

THERMODYNAMICS
TUTORIAL 5
HEAT ENGINES AND THE 2nd LAW of THERMODYNAMICS

This tutorial is set at QCF Level 4 to 5

On completion you should be able to:

- Explain the thermodynamic concepts of a heat engine
- Explain the importance of the second law of thermodynamics in engineering process analysis
- Define and evaluate the entropy substances
- Evaluate changes of entropy in reversible and irreversible thermodynamic processes.
- Apply what you have learned to Thermodynamic processes - constant pressure; constant volume; isothermal; adiabatic; polytropic.

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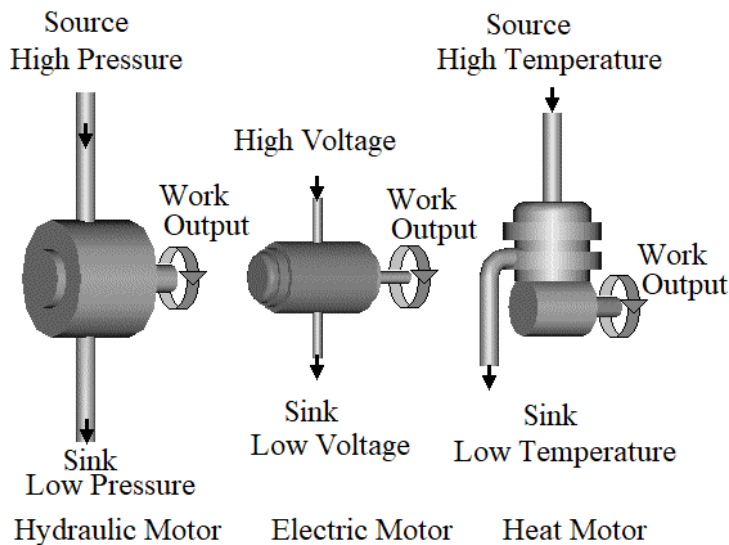
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1. Introduction

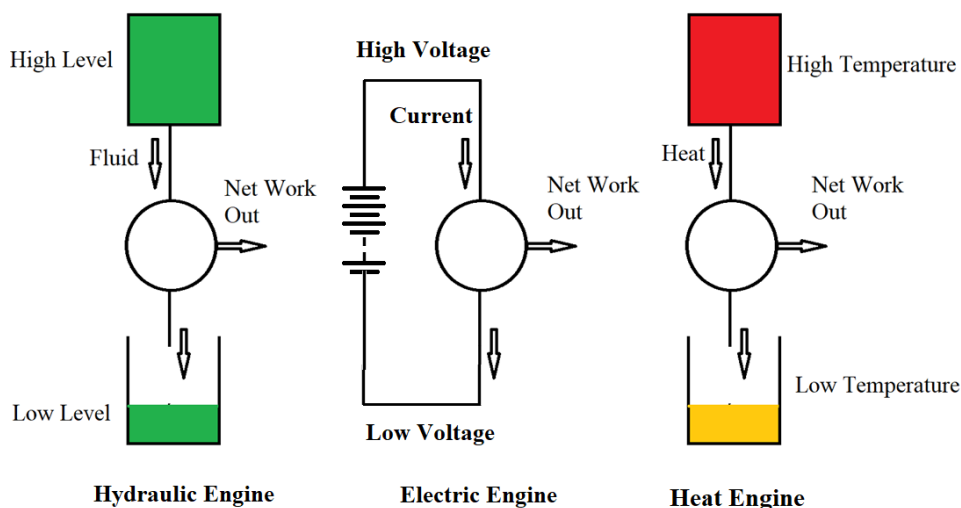
This tutorial introduces you to some very important fundamental principles in Thermodynamics. The second Law of Thermodynamics has profound consequences in the wider field of physics but here we are concerned with its meaning in the study of heat engines and the important property of Entropy that has not been discussed before in these tutorials. Let us start by considering an idealised model of a heat engine.

2. Heat Engines and Comparison to Other Engines

There are various forms of engines. For example a hydraulic engine converts gravitational energy (also known as pressure energy, flow energy or potential energy) into work or power. An electric motor or engine converts electrical energy into work or power. A heat engine is one that converts heat into work or power.



The diagram below illustrates an idealised model comparison.



2.1 Efficiency

The Efficiency of an engine is defined as

$$\text{Efficiency } \eta = \frac{W_{\text{out}}}{\text{Energy Supplied}}$$

Note the symbol is the Greek letter Eta η Now let's compare the efficiency formulae for these engines

Hydraulic Motor

Fluid power is transported by the flow Q m³/s. The energy contained in a volume Q m³ of liquid at a pressure p is the flow energy given by the expression pQ . The hydraulic motor requires a source of liquid at a high pressure p_1 and exhausts at a lower pressure p_2 . The energy supplied is p_1Q and some of this is converted into work. The energy in the low pressure liquid is p_2Q . For a perfect motor with no losses due to friction, the law of energy conservation gives the work output and efficiency as follows.

$$W_{\text{out}} = p_1Q - p_2Q = Q(p_1 - p_2)$$

$$\text{Efficiency } \eta = \frac{W_{\text{out}}}{\text{Energy Supplied}} = \frac{Q(p_1 - p_2)}{p_1Q} = \frac{(p_1 - p_2)}{p_1} = 1 - \frac{p_2}{p_1}$$

Electric Motor

Electric power is transported by the current. Electrical energy is the product of the charge Q Coulombs and the electric potential V Volts. The energy input at a high voltage is V_1Q and the energy exhausted at low voltage is V_2Q . For a perfect motor with no losses due to friction, the work output and efficiency are found from the law of energy conservation as follows.

$$W_{\text{out}} = V_1Q - V_2Q = Q(V_1 - V_2)$$

$$\text{Efficiency } \eta = \frac{W_{\text{out}}}{\text{Energy Supplied}} = \frac{Q(V_1 - V_2)}{V_1Q} = \frac{(V_1 - V_2)}{V_1} = 1 - \frac{V_2}{V_1}$$

Heat Motor

Temperature is by analogy the equivalent of pressure and electric potential. In order to complete the analogy, we need something that is equivalent to volume and electric charge that transports the energy. It is not difficult to visualise a volume of liquid flowing through a hydraulic motor. It is not impossible to visualise a flow of electrons bearing electric charge through an electric motor. It is impossible to visualise something flowing through our ideal heat engine that transports pure heat but the analogy tells us there must be something so let us suppose a new property called **Entropy** and give it a symbol S . Entropy must have units of energy per degree of temperature or Joules per Kelvin. Entropy is dealt with more fully later on.

