UNIT 61: ENGINEERING THERMODYNAMICS

Unit code: D/601/1410

QCF level: 5

Credit value: 15

OUTCOME 3

TUTORIAL No. 6 - RECIPROCATING AIR COMPRESSORS

3 **Be able to evaluate the performance of reciprocating air compressors**

Property diagrams: theoretical pressure-volume diagrams for single and multistage compressors; actual indicator diagrams; actual, isothermal and adiabatic compression curves; induction and delivery lines; effects of clearance volume

Performance characteristics: free air delivery; volumetric efficiency; actual and isothermal work done per cycle; isothermal efficiency

First law of thermodynamics: input power; air power; heat transfer to intercooler and aftercooler; energy balance

Faults and hazards: effects of water in compressed air; causes of compressor fires and explosions

In order to complete this section you should be familiar with gas laws and polytropic gas processes. You will study the principles of reciprocating compressors in detail and some principles of rotary compressors. On completion you should be able to do the following.

- Describe the working principles of reciprocating compressors.
- Describe the basic design of various other compressors.
- Define and calculate swept volume.
- Define and calculate volumetric efficiency.
- Define and calculate isothermal efficiency.
- Define and calculate indicated power.
- State the benefits of cooling.
- Calculate the heat rejected through cooling.
- Define and calculate the inter-stage pressures for multiple compressors.

INTRODUCTION

Air is an expansive substance and dangerous when used at high pressures. For this reason, most applications are confined to things requiring low pressures (10 bar or lower) but there are industrial uses for high pressure air up to 100 bar.

The common source of the air is the compressor. There are many types of compressors with different working principles and working conditions. This is a list of the main types.

- Reciprocating.
- Sliding vane compressors.
- Lobe compressors.
- Helical screws.
- Centrifugal.
- Axial turbine compressors.

The function of all of them is to draw in air from the atmosphere and produce air at pressures substantially higher. Usually a storage vessel or receiver is used with the compressor.

Compressed air has many applications. It is also used for powering pneumatically operated machines. It is used as a power medium for workshop tools such as shown in Fig.1.

Fig. 1

1. COMPRESSED AIR

1.1 ATMOSPHERIC VAPOUR

Water vapour in the atmosphere has important consequences on compressors. Atmospheric air contains *WATER VAPOUR* mixed with the other gases. The ratio of the mass of water vapour in the air to the mass of the air is called the *ABSOLUTE HUMIDITY*. The quantity of water that can be absorbed into the air at a given pressure depends upon the temperature. The hotter the air, the more water it can absorb. When the air contains the maximum possible amount of vapour it is at its dew point and rain or fog will appear. The air is then said to have 100% humidity. When the air contains no water vapour at all (dry air), it has 0% humidity. This is called the *RELATIVE HUMIDITY*. For example if the air has 40% relative humidity it means that it contains 40% of the maximum that it could contain. There are various ways to determine the humidity of air and instruments for doing this are called *HYGROMETERS*.

The importance of humidity to air compressors is as follows. When air is sucked into the compressor, it brings with it water vapour. When the air is compressed the pressure and the temperature of the air goes up and the result is that the compressed air will have a relative humidity of about 100% and it will be warm. When the air leaves the compressor it will cool down and the water vapour will condense. Water will then clog the compressor, the receiver and the pipes.

Water damages air tools; ruins paint sprays, and corrode pipes and equipment. For this reason the water must be removed and the best way is to use a well designed compressor installation and distribution network.

1.2 TYPICAL RECIPROCATING COMPRESSOR LAYOUT

Figure 2 shows the layout of a two stage reciprocating compressor typically for supplying a workshop.

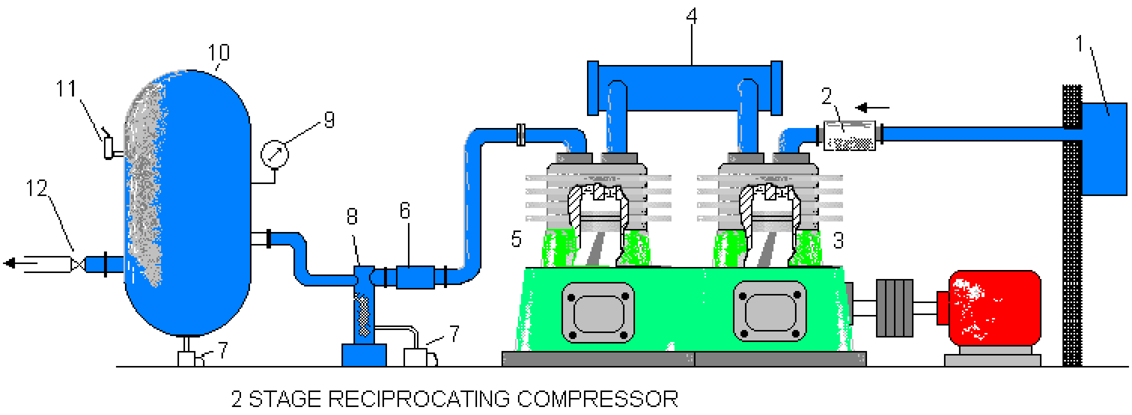

Fig.2

1. Induction box and silencer on outside of building with course screen.
2. Induction filter.
3. Low pressure stage.
4. Intercooler.
5. High pressure stage.
6. Silencer.
7. Drain trap.
8. After cooler
9. Pressure gauge.
10. Air receiver.
11. Safety pressure relief valve.
12. Stop valve

1.3 HAZARDS AND SAFETY

Dangers associated with air compressors are as follows.

- Pressure vessels may rupture.
- Oil leaks may burn or cause other accidents.
- Oil in the compressed air may explode.
- Water in the compressed air may damage equipment.

There are many regulations concerning the use, maintenance and inspection of pressure vessels. Vessels must have the safe working pressure marked on them. They must have a pressure gauge and be fitted with an isolating valve. They must also be fitted with a pressure release valve to prevent overpressure.

