APPLIED THERMODYNAMICS

TUTORIAL 4

PISTON ENGINES

In this tutorial you will do a comprehensive study of piston engine cycles and all matters associated with these engines required for the examination.

On completion of this tutorial you should be able to do the following.

- Calculate the fuel power of an engine.
- Calculate the brake power of an engine.
- Calculate the indicated power of an engine.
- Calculate the various efficiencies of an engine.
- Calculate the Mean Effective Pressure of an engine.
- Describe the standard thermodynamic cycles for spark and compression ignition engines.
- Solve standard cycles.

First we will examine engine testing methods.

1. ENGINE TESTING

1.1 FUEL POWER (F.P.)

Fuel power is the thermal power released by burning fuel inside the engine.

F.P. = mass of fuel burned per second x calorific value of the fuel.

$$\textbf{F.P.} = m_f \times \textbf{C.V.}$$

All engines burn fuel to produce heat that is then partially converted into mechanical power. The chemistry of combustion is not dealt with here. The things you need to learn at this stage follow.

1.1.1 AIR FUEL RATIO

This is the ratio of the mass of air used to the mass of fuel burned.

$$\textbf{Air Fuel Ratio} = m_a/m_f$$

STOICHIOMETRIC RATIO

This is the theoretical air/fuel ratio which is required to exactly burn the fuel.

TRUE RATIO

In reality, the air needed to ensure complete combustion is greater than the ideal ratio. This depends on how efficient the engine is at getting all the oxygen to meet the combustible elements.

The volume of air drawn into the engine is theoretically equal to the capacity of the engine (the swept volumes of the cylinders). The mass contained in this volume depends upon the pressure and temperature of the air. The pressure in particular, depends upon the nature of any restrictions placed in the inlet flow path.

Engines with carburettors work by restricting the air flow with a butterfly valve. This reduces the pressure to less than atmospheric at inlet to the cylinder and the restriction of the inlet valve adds to the affect.

Engines with turbo charging use a compressor to deliver air to the cylinders at pressures higher than atmospheric.

The actual mass of air which enters the cylinder is less than the theoretical value for various reasons such as warming from the cylinder walls, residual gas left inside and leaks from the valves and around the piston. To deal with this we use the concept of ***EFFICIENCY RATIO.***

Efficiency Ratio = Actual mass/ Theoretical mass

1.1.2 CALORIFIC VALUE

This is the heat released by burning 1 kg of fuel. There is a higher and lower value for fuels containing hydrogen. The lower value is normally used because water vapour formed during combustion passes out of the system and takes with it the latent energy. We can now define the fuel power.

FUEL POWER = Mass of fuel/s x Calorific Value

1.1.3 VOLUME FLOW RATE

A two stroke engine induces the volume of air once every revolution of the crank. A 4 stroke engine does so once every two revolutions.

Induced Volume = Capacity x speed for a 2 stroke engine

Induced volume = Capacity x speed/2 for a 4 stroke engine.

WORKED EXAMPLE No.1

A 4 stroke carburetted engine runs at 2 500 rev/min. The engine capacity is 3 litres. The air is supplied at 0.52 bar and 15°C with an efficiency ratio of 0.4. The air fuel ratio is 12/1. The calorific value is 46 MJ/kg. Calculate the heat released by combustion.

SOLUTION

Capacity = 0.003 m^3
Volume induced = 0.003 x (2 500/60)/2 = 0.0625 m^3/s
Using the gas law pV = mRT we have
Ideal air
m = pV/RT = 0.52 x 10^5 x 0.0625/(287 x 288) = 0.03932 kg/s

Actual air
m = 0.03932 x 0.4 = 0.01573 kg/s.

Mass of fuel
m_f = 0.01573/12 = 0.00131 kg/s

Heat released

Φ = calorific value x m_f = 46 000 kJ/kg x 0.00131 kg/s = 60.3 KW

SELF ASSESSMENT EXERCISE No.1

A 4 stroke carburetted engine runs at 3 000 rev/min. The engine capacity is 4 litres. The air is supplied at 0.7 bar and 10°C with an efficiency ratio of 0.5. The air fuel ratio is 13/1. The calorific value is 45 MJ/kg. Calculate the heat released by combustion.

(Answer 149 KW)

1.2 BRAKE POWER

Brake power is the output power of an engine measured by developing the power into a brake dynamometer on the output shaft. Dynamometers measure the speed and the Torque of the shaft. The Brake Power is calculated with the formula

B.P. = $2\pi NT$ where N is the shaft speed in rev/s
T is the torque in N m

You may need to know how to work out the torque for different types of dynamometers. In all cases the torque is **T = net brake force x radius**

The two main types are shown below.