The energy supplied at temperature T_1 is T_1S and the energy exhausted is T_2S . For a perfect motor with no losses due to friction, the law of energy conservation gives the work output and efficiency as follows.

$$W_{\text{out}} = T_1S - T_2S = S(T_1 - T_2)$$

$$\text{Efficiency } \eta = \frac{W_{\text{out}}}{\text{Energy Supplied}} = \frac{S(T_1 - T_2)}{T_1S} = \frac{(T_1 - T_2)}{T_1} = 1 - \frac{T_2}{T_1}$$

Note that if p_2 or V_2 or T_2 are zero we theoretically have an efficiency of 1 or 100%

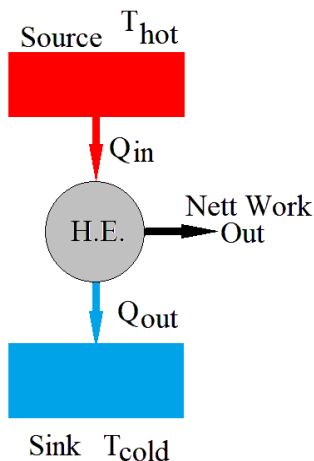
In a perfect motor the energy conversion process is 100% efficient but we may not have converted all the energy supplied into work and unused energy may be wasted into the sink. In the case of the electric motor, the lowest value for V_2 (so far as we know) is ground voltage zero, so theoretically we can obtain 100% efficiency by exhausting the electric charge with no residual energy.

In the case of the hydraulic motor, the lowest pressure we can exhaust to is atmospheric pressure so we always waste some energy in the exhausted liquid as it is impossible to exhaust at absolute zero pressure.

In the case of the heat engine or motor, the lowest temperature to which we can exhaust is ambient conditions, typically 300 K, so there is a lot of residual energy in the exhaust. Only by exhausting to absolute zero, can we extract all the energy.

2.2. Model Heat Engine

A model heat engine is usually represented by the following diagram. (Note that the word engine is usually preferred to motor).



The energy transfer from the hot source is Q_{in} Joules.

The energy transfer rate from the hot source is Φ_{in} Watts.

The energy transfer to the cold sink is Q_{out} Joules and the energy transfer rate is Φ_{out} Watts.

The work output is W Joules and the power output is P Watts.

By considering the total conservation of energy, it follows that the energy converted into work must be

$$W = Q_{in} - Q_{out} \text{ Joules or } P = \Phi_{in} - \Phi_{out} \text{ Watts}$$

The efficiency of any machine is the ratio Output/Input so the thermal efficiency of a heat engine may be developed as follows.

$$\text{Efficiency } \eta_{th} = \frac{W}{Q_{in}} \quad W = Q_{in} - Q_{out} \quad \eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

In terms of energy transfer rates is Watts this is

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}}$$

It follows from our analogy that $Q_{in} = ST_1$ and $Q_{out} = ST_2$ and confirms

$$\eta = 1 - \frac{T_2}{T_1}$$

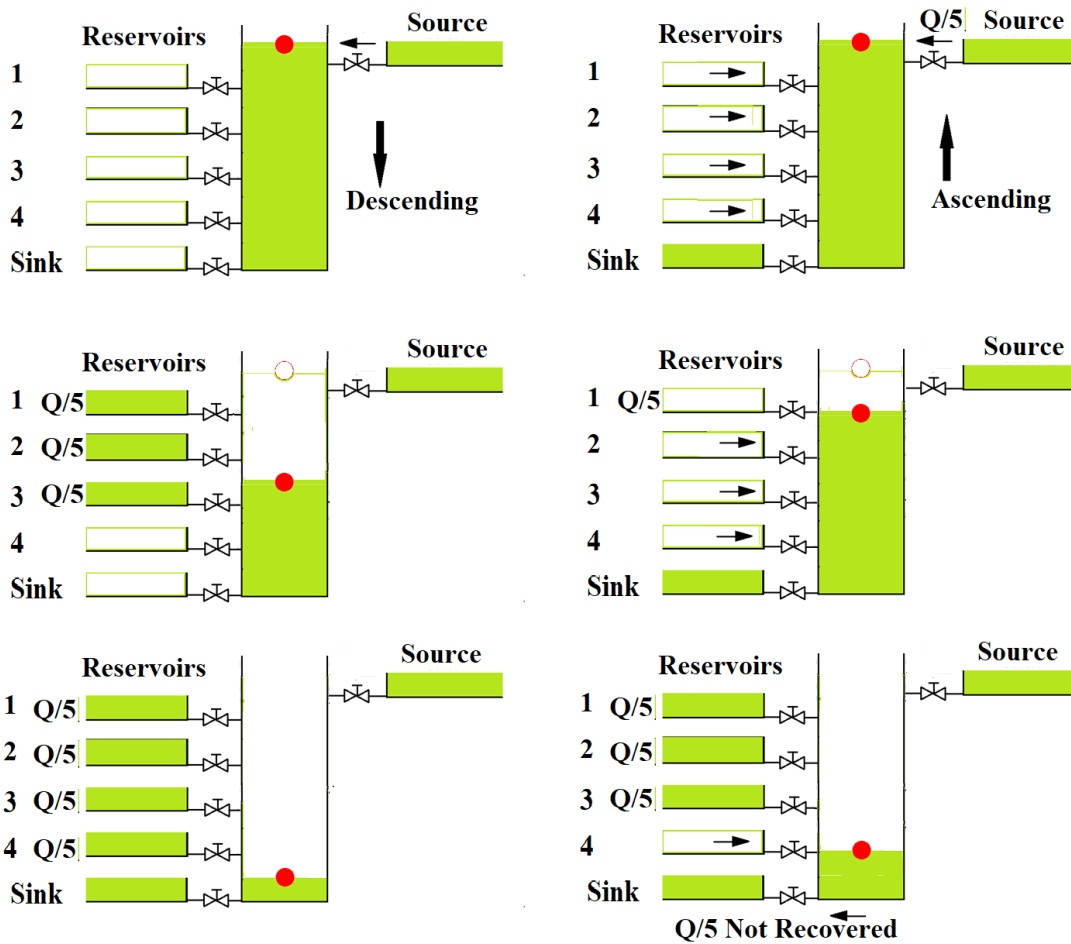
SELF ASSESSMENT EXERCISE No. 1

1. A heat engine is supplied with 60 MW of energy and produces 20 MW of power. What is the thermal efficiency and the heat lost?
2. A heat engine is supplied with 40 kJ of energy that it converts into work with 25% efficiency. What is the work output and the heat lost?