In particular, with reciprocating compressors, if water accumulates in the cylinder, it may fill the space so completely that it prevents the piston reaching the end of its travel and cause damage to the piston and head.

Oil in the cylinder can explode during compression. Normally the operating pressure is not high enough to produce the temperature required. However, if the outlet becomes blocked (e.g. the valve sticks or the outlet pipe is closed with an isolating valve), then the danger exists.

Oil or water in the air can also cause damage when supplied to some kinds of tools. For this reason a good installation fully conditions the air to remove water, dirt and oil.

The following is a list of precautions to be taken against fires and explosions.

- Avoid overheating.
- Keep discharge temperatures within the recommended limits.
- Keep the deposit formation to a minimum by using the correct lubricant.
- Ensure efficient filtration of the air. This reduces wear on the valves and pistons and reduces deposits of carbonised particles.
- Avoid over-feeding oil to the cylinders.
- Minimise the carry over of oil between stages.
- Avoid high temperatures and low airflow when idling.
- Keep the coolers in good condition.
- Do not use solvents for cleaning or use anywhere near to an installation as the vapour given off can ignite.
- Do not allow naked flames (e.g. smoking) near to an installation when it is opened.

1.4 FREE AIR DELIVERY

When a gas such as air flows in a pipe, the mass of the air depends upon the pressure and temperature. It would be meaningless to talk about the volume of the air unless the pressure and temperature are considered. For this reason the volume of air is usually stated as FREE AIR DELIVERY or FAD. In other words FAD refers to the volume the air would have if let out of the pipe and returned to atmospheric pressure and temperature.

The FAD is also the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at a higher pressure. Suppose atmospheric conditions are $p_a T_a$ and V_a (the FAD) and the compressed conditions are p, V and T. Applying the gas law we have

$$\frac{pV}{T} = \frac{p_a V_a}{T_a} \qquad V_a = \frac{pVT_a}{Tp_a} = F.A.D$$

2. CYCLE FOR RECIPROCATING COMPRESSOR

2.1 THEORETICAL CYCLE

Figure 3 shows the basic design of a reciprocating compressor.

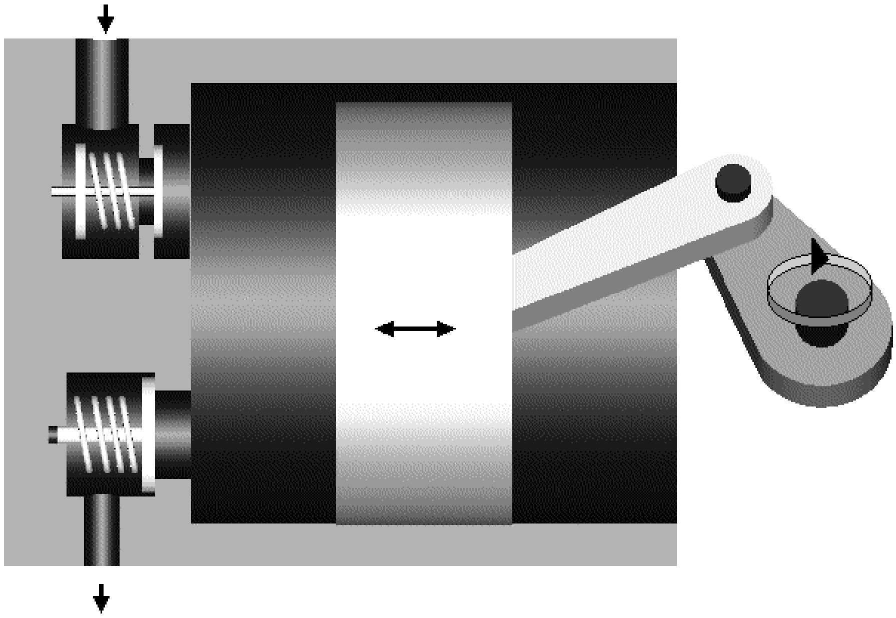

Fig. 3

Figure 4 shows the pressure – volume diagram for an ideal reciprocating compressor.

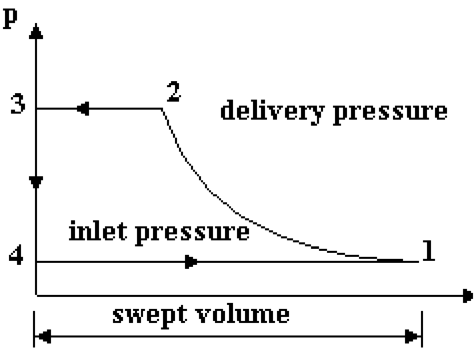

Fig.4

The piston reciprocates drawing in gas, compressing it and expelling it. If the piston expels all the air and there is no restriction at the valves, the pressure - volume cycle is as shown. Gas is induced from 4 to 1 at the inlet pressure. It is then trapped inside the cylinder and compressed according the law $pV^n = C$. At point 2 the pressure reaches the same level as that in the delivery pipe and the outlet valve pops open. Air is then expelled at the delivery pressure. The delivery pressure might rise very slightly during expulsion if the gas is being compacted into a fixed storage volume. This is how pressure builds up from switch on.

2.2 VOLUMETRIC EFFICIENCY

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.

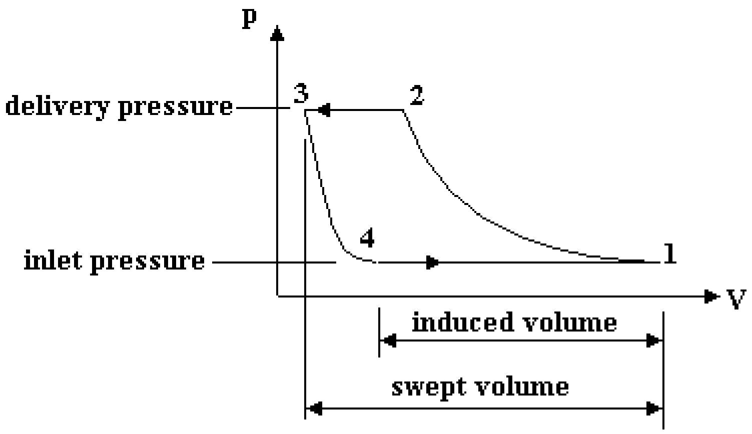

Fig. 5

The volumetric efficiency is defined by the following equation.

