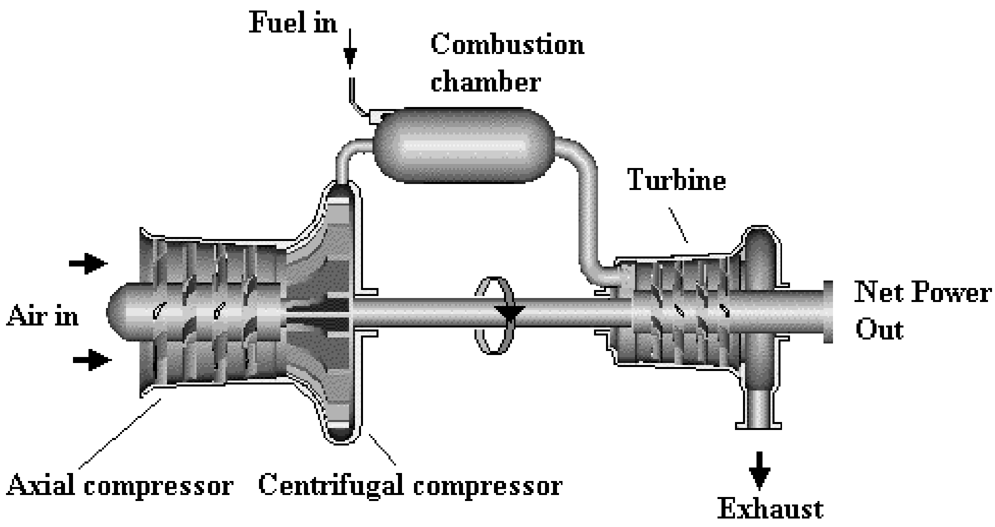
$$P = \Delta H/s = \dot{m}\ c_p\,\Delta T = 4 \times 1.005 \times (562.5 - 1\,173) = -\,2\,454.21 \text{ kW}$$

$$P = 2.454 \text{ MW} \quad \text{(Leaving the system)}$$

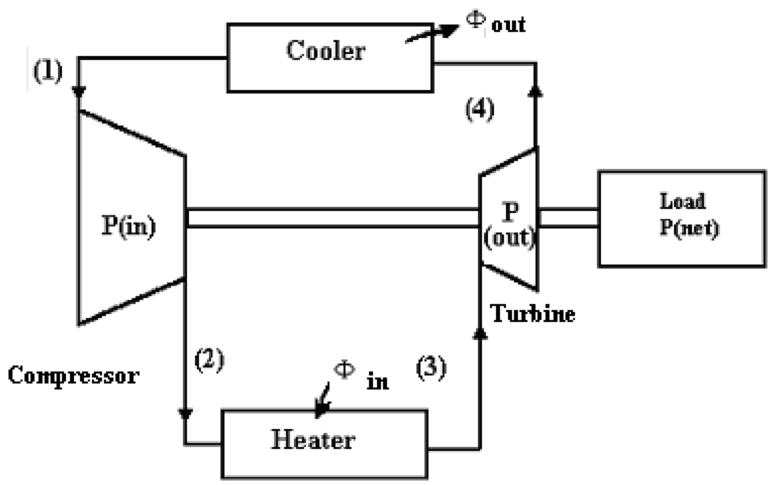
P(out) = 2.454 MW

6.2 The Ideal Gas Turbine Cycle

This is the only engine cycle with steady flow processes in this tutorial. The ideal and basic cycle is called the ***Joule*** cycle and is also known as the constant pressure cycle because the heating and cooling processes are conducted at constant pressure. The diagram below shows the main layout of a basic cycle.



The cycle in block diagram form is shown below.



There are 4 ideal processes in the cycle.

1 - 2 Reversible adiabatic (isentropic) compression requiring power input.

$$P_{in} = \Delta H/s = \dot{m}\, c_p\, (T_2 - T_1)$$

2 - 3 Constant pressure heating requiring heat input.

$$\Phi_{in} = \Delta H/s = \dot{m}\, c_p\, (T_3 - T_2)$$

3 - 4 Reversible adiabatic (isentropic) expansion producing power output.

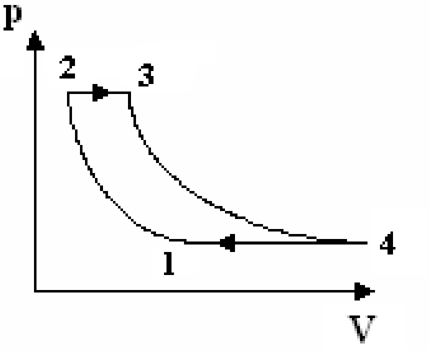
$$P_{out} = \Delta H/s = \dot{m}\, c_p\, (T_3 - T_4)$$

4 - 1 Constant pressure cooling back to the original state requiring heat removal.