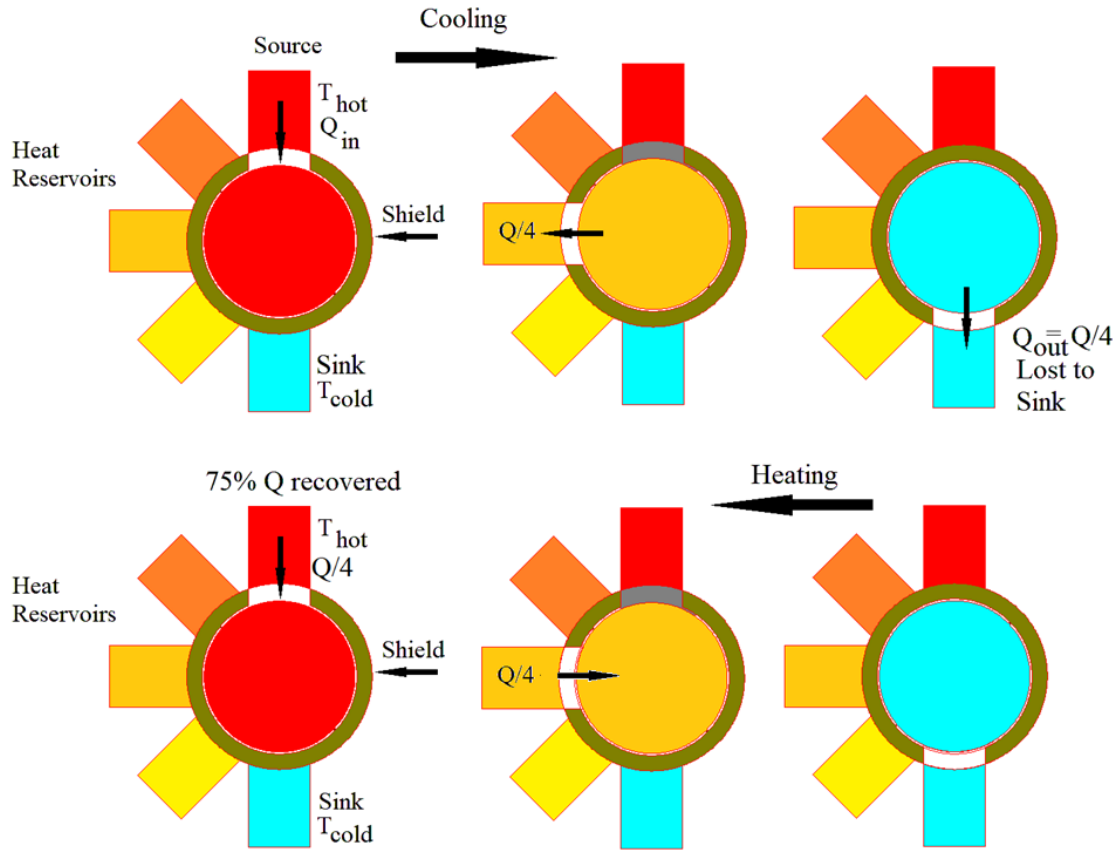
3. Reversibility

Let's look at a hydraulic analogy of a reversible process. A simple version of this is used to save water on a lock for lowering and raising boats from one level to another.

Starting with an empty lock, it is filled to the top from the source with a quantity of water Q . The level in the lock is then lowered by emptying it in stages discharging equal amounts into the reservoirs (holding tanks) and the last water is drained into the sink and lost. In order to refill the lock, the reservoirs are opened one at a time putting back the water stored but this does not raise the level back to top and water has to be added from the source to replace that lost in the sink. In the example illustrated there are 5 stages and you can see that 80% of the water, and hence the energy is recovered so the process is 80% efficient and 80% reversible.



Now consider the equivalent thermodynamic system. In the illustration a central body is exposed to a source of heat at the hottest temperature T_{hot} and heated up to that temperature. The body is then cooled in stages by rotating the shield and allowing it to give up heat to thermal storage reservoirs at lower temperatures and finally it is cooled to the lowest temperature T_{cold} when it reaches the sink. In this example $\frac{1}{4}$ of the heat supplied ($\frac{1}{4} Q_{in}$) is lost to the sink (Q_{out}). If the process is reversed heat is given back to the central body in stages raising the temperature but in order to get back to T_{hot} $Q/4$ must be added from the source. 75% of the heat is recovered and the process is 75% efficient and 75% reversible.

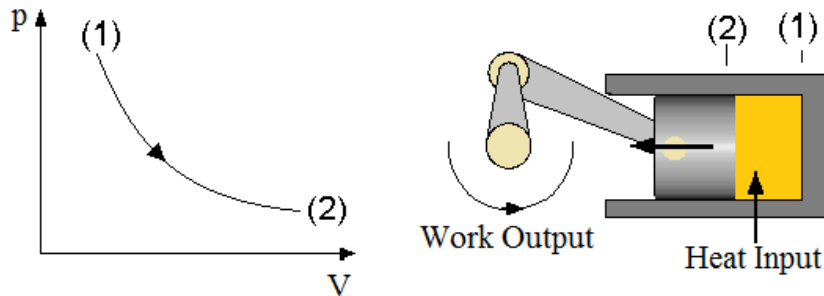


You might realise that in both examples the efficiency and reversibility of the process depend on the number of intermediate stages. More and smaller steps would give a smaller temperature drop each time and would increase the efficiency. One step would be zero efficient and totally irreversible. An infinite number of steps would be 100% efficient and fully reversible. In this case the temperature difference would be zero for each step and this is impractical.

If we could transfer heat back and forth at constant temperature then the process would be *isothermal*. This can happen when a substance is both liquid and vapour and adding heat causes some of the liquid to evaporate or removing heat causes some of the vapour to condense. This does happen at constant temperature (the boiling point).

A more practical way of transferring heat isothermally to or from a gas is to combine it with doing work. If we heat a gas the temperature will change and the gas will expand. If we let the expansion do work say by moving a piston as illustrated, then any temperature change is reduced by extracting work energy and in this way the expansion can be isothermal obeying the gas law $pV = C$.

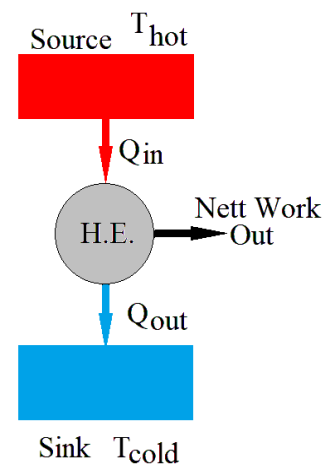
The reverse of this is an isothermal compression.



3. The Carnot Principle

Sadi Carnot (born 1796) reasoned that if an engine used isothermal expansions and compressions, then the efficiency of the engine would be the maximum possible.

Let's now consider that such an engine works between a source at temperature T_h and a sink at temperature T_c . Heat passes to the engine isothermally and work is extracted from the engine. The remainder of the energy is rejected to the sink. This is idealised as shown.