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

It may be shown that this reduces to $\eta_{vol} = 1 - c\left[\left(\frac{p_2}{p_1}\right)^{(1/n)} - 1\right]$

This efficiency is made worse if leaks occur past the valves or piston.

The clearance ratio is defined as c = Clearance volume/Swept volume.

Ideally the process 2 to 3 and 4 to 1 are isothermal. That is to say, there is no temperature change during induction and expulsion.

WORKED EXAMPLE No.1

Gas is compressed in a reciprocating compressor from 1 bar to 6 bar. The FAD is 13 dm^3/s. The clearance ratio is 0.05. The expansion part of the cycle follows the law $pV^{1.2}$ =C. The crank speed is 360 rev/min. Calculate the swept volume and the volumetric efficiency.

SOLUTION

Swept Volume = V   Clearance volume = 0.05 V

Consider the expansion from 3 to 4 on the p-V diagram.

p_4 = 1 bar p_3 = 6 bar. $p_3V_3^{1.2} = p_4V_4^{1.2}$

$6(0.05V)^{1.2} = 1(V_4^{1.2})$

V_4 = 0.222V or 22.2%% of V

F.A.D. = 0.013 m^3/s.

V_1 = V + 0.05V = 1.05V

Induced volume = V_1 - V_4 = 1.05V - 0.222V = 0.828V

Induced volume = 0.013 m^3/s

V = 0.013/0.828 = 0.0157 m^3/s

Crank speed = 6 rev/s so the swept volume = 0.0157/6 = 2.62 dm^3.

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

$$\eta_{vol} = \frac{0.828V}{V} = 82.8\,\%$$

WORKED EXAMPLE No.2

Show that the volumetric efficiency of an ideal single stage reciprocating compressor with a clearance ratio is c is given by the expression below.

$$\eta_{vol} = 1 - c\left[\left(\frac{p_H}{p_L}\right)^{(1/n)} - 1\right]$$

p_L is the inlet pressure and p_H the outlet pressure.

SOLUTION

Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$

Clearance volume = V_3

$$c = \frac{V_3}{V_1 - V_3}$$

$$V_1 - V_3 = \frac{V_3}{c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(A)$$

$$\frac{V_1}{V_3} - 1 = \frac{1}{c}$$

$$\frac{V_1}{V_3} = \frac{1}{c} + 1 = \frac{(1+c)}{c} \ldots\ldots\ldots\ldots\ldots\ldots(B)$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} \quad \text{Substitute (A) for the bottom line.}$$

$$\eta_{vol} = \frac{c(V_1 - V_4)}{V_3} = c\left\{\frac{V_1}{V_3} - \frac{V_4}{V_3}\right\} \quad \text{Substitute (B)}$$

$$\eta_{vol} = c\left\{\left(\frac{(1+c)}{c}\right) - \frac{V_4}{V_3}\right\} = 1 + c - c\frac{V_4}{V_3}$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n} = \left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = 1 + c - c\left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\}$$

WORKED EXAMPLE No.3

A single stage reciprocating compressor has a clearance volume of 20 cm^3. The bore and stroke are 100 mm and 80 mm respectively. The compression and expansion processes have a polytropic index of 1.25. The inlet and outlet pressures are 1 and 6 bar respectively.

Determine the volumetric efficiency.

SOLUTION

The swept volume is the product of stroke and bore area.

$$\text{S.V.} = 80 \text{ x } \frac{\pi \text{ x } 100^2}{4} = 628.3 \text{ x } 10^3 \text{mm}^3 \text{or } 628.3 \text{ cm}^3$$

The Clearance volume is 20 cm^3.

$$c = \frac{20}{628.3} = 0.03183$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\} = 1 - 0.03183\left\{\left(\frac{6}{1}\right)^{1/1.25} - 1\right\}$$

$$\eta_{vol} = 1 - 0.03183\{4.192 - 1\} = 1 - 0.1016 = 0.898 \text{ or } 89.8\%$$

2.3 REAL p-V DIAGRAMS

In real compressors the warm cylinder causes a slight temperature rise over the induction from 4 to 1. The gas is restricted by the valves and p_1 is slightly less than p_4. The valves also tend to move so the real cycle looks more like figure 6

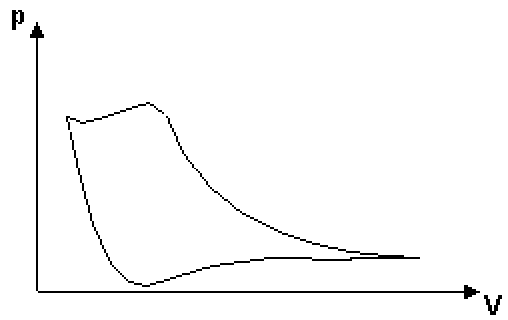

Fig. 6

WORKED EXAMPLE No.4

A single stage reciprocating compressor produces a FAD of 2 dm^3/s at 420 rev/min. The inlet pressure is 1 bar. The polytropic index is 1.2 for the compression and expansion. The outlet pressure is 8 bar. The clearance volume is 10 cm^3.

Determine the volumetric efficiency.

SOLUTION

First find the induced volume. This is the free air drawn in for each revoution.

$$F.A.D.\ \text{per rev.} = \frac{2x60}{420} = 0.2857\ \text{dm}^3/\text{rev}$$

This is the induced volume $V_1 - V_4$

The clearance volume is $V_3 = 10\ \text{cm}^3$ or $0.01\ \text{dm}^3$.