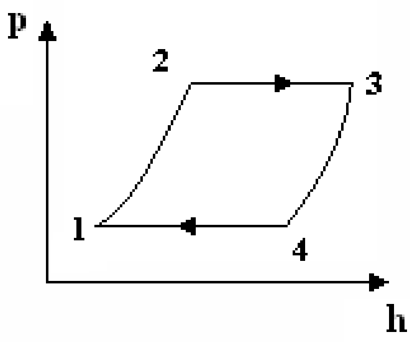
$$\Phi_{out} = \Delta H/s = \dot{m}\, c_p\, (T_4 - T_1)$$

The cycle can be drawn as a pressure – volume diagram, a pressure - enthalpy diagram and a temperature-entropy diagram shown below.

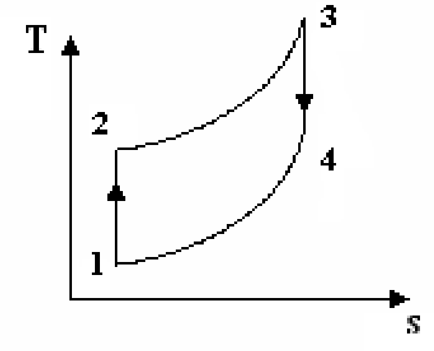
Since flow processes are used enthalpy plays a major part of the work and p – h diagrams become important.



p - V diagram



p - h diagram



T - s diagram

6.3 Efficiency

The efficiency is found by applying the first law of thermodynamics.

$$\Phi_{net} = P_{net}$$

$$\Phi_{in} - \Phi_{out} = P_{out} - P_{in}$$

Remember that the energy transfer rates for all the ideal processes is given by

$$P = \dot{m}\Delta h \quad \text{and} \quad Q = \dot{m}\Delta h \text{ and for a perfect gas } \Delta h = c_p \Delta T$$

$$\eta_{th} = \frac{P_{net}}{\Phi_{in}} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{\dot{m}c_p(T_4 - T_1)}{\dot{m}c_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

It assumed that the mass and the specific heats are the same for the heater and cooler.

It is easy to show that the temperature ratio for the turbine and compressor are the same.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = (r_p)^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{1-\frac{1}{\gamma}} = (r_p)^{1-\frac{1}{\gamma}} = \frac{T_2}{T_1}$$

r_p is the pressure compression ratio for the turbine and compressor.

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\left(\frac{T_3 T_1}{T_2} - T_1\right)}{\left(\frac{T_2 T_4}{T_1} - T_2\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \qquad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_p^{1-\frac{1}{\gamma}}} = 1 - r_p^{-0.286} \text{ since } \gamma = 1.4$$

This shows that the efficiency depends only on the pressure ratio which in turn affects the hottest temperature in the cycle.

WORKED EXAMPLE No. 7

A gas turbine uses the Joule cycle. The pressure ratio is 6/1. The inlet temperature to the compressor is 10^oC. The flow rate of air is 0.2 kg/s. The temperature at inlet to the turbine is 950^oC. Calculate the following.

i. The cycle efficiency.

ii. The heat transfer into the heater.

iii. The net power output.

$\gamma = 1.4$ $\quad\quad\quad$ $c_p = 1.005$ kJ/kg K

SOLUTION

$$\eta_{th} = 1 - r_p^{-0.286} = 1 - 6^{-0.286} = 0.4 \text{ or } 40\%$$

$$T_2 = T_1 r_p^{0.286} = 283 \times 6^{0.286} = 472.4 \text{ K}$$

$$\Phi_{in} = \dot{m}\, c_p(T_3 - T_2) = 0.2 \times 1.005(1\,223 - 472.4) = 150.8 \text{ kW}$$

$$P_{net} = \Phi_{in} \times \eta_{th} = 150.8 \times 0.4 = 60.3 \text{ kW}$$

SELF ASSESSMENT EXERCISE No. 4

1. A gas turbine expands 6 kg/s of air from 8 bar and 700^oC to 1 bar isentropically.

   Calculate the exhaust temperature and the power output. $\gamma = 1.4$ $\;$ $c_p = 1.005$ kJ/kg K
   (Answers 537.1 K and 2.628 MW)

2. A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10^oC. After constant pressure heating, the pressure and temperature are 7 bar and 700^oC respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

   1. The cycle efficiency.

   2. The heat transfer into the heater.

   3. The net power output.

   $\gamma = 1.4$ $\quad\quad\quad$ $c_p = 1.005$ kJ/kg K

   (Answers 42.7 % , 206.7 kW and 88.26 kW)