The first Law of Thermodynamics tells us that:

$$Q_{in} = W_{out} + Q_{out} \quad \text{and} \quad W_{out} = Q_{in} - Q_{out}$$

The efficiency of the engine at converting heat into work is the thermal efficiency:

$$\eta_{th} = \frac{W_{out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

The heat transfer is directly proportional to the temperatures so Carnot reasoned that the maximum possible efficiency of a heat engine is:

$$\eta_{th} = \eta_{carnot} = 1 - \frac{T_c}{T_h}$$

These led physicists to realise that there must be an absolute scale of temperature. Clearly the value of the above equation is different if $^{\circ}\text{C}$ is used instead of the absolute temperature in Kelvins.

4. The Second Law of Thermodynamics

This is not a law that can simply be written down as an equation. One of the main outcomes of the law is:

No heat engine can have a thermal efficiency of 100%

In fact they come nowhere near to 100%

The forgoing work raises many questions and leads us to consider a new fluid property called Entropy. Another outcome of the law is:

Every action increases the entropy of the universe

This needs a lot more explaining and even involves the idea of the direction of time but we will only look at the use in thermodynamics.

The idea that heat can only be transferred back and forth between two bodies with 100% efficiency and be 100% reversible is clearly impossible to achieve.

We understand that no engine can give out more work than the heat it receives and that no engine can function without losing some heat to the sink. In other words no heat engine can exceed nor equal 100% efficiency. The best possible is that given by:

$$\eta_{th} = 1 - \frac{T_c}{T_h}$$

The 2nd Law also leads to concepts of absolute temperature and Entropy. We will now see what is involved in devising an engine cycle to convert heat into a nett amount of work.

WORKED EXAMPLE No. 1

Calculate the efficiency for a theoretical Carnot engine that is supplied with heat at 1 200°C and rejects heat to a sink at 20°C

SOLUTION

$$\eta_{carnot} = 1 - \frac{T_c}{T_h} = 1 - \frac{20 + 273}{1200 + 273} = 1 - \frac{293}{1473} = 0.8$$

The efficiency is 80%

WORKED EXAMPLE No. 2

A heat engine draws heat from a combustion chamber at 300°C and exhausts to atmosphere at 10°C. What is the maximum possible thermal efficiency that could be achieved theoretically?

SOLUTION

The maximum efficiency possible is the Carnot efficiency. Remember to use absolute temperatures.

$$\eta_{carnot} = 1 - \frac{T_c}{T_h} = 1 - \frac{10 + 273}{300 + 273} = 1 - \frac{283}{573} = 0.505$$

The maximum possible efficiency is 50.5%

SELF ASSESSMENT EXERCISE No. 2

1. A heat engine works between temperatures of 1 100°C and 120°C.
It is claimed that it has a thermal efficiency of 75%. Is this possible?
(Answer the maximum efficiency cannot exceed 71%)
2. Calculate the Carnot efficiency for an Engine working between 1 200°C and 200° C.
(Answer 67.9%)

7. Entropy

7.1 Definition

You should already be familiar with the theory of work laws in closed systems. You should know that the area under a pressure-volume diagram for a reversible expansion or compression gives the work done during the process. This might take us to the idea that there is a property which can be plotted against temperature such that the area under the graph gives us the heat transfer Q . This property is *entropy*. The symbol for entropy is S and for specific entropy s .

Consider a $p - V$ and $T - S$ graph for a reversible expansion.

From the $p - V$ graph we have

$$W = \int p \, dV$$

From the $T - s$ graph we have

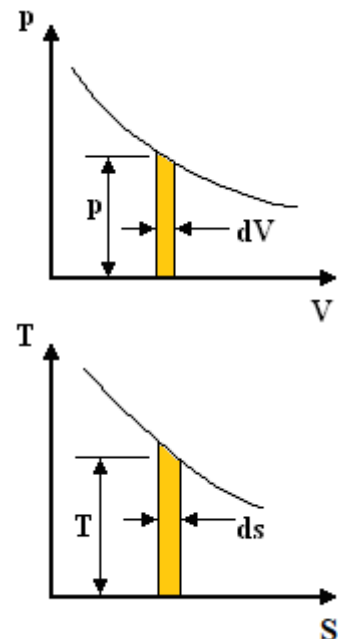
$$Q = \int T \, dS$$

This is the way entropy was developed for thermodynamics and from the above we get the definition:

$$dS = \frac{dQ}{T}$$

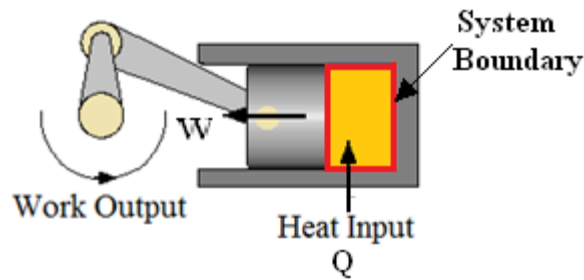
The units of entropy are J/K and for specific entropy $J/kg \, K$

It should be pointed out that there are other definitions of entropy but this one is the most meaningful for thermodynamics. A suitable integration will enable you to solve the entropy change for a fluid process.



8. Entropy Changes for a Perfect Gas in Closed Systems

Consider a closed system expansion of a fluid against a piston with heat and work transfer taking place.



Applying the non-flow energy equation we have

$$Q + W = \Delta U$$

Differentiating we have

$$dQ + dW = dU$$

Since $dQ = TdS$ and $dW = -pdV$ then

$$TdS - pdV = dU$$

$$TdS = dU + pdV$$

This expression is the starting point for all derivations of entropy changes for any fluid (gas or vapour) in closed systems. It is normal to use specific properties so the equation becomes

$$T ds = du + p dv$$

From the gas law $p v = RT$ we may substitute for p and the equation becomes

$$T ds = du + R T \frac{dv}{v}$$

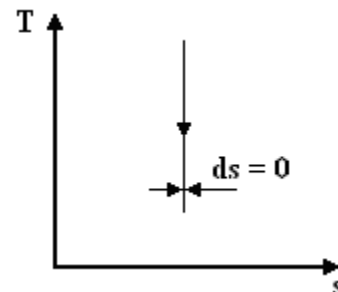
Rearranging and substituting $du = c_v dT$ we have

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v} \dots \dots (1)$$

s is specific entropy v is specific volume. u is specific internal energy.

8.1 Isentropic Processes

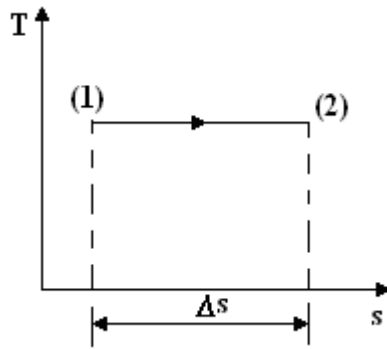
The word *Isentropic* means constant entropy and this is a very important thermodynamic process. It occurs in particular when a process is reversible and adiabatic. This means that there is no heat transfer to or from the fluid and no internal heat generation due to friction. In such a process it follows that if dQ is zero then dS must be zero. Since there is no area under the T - S graph, then the graph must be a vertical line as shown.