Next we need to find V_4

$$p_3 V_3^n = p_4 V_4^n$$

$$8 \text{ x } 0.01^{1.2} = 1 \text{x} V_4^{1.2} \quad \text{hence} \quad V_4 = 0.0566\ \text{dm}^3$$

$$V_1 = 0.2857 + 0.0566 = 0.3423 \text{m}^3$$

$$\text{Swept volume} = V_1 - V_3 = 0.3323\ \text{dm}^3$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{0.2857}{0.3323} = 0.859 \text{ or } 85.9\%$$

2.4 INDICATED POWER

The indicated work per cycle is the area enclosed by the p - V diagram. The easiest way to find this is by integrating with respect to the pressure axis.

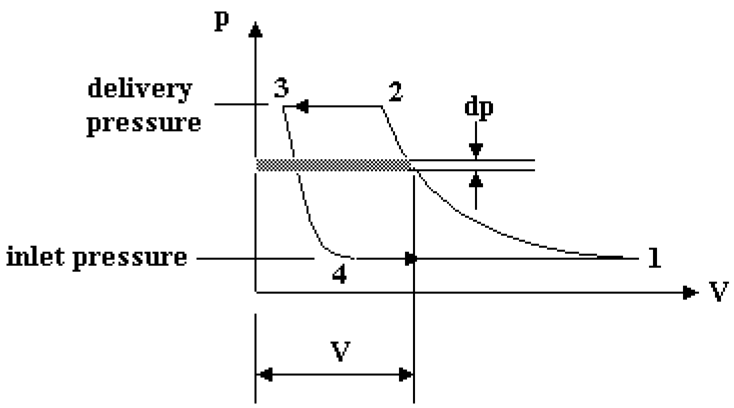

Fig.7

2.4.1 POLYTROPIC PROCESSES

If the processes 1 to 2 and 3 to 4 are polytropic $pV^n = C$. $V = C^{1/n} p^{-1/n}$

The work done is given by $W = \int V dp$

Consider the expression

$$\int V dp = C^{1/n}\int p^{-1/n} dp = \left[\frac{C^{1/n} p^{1-1/n}}{(1-1/n)}\right] = \frac{n}{n-1}\left(C^{1/n} p^{1-1/n}\right)$$

$C = pV^n \qquad c^{\frac{1}{n}} = p^{\frac{1}{n}} V$ substitutein and

$$\int V dp = \left[\frac{nVp^{1/n} p^{1-1/n}}{(n-1)}\right] = \left[\frac{npV}{(n-1)}\right]$$

Between the limits of p2 and p1 this becomes $W_{1\text{-}2} = \frac{n[p_2V_2 - p_1V_1]}{(n-1)}$

Between the limits p4 and p3 this becomes $W_{4\text{-}3} = \frac{n[p_4V_4 - p_3V_3]}{(n-1)}$

Subtract one from the other to find the indicated work.

$$W = \frac{n[p_2V_2 - p_1V_1]}{(n-1)} - \frac{n[p_3V_3 - p_4V_4]}{(n-1)}$$

$$W = \left(\frac{n}{n-1}\right)\left[p_1V_1\left\{\left(\frac{p_2V_2}{p_1V_1}\right)-1\right\} - p_4V_4\left\{\left(\frac{p_3V_3}{p_4V_4}\right)-1\right\}\right]$$

Substitute the relationships $\frac{V_2}{V_1} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$ and $\frac{V_3}{V_{41}} = \left(\frac{p_3}{p_4}\right)^{-\frac{1}{n}}$

$$W = \left(\frac{n}{n-1}\right)\left[p_1V_1\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} - p_4V_4\left\{\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$W = \left(\frac{n}{n-1}\right)\left[p_1V_1\left\{r_p^{\frac{n-1}{n}} - 1\right\} - p_4V_4\left\{r_p^{\frac{n-1}{n}} - 1\right\}\right]$$

$$W = \left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[p_1V_1 - p_4V_4]$$

since $p_1 = p_4$

$$W = p_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[V_1 - V_4]$$

$$W = p_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[\Delta V] \text{ where } \Delta V \text{ is the induced volume.}$$

The corresponding induced mass is $m = \frac{p_1 \Delta V}{RT_1}$

$$W = mRT_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}$$

If the clearance volume is ignored $\Delta V = V_1$.

$$W = \left(\frac{n}{n-1}\right)p_1V_1\left\{r_p^{\frac{n-1}{n}} - 1\right\} \quad \text{or} \quad W = mRT_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}$$

2.4.2 ISOTHERMAL PROCESSES

If the processes 1 to 2 and 3 to 4 are isothermal then $pV = C$. $V = C^1 p^{-1}$

The work done is given by $W = \int V dp$

Consider the expression

$$\int V dp = C\int p^{-1} dp = C \ln p$$

Between the limits of p2 and p1 this becomes $p_1V_1 \ln \frac{p_2}{p_1}$

Between the limits p4 and p3 this becomes $p_4V_4 \ln \frac{p_3}{p_3}$

The indicated work (input) is then

$$W = p_1V_1\ln\left(\frac{p_2}{p_1}\right) - p_4V_4\ln\left(\frac{p_3}{p_4}\right)$$
$$W = p_1V_1\ln(r_p) - p_4V_4\ln(r_p)$$
$$W = \ln(r_p)(p_1V_1 - p_4V_4)$$
since $p_1 = p_2$ we get the following.
$$W = p_1\ln(r_p)(V_1 - V_4) = p_1\ln(r_p)\Delta V$$
$$W = \ln(r_p)mRT_1$$

If the clearance volume is neglected $\Delta V=V_1$

$$W = \ln(r_p)(p_1V_1)$$
$$W = \ln(r_p)mRT_1$$

m is the mass induced and expelled each cycle and W is the indicated work per cycle. The indicated power is found by multiplying W by the strokes per second.

I.P. = W x N where N is the shaft speed in Rev/s

2.5 ISOTHERMAL EFFICIENCY

The minimum indicated power is obtained when the index n is a minimum. The ideal compression is hence isothermal with n=1. The isothermal efficiency is defined as

$$\eta_{iso} = \frac{\text{ISOTHERMAL WORK}}{\text{POLYTROPIC WORK}}$$

$$\eta_{iso} = \frac{p_1 \ln(r_p)(\Delta V)}{p_1\left(\dfrac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[\Delta V]}$$

$$\eta_{iso} = \frac{(n-1)\ln(r_p)}{n\left\{r_p^{\frac{n-1}{n}} - 1\right\}}$$

WORKED EXAMPLE No.5

A single stage reciprocating compressor draws in air at atmospheric pressure of 1.01 bar and delivers it at 9.5 bar. The polytropic index is 1.18 for the compression and expansion.

The swept volume is 1.5 dm^3 and the clearance volume is 0.10 dm^3. The speed is 500 rev/min.

Determine the following.

i. The volumetric efficiency.
ii. The free air delivery.
iii. The indicated power.
iv. The isothermal efficiency.

SOLUTION

$$\text{The clearance ratio } c = \frac{\text{clearance volume}}{\text{swept volume}} = \frac{0.1}{1.5} = 0.0667$$

$$\text{Pressure ratio } r_p = \frac{p_2}{p_1} = \frac{9.5}{1.01} = 9.406$$

$$\eta_v = 1 - c\left[\left(r_p^{\frac{1}{n}}\right) - 1\right]$$

$$\eta_v = 1 - 0.0667\left[\left(9.406^{\frac{1}{1.18}}\right) - 1\right] = 0.621$$

$$\text{Induced Volume} = \eta_v \text{ x Swept Volume} = 0.621 \text{ x } 1.5 = 0.9315 \text{ dm}^3$$

$$\text{FAD per stroke} = \text{Induced volume} = 0.9318 \text{ dm}^3$$

$$FAD \text{ per minute} = 0.9315 \text{ x } 500 = 465.8 \text{ dm}^3/\text{min} \text{ or } 0.4658 \text{ m}^3/s$$

$$\text{Indicated Power} = p_1\left(\frac{n}{n-1}\right)\left[r_p^{\frac{n-1}{n}} - 1\right] \text{ x FAD}$$

$$\text{I.P.} = 1.01 \text{ x } 10^5\left(\frac{1.18}{0.18}\right)\left[9.406^{\frac{0.18}{1.18}} - 1\right] \text{ x } \frac{0.4658}{60}$$

$$I.P. = 5141.3\left[9.406^{0.1525} - 1\right]$$

$$I.P. = 5141.3[1.407 - 1] = 2096 \text{ Watt}$$

$$\eta_{iso} = \frac{(n-1)\ln(r_p)}{n\left\{r_p^{\frac{n-1}{n}} - 1\right\}} = \frac{0.18\ln 9.406}{1.18\left\{9.406^{\frac{0.18}{1.18}} - 1\right\}} = \frac{0.3419}{0.407} = 0.84$$

SELF ASSESSMENT EXERCISE No.1

1. A reciprocating air compressor operates between 1 bar and 8 bar. The clearance volume is 15 cm^3 and the swept volume is 900 cm^3. The index of compression and expansion is 1.21. Calculate the following.

   i. The ideal volumetric efficiency. (92.4%)

   ii. The ideal indicated work per cycle. (208.2 J)

   iii. The Isothermal work per cycle. (172.9 J)

   iv. The Isothermal efficiency. (83%)

2. A reciprocating air compressor following the ideal cycle has a free air delivery of 60 dm^3/s. The clearance ratio is 0.05. The inlet is at atmospheric pressure of 1 bar. The delivery pressure is 7 bar and the compression is polytropic with an index of 1.3. Calculate the following.

   i. The ideal volumetric efficiency. (82.7%)

   ii. The ideal indicated power. (14.74 KW)

   iii. The Isothermal efficiency. (79.2 %)

3. A single stage reciprocating compressor draws in air at atmospheric pressure of 1.0 bar and delivers it at 12 bar. The polytropic index is 1.21 for the compression and expansion. The swept volume is 2.0 dm^3 and the clearance volume is 0.16 dm^3. The speed is 600 rev/min. Determine the following.

   i. The ideal volumetric efficiency. (45.6%)

   ii. The free air delivery. (547.6 dm^3/min)

   iii. The ideal indicated power. (2.83 kW)

   iv. The Isothermal efficiency. (80%)

3 MULTIPLE STAGE COMPRESSORS

The main advantage to compressing the air in stages is that the air may be cooled between each stage and the overall compression is nearer to being isothermal. This reduces the power requirement and allows removal of water from the air. Two stage compressions are common but when very high pressure is required, more stages may be used.

3.1 THE EFFECT OF INTER-COOLING ON THE INDICATED WORK

Consider the p - V diagram for a compressor with two stages.

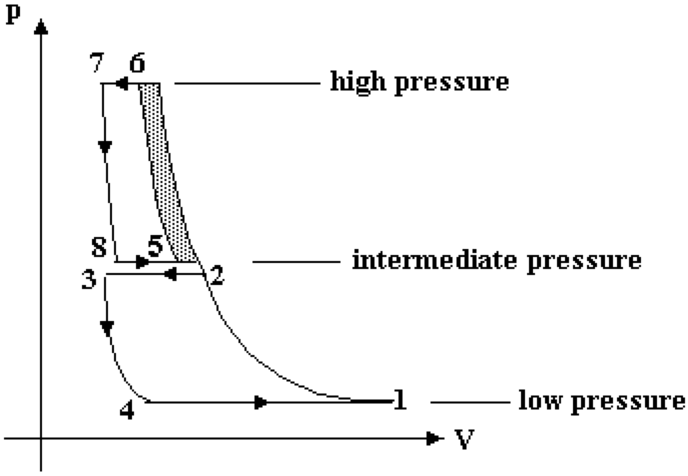

Fig.8

The cycle 1 to 4 is a normal cycle conducted between p_L and p_M. The air is expelled during process 3 to 4 at p_M and constant temperature. The air is then cooled at the intermediate pressure and this causes a contraction in the volume so that the induced volume V_8 to V_5 is smaller than the expelled volume V_2 to V_3. The high pressure cycle is then a normal cycle conducted between p_M and p_H.