There are other cases where the entropy is constant. For example, if there is friction in the process generating heat but this is lost through cooling, then the nett result is zero heat transfer and constant entropy. You do not need to be concerned about this at this stage.

Entropy is used in the solution of gas and vapour problems. We should now look at practical applications of this property and study the entropy changes which occur in closed and steady flow systems for perfect gases and vapours.

8.2 Isothermal Process



In this case temperature is constant. Starting with equation (1)

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

Since $dT = 0$ then

$$\Delta s = s_2 - s_1 = R \ln \frac{v_2}{v_1}$$

A quicker alternative derivation for those familiar with the work laws is:

$$Q + W = \Delta U \quad \text{but } \Delta U = 0 \text{ so } Q = -W$$

$$W = -mRT \ln \frac{V_2}{V_1} \quad Q = \int T ds = T \Delta S$$

T is constant so

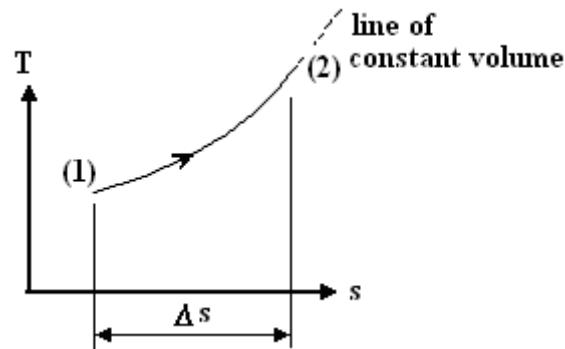
$$\Delta S = \frac{Q}{T} = -\frac{W}{T} = m R \ln \frac{V_2}{V_1}$$

$$\Delta S = m R \ln \frac{V_2}{V_1}$$

$$\Delta s = R \ln \frac{V_2}{V_1} \quad \text{and since } \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

$$\Delta s = R \ln \frac{p_1}{p_2}$$

8.3 Constant Volume Process



Starting again with equation (1) we have

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

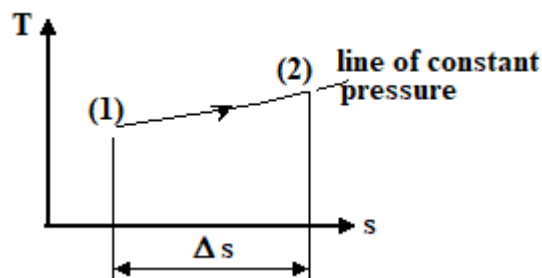
In this case $dv = 0$ so

$$ds = c_v \frac{dT}{T}$$

Integrating between limits (1) and (2)

$$\Delta s = c_v \ln \frac{T_2}{T_1}$$

8.4 Constant Pressure Process



Starting again with equation (1) we have

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

In this case we integrate and obtain

$$\Delta s = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

For a constant pressure process,

$$\frac{v}{T} = \text{constant} \quad \text{and} \quad \frac{v_2}{v_1} = \frac{T_2}{T_1}$$

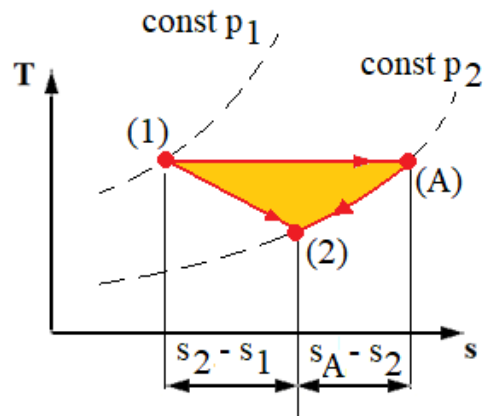
The expression becomes

$$\Delta s = c_v \ln \frac{T_2}{T_1} + R \ln \frac{T_2}{T_1} = (c_v + R) \ln \frac{T_2}{T_1} = c_p \ln \frac{T_2}{T_1}$$

It was shown in an earlier tutorial that $R = c_p - c_v$

8.5 Polytropic Process

This is the most difficult of all the derivations here. Since all the forgoing are particular examples of the polytropic process then the resulting formula should apply to them also.



The polytropic expansion is from (1) to (2) on the T- s diagram with different pressures, volumes and temperatures at the two points. The derivation is done in two stages by supposing the change takes place first at constant temperature from (1) to (A) and then at constant pressure from (A) to (2). You could use a constant volume process instead of constant pressure if you wish.

$$s_2 - s_1 = (s_A - s_1) - (s_A - s_2) = (s_A - s_1) + (s_2 - s_A)$$

For the constant temperature process

$$s_A - s_1 = R \ln \frac{p_1}{p_A}$$

For the constant pressure process

$$s_2 - s_A = c_p \ln \frac{T_2}{T_A}$$

Hence

$$\Delta s = s_2 - s_1 = R \ln \frac{p_1}{p_A} + c_p \ln \frac{T_2}{T_A}$$

Since $p_A = p_2$ and $T_A = T_1$ then

$$\Delta s = s_2 - s_1 = R \ln \frac{p_1}{p_2} + c_p \ln \frac{T_2}{T_1}$$

Divide through by R

$$\frac{\Delta s}{R} = \ln \frac{p_1}{p_2} + \frac{c_p}{R} \ln \frac{T_2}{T_1}$$

From the relationship between c_p , c_v , R and γ we have

$$\frac{c_p}{R} = \frac{\gamma}{\gamma - 1}$$

Hence

$$\frac{\Delta s}{R} = \ln \frac{p_1}{p_2} + \frac{\gamma}{\gamma - 1} \ln \frac{T_2}{T_1} = \ln \frac{p_1}{p_2} \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma - 1}}$$

This formula is for a polytropic process and should work for isothermal, constant pressure, constant volume and adiabatic processes also. In other words, it must be the derivation for the entropy change of a perfect gas for any closed system process.

WORKED EXAMPLE No. 3

A perfect gas is expanded from 5 bar to 1 bar by the law $pV^{1.2} = C$. The initial temperature is 200°C . Calculate the change in specific entropy.

$R = 287 \text{ J/kg K}$ $\gamma = 1.4$.

SOLUTION

$$T_2 = 473 \left(\frac{1}{5}\right)^{1-\frac{1}{1.2}} = 361.7 \text{ K}$$

$$\frac{\Delta s}{R} = \ln \frac{p_1}{p_2} \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \ln(5) \left(\frac{361.7}{472}\right)^{\frac{1.4}{1.4-1}} = 0.67$$

$$\Delta s = 0.671 \times 287 = 192.5 \text{ J/kg K}$$

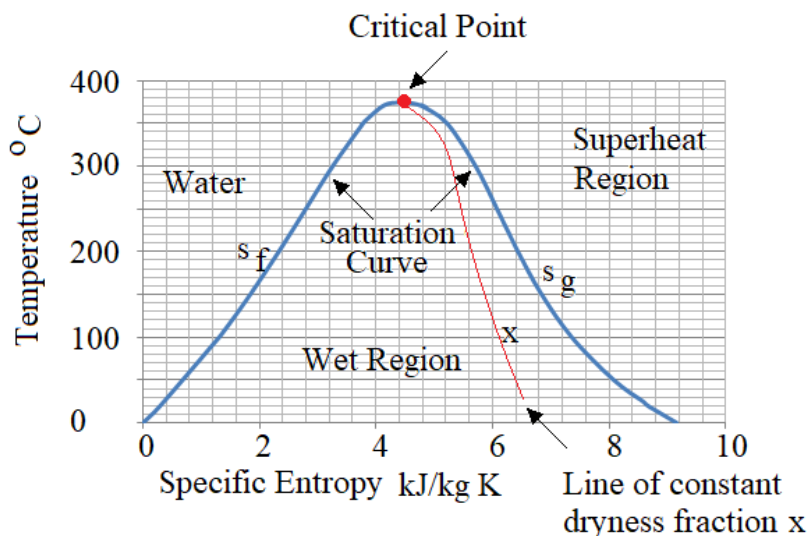
SELF ASSESSMENT EXERCISE No. 3

1. Calculate the specific entropy change when a perfect gas undergoes a reversible isothermal expansion from 500 kPa to 100 kPa. $R = 287 \text{ J/kg K}$. (Answer $+461.9 \text{ J/kg K}$).
2. Calculate the total entropy change when 2 kg of perfect gas is compressed reversibly and isothermally from 9 dm^3 to 1 dm^3 . $R=300 \text{ J/kg K}$. (Answer -1.32 kJ/k)
3. Calculate the change in entropy when 2.5 kg of perfect gas is heated from 20°C to 100°C at constant volume. Take $c_v= 780 \text{ J/kg K}$ (Answer 470 J/K)
4. Calculate the total entropy change when 5 kg of gas is expanded at constant pressure from 30°C to 200°C . $R = 300 \text{ J/kg K}$ $c_v= 800 \text{ J/kg K}$ (Answer 2.45 kJ/K)
5. A perfect gas is expanded from 5 bar to 1 bar by the law $pV^{1.6} = C$. The initial temperature is 200°C . Calculate the change in specific entropy.
 $R = 287 \text{ J/kg K}$ $\gamma = 1.4$. (Answer -144 J/kg K)
6. A perfect gas is expanded reversibly and adiabatically from 5 bar to 1 bar by the law $pV^{\gamma} = C$. The initial temperature is 200°C . Calculate the change in specific entropy using the formula for a polytropic process. $R = 287 \text{ J/kg K}$ $\gamma = 1.4$. (The answer should be zero since the process is constant entropy).

If you want to solve problems involving vapours, especially steam, you need to become familiar with property charts in order to sketch the process and evaluate the properties using entropy. This is covered next.

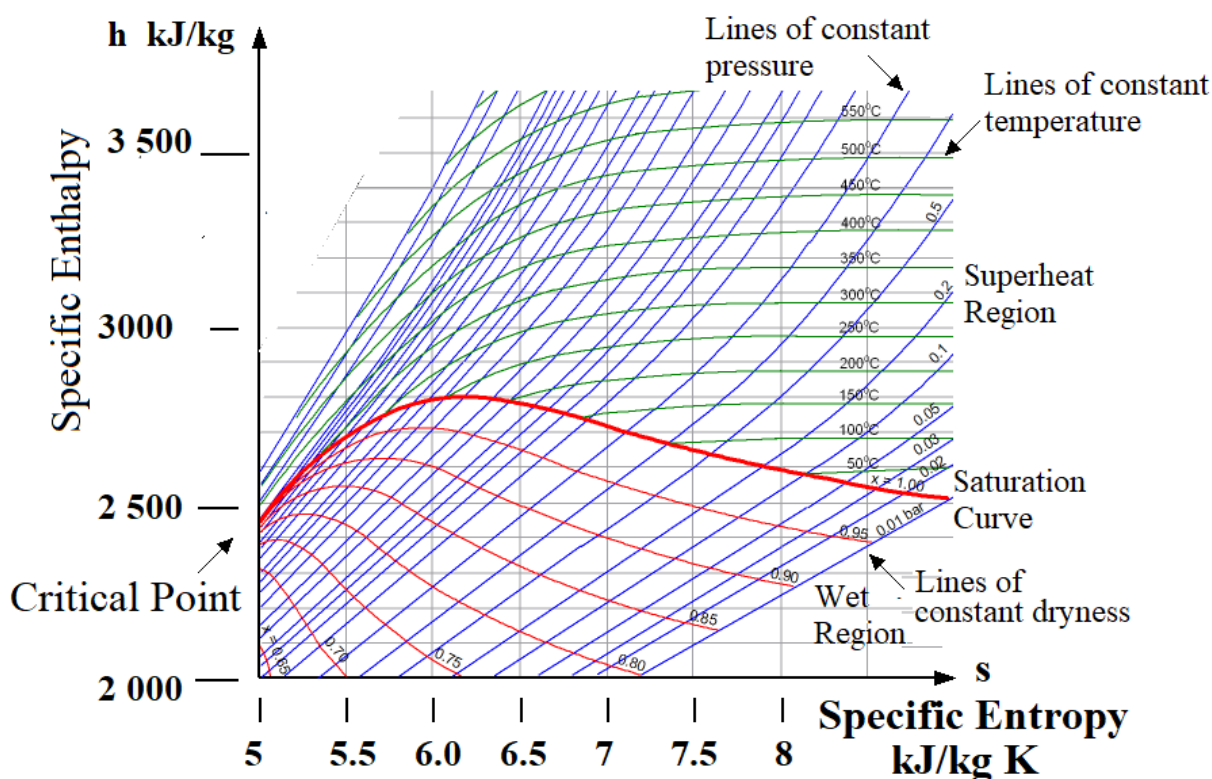
9. Temperature - Entropy ($T-s$) Diagram for Vapours

If you plot the specific entropy for saturated liquid (s_f) and for dry saturated vapour (s_g) against temperature, you would obtain the saturation curve. Lines of constant dryness fraction and constant pressure may be shown. The diagram below is for steam.