The shaded area of the diagram represents the work saved by using the intercooler. The optimal saving is obtained by choosing the correct intermediate pressure. This may be found as follows.

3.2 OPTIMAL INTER-STAGE PRESSURE

$W = W_1 + W_2$ where W_1 is the work done in the low pressure stage and W_2 is the work done in the high pressure stage.

$$W = \frac{mRn(T_2 - T_1)}{(n-1)} + \frac{mRn(T_6 - T_5)}{(n-1)}$$

$$\text{Since } T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(1-1/n)} \; and \; T_6 = T_5\left(\frac{p_6}{p_5}\right)^{(1-1/n)}$$

then assuming the same value of n for each stage

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_2}{p_1}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_6}{p_5}\right\}^{1-(1/n)} - 1\right]$$

Since $p_2 = p_5 = p_m$ and $p_6 = p_H$ and $p_1 = p_L$

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_M}{p_L}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_H}{p_M}\right\}^{1-(1/n)} - 1\right]$$

For a minimum value of W we differentiate with respect to p_M and equate to zero.

$$\frac{dW}{dp_M} = mRT_1 p_L^{(1-n)/n} p_M^{-1/n} - mRT_5 p_H^{(n-1)/n} p_M^{(1-2n)/n}$$

If the intercooler returns the air to the original inlet temperature so that $T_1 = T_5$, then equating to zero reveals that for minimum work

$$\mathbf{p_M = (p_L p_H)^{1/2}}$$

It can further be shown that when this is the case, the work done by both stages is equal.

When K stages are used, the same process reveals that the minimum work is done when the pressure ratio for each stage is $\mathbf{(p_L/p_H)^{1/K}}$

4 ROTARY COMPRESSORS

Figure 9 shows three types of rotary compressors. From left to right - the vane type, centrifugal type and axial flow type. Figure 10 shows a screw compressor.

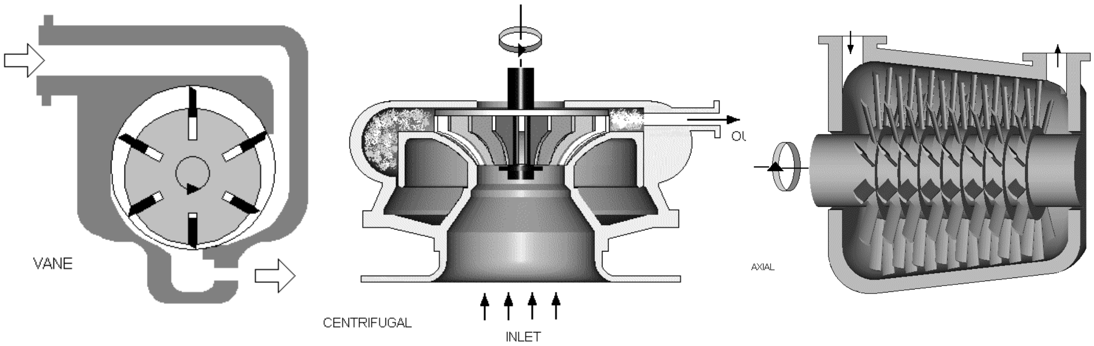

Fig. 9

4.1 VANE TYPE

The vanes fit in slots in the rotor. The rotor is eccentric to the bore of the cylinder. When the rotor is turned, centrifugal force throws the vanes out against the wall of the cylinder. The space between the vanes grows and shrinks as the rotor turns so if inlet and outlet passages are cut in the cylinder at the appropriate point, air is drawn in, squeezed and expelled. This type of compressor is suitable for small portable applications and is relatively cheap. Vane compressors often use oil to lubricate and cool the air and a system similar to that shown for the screw compressor (figure 10) is used.

4.2 CENTRIFUGAL TYPE

The rotor has a set of vanes shaped as shown in the diagram. When the rotor spins, the air between the vanes is thrown outwards by centrifugal force and gathered inside the casing. As the air slows down in the casing, the kinetic energy is converted into pressure. The shape of the casing is important and is basically an eccentric passage surrounding the rotor edge. Fresh air is drawn in from the front of the rotor.

These types of compressor are suitable for medium and large flow rates. Pressure up to 25 bar may be obtained by using several stages or using them as the second stage of an axial flow type. Very large compressors are used to supply large volumes of air at low pressure to combustion chambers and blast furnaces.

4.3 AXIAL FLOW TYPE

The axial flow compressor is basically many rows of fan blades arranged along the axis. Each row gives the air kinetic energy. Not shown on the diagram are fixed vanes in between each row that slows the air down again and raises the pressure. In this way the pressure gradually increases as the air flows along the axis from inlet to outlet. Often a centrifugal stage is situated at the end in order to give it a boost and change the direction of flow to the side. Typical industrial compressors can provide 70 m^3/s at 15 bar. They are not suitable for small flow rates (below 15 m^3/s).

Axial flow compressors are commonly used in jet and gas turbine engines but they have many applications where large flow rates and medium to high pressure are required.

4.4 SCREW TYPES

Two rotors have helical lobes cut on them in such a way that when they mesh and rotate in opposite directions, air is drawn along the face of the lobes from input to output. Oil is used liberally to seal the air. The oil also acts as a coolant and the diagram shows how the oil and air are separated and then cooled in a radiator. The oil is re-circulated.