9.1 Specific Enthalpy-Specific Entropy ($h-s$) Diagram.

This diagram is especially useful for steady flow processes. The diagram is obtained by plotting h_g against s_g and h_f against s_f to obtain the characteristic saturation curve. The two curves meet at the critical point C. Lines of constant pressure, temperature and dryness are superimposed on the diagram. This is an extremely useful chart and it is available commercially. If any two coordinates are known, a point can be obtained on the chart and all other relevant values may be read off. $h-s$ charts are especially useful for solving isentropic processes because the process is a vertical line on this graph.



9.2. Isentropic Processes with Steam

The word *Isentropic* means constant entropy and this is a very important thermodynamic process. It occurs in particular when a process is reversible and adiabatic. This means that there is no heat transfer to or from the fluid and no internal heat generation due to friction. In such a process it follows that if dQ is zero then dS must be zero. Since there is no area under the T-S graph, the graph must be a vertical line.

There is another case where the entropy is constant. For example, if there is friction in the process generating heat but this is lost through cooling, then the nett result is zero heat transfer and constant entropy. You do not need to be concerned about this at this stage.

WORKED EXAMPLE No. 4

Steam at 2 bar and 150°C is expanded reversibly and adiabatically to 1 bar. Calculate the final dryness fraction and the enthalpy change.

SOLUTION

$$h_1 \text{ at 2 bar and } 150^\circ\text{C} = 2770 \text{ kJ/kg}$$

$$s_1 \text{ at 2 bar and } 150^\circ\text{C} \text{ is } 7.280 \text{ kJ/kg K.}$$

Because the process is adiabatic and reversible, the entropy remains the same.

$$s_2 \text{ at 1 bar and assumed wet is } s_f + x s_{fg} = s_1$$

$$7.280 = 1.303 + x(6.056)$$

$$x = 0.987$$

$$h_2 \text{ at 1 bar and } 0.987 \text{ dry} = h_f + x h_{fg}$$

$$h_2 = 417 + 0.987(2258) = 2645.6 \text{ kJ/kg}$$

$$\Delta h = 2645.6 - 2770 = -124.4 \text{ kJ/kg}$$

WORKED EXAMPLE No. 5

A steam turbine expands 60 kg/s from 40 bar and 300°C to 4 bar reversibly and adiabatically (isentropic). Calculate the theoretical power output.

SOLUTION

$$\dot{\Phi} + P = \Delta E \text{ per second (S F E E)}$$

The process is adiabatic. $\dot{\Phi} = 0$ and the only energy term to use is enthalpy.

$$P = \Delta H \text{ per second.}$$

$$h_1 \text{ at 40 bar and } 300^\circ\text{C} = 2963 \text{ kJ/kg}$$

$$s_1 \text{ at 40 bar and } 300^\circ\text{C} \text{ is } 6.364 \text{ kJ/kg K.}$$

$$s_2 \text{ at 4 bar and assumed wet is } s_f + x s_{fg} = s_1$$

$$6.364 = 1.776 + x(5.121)$$

$$x = 0.896$$

$$h_2 \text{ at 4 bar and } 0.896 \text{ dry} = h_f + x h_{fg}$$

$$h_2 = 605 + 0.896(2134) = 2517 \text{ kJ/kg}$$

$$P = \Delta H \text{ per second} = 60(2517 - 2963) = -26756 \text{ kW (out of system)}$$

SELF ASSESSMENT EXERCISE No. 4

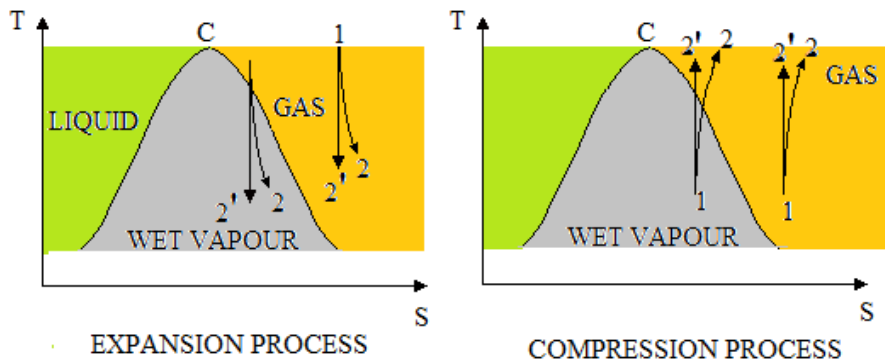
1. A turbine expands 40 kg/s of steam from 20 bar and 250°C reversibly and adiabatically to 0.5 bar. Calculate the theoretical power output.
2. A turbine expands 4 kg/s of steam from 50 bar and 300°C reversibly and adiabatically to 0.1 bar. Calculate the theoretical power output.
3. A turbine expands 20 kg/s of steam from 800 bar and 400°C reversibly and adiabatically to 0.2 bar. Calculate the theoretical power output.
4. A turbine expands 1 kg/s of steam reversibly and adiabatically. The inlet conditions are 10 bar and dry saturated. The outlet pressure is 3 bar. Calculate the theoretical power output.

10. Isentropic Efficiency

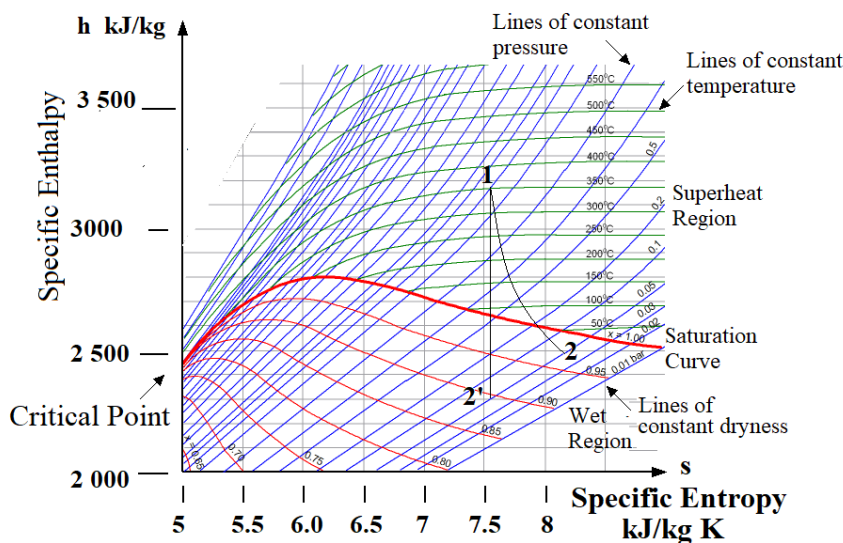
In processes involving the expansion and compression of gas and vapour friction has an important effect that has to be taken into account when solving problems involving engine cycles. It also has to be considered in other processes such as expansion in a nozzle. Real gases do not behave exactly like ideal gases and the molecules do not move around freely but convert energy into internal heat and this will

- generate heat which is in effect a heat transfer,
- increase the entropy,
- make the final enthalpy bigger than it would otherwise be,
- make the final temperature bigger than it would otherwise be if it is a gas or superheated vapour,

An adiabatic process with friction has no external heat transfer (Φ Watts or Q Joules) but the internal heat generated causes an increase in entropy. This is illustrated on the T - s sketches below. Consider the expansion and compression processes on figure A and B. Note that a compression or expansion might cross the saturation curve from wet vapour to gas or vice versa or it may stay entirely as a gas.



The same points are also apparent on the h - s diagram used with steam (or other vapours). The figure shows a vapour expansion from (1) to (2) with the ideal being from (1) to (2'). Note how it ends up dryer at the same pressure with an increase in entropy. Vapour is not normally compressed so this is not shown.



The ideal change in enthalpy is $h_2' - h_1$ The actual change is $h_2 - h_1$

The isentropic efficiency is defined as

$$\eta_{is} = \frac{\Delta h(\text{actual})}{\Delta h(\text{ideal})} = \frac{h_2 - h_1}{h_2' - h_1} \text{ for an expansion}$$

$$\eta_{is} = \frac{\Delta h(\text{ideal})}{\Delta h(\text{actual})} = \frac{h_2' - h_1}{h_2 - h_1} \text{ for a compression}$$

In the case of a perfect gas $h = c_p T$ hence

$$\eta_{is} = \frac{\Delta h(\text{actual})}{\Delta h(\text{ideal})} = \frac{T_2 - T_1}{T_2' - T_1} \text{ for an expansion}$$

$$\eta_{is} = \frac{\Delta h(\text{ideal})}{\Delta h(\text{actual})} = \frac{T_2' - T_1}{T_2 - T_1} \text{ for a compression}$$

Note that for an expansion this produces a negative number on the top and bottom lines that cancels out.

WORKED EXAMPLE No. 6

A steam turbine takes steam at 70 bar and 500°C and expands it to 0.1 bar with an isentropic efficiency 0.9. The process is adiabatic.

The power output of the turbine is 35 MW. Determine the enthalpy at exit and calculate the flow rate of steam in kg/s.

Note you need the tables and h-s chart for steam.

SOLUTION

$$h_1 = 3410 \text{ kJ/kg (tables)} \quad s_1 = 6.796 \text{ kJ/kg K} \quad \text{for an ideal expansion } s_1 = s_2'$$

Assuming that the steam becomes wet during the expansion, then

$$s_2' = s_f + x' s_{fg} \text{ at 0.1 bar}$$

$$6.796 = 0.649 + x' 7.500 \text{ (tables)} \quad x' = 0.8196$$

Note if x' is larger than 1 then the steam is still superheated and the solution does not involve x .

$$\text{Now find } h_2'. \quad h_2' = h_f + x' h_{fg} \text{ at 0.1 bar}$$

$$h_2' = 192 + (0.8196)(2392) = 2152.2 \text{ kJ/kg.}$$

$$\text{Ideal change in enthalpy} \quad \Delta h' = 2152.2 - 3410 = -1257.5 \text{ kJ/kg}$$

$$\text{Actual change in enthalpy} \quad \Delta h = 0.9(-1257.5) = -1131.7 \text{ kJ/kg}$$

$$\text{Actual change in enthalpy} \quad \Delta h = (h_2 - h_1) = -1131.7 \text{ kJ/kg}$$

$$h_2 - 3410 = -1131.7 \text{ kJ/kg}$$

$$h_2 = 2278.3 \text{ kJ/kg}$$

From the steady flow energy equation (with which you should already be familiar) we have

$$\Phi + P = \Delta H/s$$

Since there is no heat transfer then this becomes

$$P = \Delta H/s = \dot{m} (h_2 - h_1) = \dot{m} (-1131.7) = -35000 \text{ kW}$$

$$\text{hence} \quad \dot{m} = 30.926 \text{ kg/s}$$

(Note the sign convention used here is negative for energy leaving the system)

WORKED EXAMPLE No. 7

A gas turbine expands gas from 1 MPa pressure and 600°C to 100 kPa pressure. The isentropic efficiency is 0.92. The mass flow rate is 12 kg/s. Calculate the exit temperature and the power output.

Take $c_v = 718 \text{ J/kg K}$ and $c_p = 1\,005 \text{ J/kg K}$

SOLUTION

The process is adiabatic so the ideal temperature $T_{2'}$ is given by

$$T_{2'} = T_1 (r_p)^{1-1/\gamma}$$

r_p is the pressure ratio $r_p = p_2/p_1 = 0.1$

$$\gamma = c_p / c_v = 1.005/0.718 = 1.4$$

$$T_{2'} = 873(0.1)^{1-1/1.4} = 451.9 \text{ K}$$

Now we use the isentropic efficiency to find the actual final temperature.

$$\eta_{is} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$0.92 = \frac{T_2 - 873}{451.9 - 873}$$

$$T_2 = 485.6 \text{ K}$$

Now we use the SFEE to find the power output.

$$\Phi + P = \dot{m}c_p(T_2 - T_1)$$

The process is adiabatic $\Phi = 0$.

$$P = 12(1.005)(485.6 - 873) = -4\,672 \text{ kW (out of system)}$$

SELF ASSESSMENT EXERCISE No. 5

1. Steam is expanded adiabatically in a turbine from 100 bar and 600°C to 0.09 bar with an isentropic efficiency of 0.88. The mass flow rate is 40 kg/s.

Calculate the enthalpy at exit and the power output.

(Ans. 51 MW)

2. A gas compressor compresses gas adiabatically from 1 bar and 15°C to 10 bar with an isentropic efficiency of 0.89. The gas flow rate is 5 kg/s.

Calculate the temperature after compression and the power input.

(Ans. -1.513 MW)

Take $c_v = 718 \text{ J/kg K}$ and $c_p = 1005 \text{ J/kg K}$

3. A turbine is supplied with 3 kg/s of hot gas at 10 bar and 920°C. It expands adiabatically to 1 bar with an isentropic efficiency of 0.92. Calculate the final temperature and the power output. $c_p = 1.005 \text{ kJ/kg K}$ $\gamma = 1.4$

(663 K and 1.6 MW)

4. A turbine is supplied with 7 kg/s of hot gas at 9 bar and 850°C that it expands adiabatically to 1 bar with an isentropic efficiency of 0.87. Calculate the final temperature and the power output. $c_p = 1.005 \text{ kJ/kg K}$ $\gamma = 1.4$

(667 K and 3.2 MW)