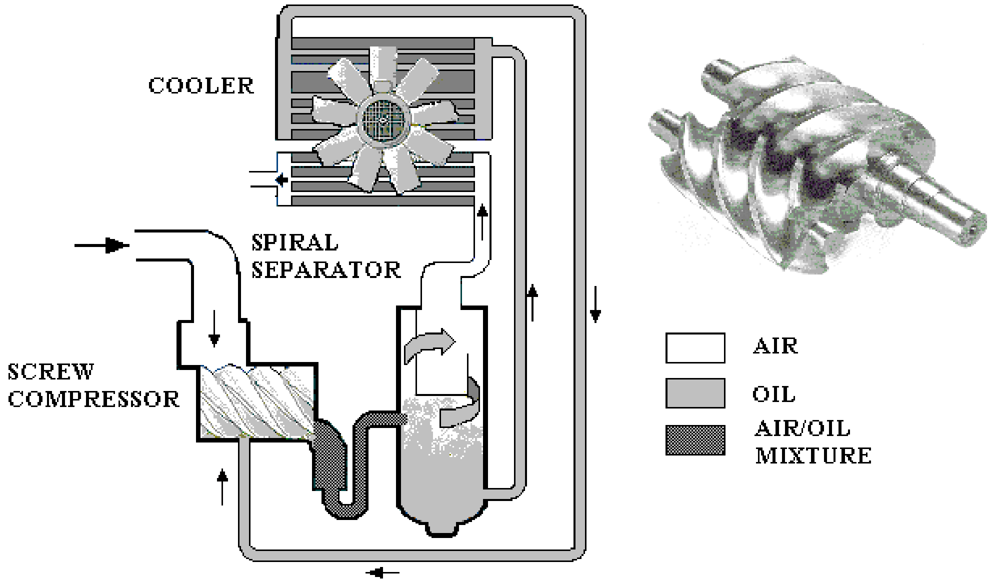

Fig. 10

4.5 LOBE TYPES

Lobe compressors are commonly used as superchargers on large engines. Figure 11 shows the basic design. Air is carried around between the lobes and the outer wall and is expelled when the lobes come together. These compressors are not suitable for high pressures but flow rates around 10 000 m^3/hr are achievable.

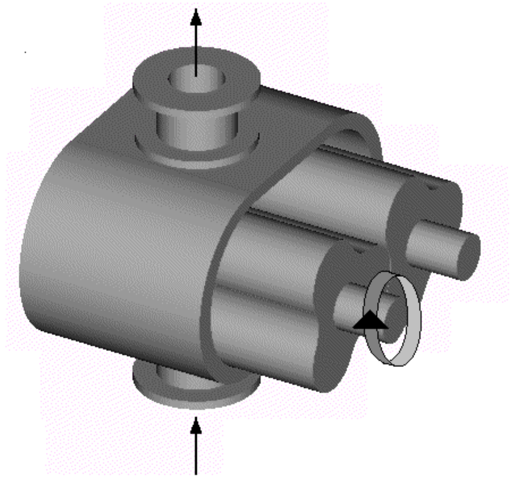

Figure 11

5. COOLERS

Coolers are used for the following reasons.

- To reduce the indicated work in multiple stage compressors.
- To condense water from the air.

For reciprocating compressors, the cooling takes place in the following places.

- The cylinder.
- Between stages.
- After final compression.

5.1 CYLINDER COOLING

The cylinders may be cooled with air and are designed with cooling fins on the outside. Circulating water through a cooling jacket produces more effective cooling.

5.2 INTER COOLING

These are usually simple heat exchangers with water-cooling. Drain traps are fitted to them to remove the water that condenses out of the air. Indicated work is saved as explained in section 3.

Assuming that no heat is lost to the surroundings, the 1st. Law may be applied to the air and water side to produce the following heat balance.

$\Phi = m_a\ c_a\ \Delta T_a = m_w\ c_w\ \Delta T_w$

m_a is the mass flow rate of air.
m_w is the mass flow rate of water.
ΔT_a is the temperature change of the air.
ΔT_w is the temperature change of the water.
c_a is the specific heat capacity of air.
c_w is the specific heat capacity of water.

5.3 AFTER COOLING

The only purpose of an after cooler is to cool the air to around ambient conditions and condense water from the air. This is usually another water-cooled heat exchanger and the same heat balance may be applied.

WORKED EXAMPLE No.6

A single acting reciprocating compressor runs at 360 rev/min and takes in air at 1 bar and 15^oC and compresses it in 3 stages to 64 bar. The free air delivery is 0.0566 m^3/s. There is an intercooler between each stage, which returns the air to 15^oC. Each stage has one piston with a stoke of 100 mm. Calculate the following.

i. The ideal pressure between each stage.
ii The ideal indicated power per stage.
iii The heat rejected from each cylinder.
iv The heat rejected from each intercooler.
v The isothermal efficiency.
vi The swept volume of each stage.
vii The bore of each cylinder.

Ignore leakage and the effect of the clearance volume. The index of compression is 1.3 for all stages.

SOLUTION

Pressure ratio for each stage = $(64/1)^{1/3} = 4$

Hence the pressure after stage 1 is 1 x 4 = 4 bar.

The pressure after the second stage is 4 x 4 = 16 bar

The final pressure is 16 x 4 = 64 bar.

$T_1 = 288$ K. $m = p_1V/RT_1 = 1 \times 10^5 \times 0.0566/(287 \times 288) = 0.06847$ kg/s

$T_2 = 288(4)^{0.3/1.3} = 396.5$ K

The indicated power for each stage is the same so it will be calculated for the 1st. stage.

I.P. = $mRn\, T_1\{(p_2/p_1)^{(1-1/n)} - 1\}/(n-1)$ since m is the mass compressed.

I.P. = $0.06847 \times 287 \times 1.3 \times 288\{4^{0.3/1.3} - 1\}/(1.3-1) = 9.246$ kW

CYLINDER COOLING

Consider the energy balance over the first stage.

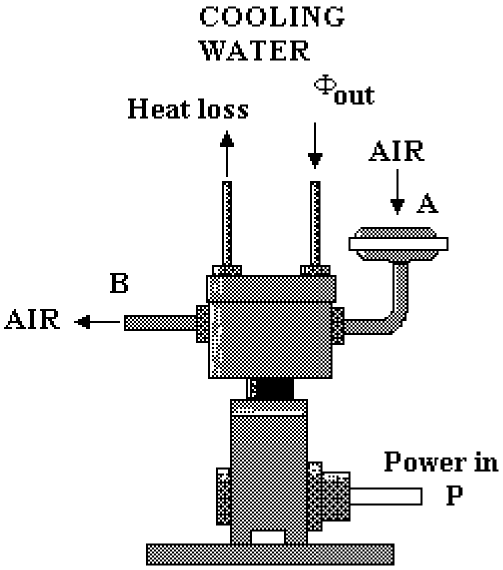

Fig. 12

Balancing the energy we have

$H_A + P_{(in)} = H_B + \Phi\ (out)$

$\Phi\ _{(out)} = P_{(in)} - mC_p(T_B - T_A)$

$\Phi\ _{(out)} = 9.246 - 0.06847 \times 1.005\ (396.5 - 288)$

$\Phi\ _{(out)} = 1.78$ kW (rejected from each cylinder)

INTERCOOLER

Now consider the Intercooler. No work is done and the temperature is cooled from T_2 to T_5.

$\Phi\ _{(out)} = mc_p(T_C - T_D) = 0.0687 \times 1.005\ (396.5 - 288) = 7.49$ kW

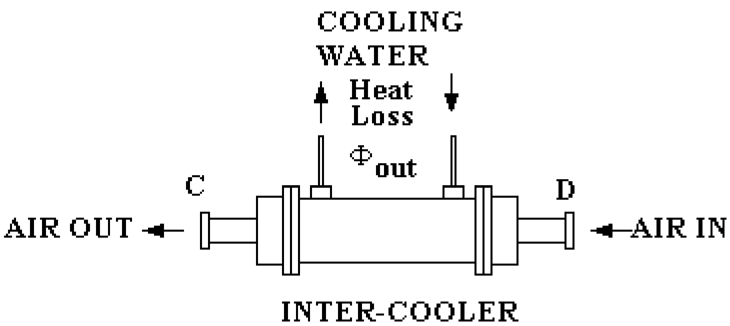

Fig.13

ISOTHERMAL EFFICIENCY

The ideal isothermal power = $mRT_1\ln(p_1/p_2)$ per stage.

P(isothermal) = 0.06847 x 287 x 288 ln4 = 7.846 kW

$\eta_{(iso)}$ = 7.846/9.246 = 84.9 %

SWEPT VOLUMES

Consider the first stage.

The F.A.D. is 0.0566 m^3/s. In the ideal case where the air is drawn in at constant temperature and pressure from the atmosphere, the FAD is given by

FAD = Swept Volume x Speed   and the speed is 6 rev/s

If the clearance volume is ignored, the FAD gives the swept volume.

S.V. (1st. Stage) = 0.0566/6 = 0.00943 m^3

S.V. = Bore Area x Stroke

$0.00943 = \pi D^2/4 \times 0.1 \quad D_1 = 0.347$ m.

Now consider the second stage. The air is returned to atmospheric temperature at inlet with a pressure of 4 bar. The volume drawn is hence 1/4 of the original FAD.

The swept volume of the second stage is hence 0.00943/4 = 0.00236 m^3.

$0.00236 = \pi D^2/4 \times 0.1$ hence $D_2 = 0.173$ m

By the same reasoning the swept volume of the third stage is

SV(3rd stage) = 0.00943/16 = 0.000589 m^3.

$0.000589 = \pi D^2/4 \times 0.1 \quad D_3 = 0.0866$ m

SELF ASSESSMENT EXERCISE No.2

1. A two stage compressor draws in 8 m^3/min from atmosphere at 15°C and 1.013 bar. The air is compressed with an index of compression of 1.27 to the inter-stage pressure of 6 bar. The intercooler must cool the air back to 15 °C.

   Look up the appropriate mean value of c_p in your tables.

   Calculate the heat that must be extracted from the air in kW. (21.8 kW)

2. A single acting 2 stage compressor draws in 8.5 m^3/min of free air and compresses it to 40 bar. The compressor runs at 300 rev/min. The atmospheric conditions are 1.013 bar and 15°C. There is an intercooler between stages that cools the air back to 15°C. The polytropic index for all compressions is 1.3. The volumetric efficiency is 90% for the low pressure stage and 85% for the high pressure stage. Calculate the following.

   i. The intermediate pressure for minimum indicated work. (6.37 bar)
   ii. The theoretical indicated power for each stage. (32.8 kW)
   iii. The heat rejected in each cylinder. (6.3 kW for both)
   iv. The heat rejected by the intercooler. (26.5 kW)
   v. The swept volumes of both stages. (31.1 and 5.24 dm^3)

3. A two stage reciprocating air compressor works between pressure limits of 1 and 20 bar. The inlet temperature is 15°C and the polytropic index is 1.3. Inter-cooling between stages reduces the air temperature back to 15°C. Both stages have the same stroke. Neglect the effect of the clearance volume.

   Calculate he following.

   i. The free air delivery for each kWh of indicated work. (10.06 m^3)
   ii. The mass of air that can be compressed for each kW h of indicated work. (12.17 kg)
   iii. The ratio of the cylinder diameters. (2.115)

   (Note 1 kW h is 3.6 MJ)

4. A single acting compressor draws in atmospheric air at 1 bar and 15 ^oC. The air is compressed in two stages to 9 bar. The compressor runs at 600 rev/min.

The installation has an inter-cooler that reduces the air temperature to 30 ^oC at inlet to the second stage. The polytropic index for all compressions is 1.28.

The clearance volume for each stage is 4% of the swept volume. The low pressure cylinder is 300 mm diameter and the stroke for both stages is 160 mm.

Calculate the following.

i. The optimal inter-stage pressure. (3 bar)
ii. The volumetric efficiency of each stage. (98.9)%
iii. The free air delivery. (7.77 m^3/min)
iv. The induced volume of the high pressure stage. (4.545 dm^3)
v. The diameter of the high pressure cylinder. (191.2 mm)
vi. The indicated power for each stage. (16 and 16.93 kW)