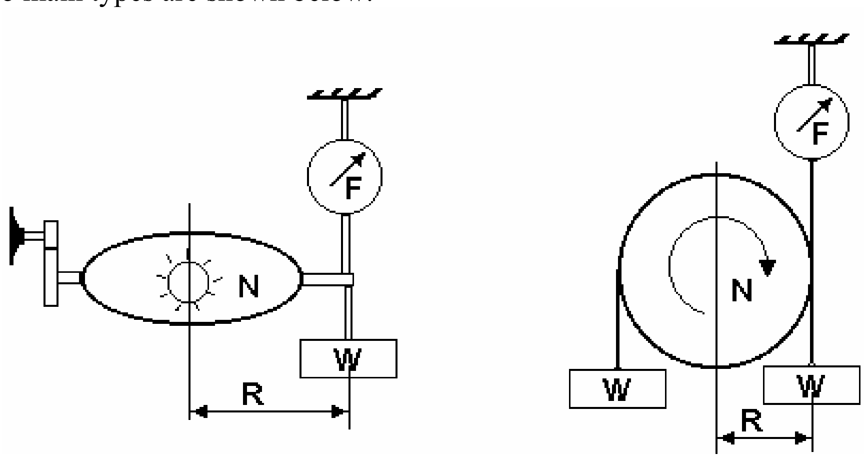


Figure 1 Figure 2

Figure 1 shows a hydraulic dynamometer which absorbs the engine power with an impeller inside a water filled casing. Basically it is a pump with a restricted flow. The power heats up the water and produces a torque on the casing. The casing is restrained by a weight pulling down and a compression spring balance pushing down also. The torque is then (F + W) x R.

Figure 2 shows a friction drum on which a belt rubs and absorbs the power by heating up the drum which is usually water cooled. The belt is restrained by a spring balance and one weight. The second equal weight acts to cancel out the other so the torque is given by T = F R.

Another form of dynamometer is basically an electric generator that absorbs the load and turns it into electric power that is dissipated in a bank of resistor as heat.

1.3 INDICATED POWER

This is the power developed by the pressure of the gas acting on the pistons. It is found by recording and plotting the pressure against volume inside the piston. Such diagrams are called indicator diagrams and they are taken with engine indicators. The diagram shows a typical indicator diagram for an internal combustion engine. Modern systems use electronic pressure and volume transducers and the data is gathered and stored digitally in a computer and then displayed and processed.

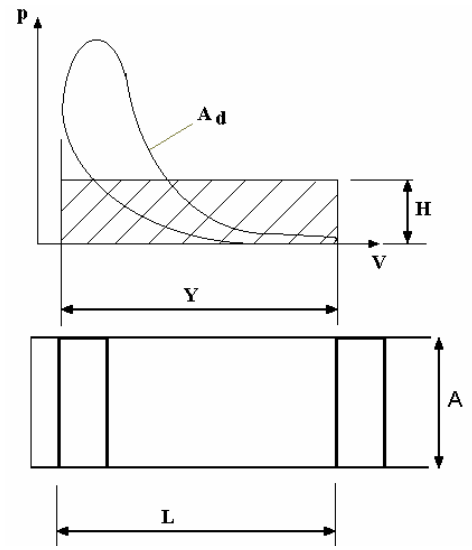


Figure 3

The average force on the piston throughout one cycle is F where F = MEP x Area of piston = pA

The Mean Effective Pressure p is the mean pressure during the cycle.
The work done during one cycle is W = Force x distance moved = F L = pAL
L is the stroke and this is twice the radius of the crank shaft.
The number of cycles per second is N.
The Indicated Power is then **I.P. = pLAN per cylinder.**
Note for a 4 stroke engine N = 1/2 the shaft speed.

MEAN EFFECTIVE PRESSURE

The mean effective pressure is found from the indicator diagram as follows.
The area enclosed by the indicator diagram represents the work done per cycle per cylinder. Let this area be A_d mm^2. The average height of the graph is H mm. The base length of the diagram is Y mm. The hatched area is equal to A_d and so

$$A_d = YH$$

$$H = A_d/Y$$

In order to convert H into pressure units, the pressure scale (or spring rate) of the indicator measuring system must be known. Let this be S_p kPa/mm. The MEP is then found from

$$\textbf{MEP} = \mathbf{S_p\ H}$$

This is also known as the ***Indicated Mean Effective Pressure*** because it is used to calculate the Indicated Power. There is also a ***Brake Mean Effective Pressure (BMEP***) which is the mean pressure which would produce the brake power.

BP = (BMEP) LAN

The BMEP may be defined from this as **BMEP = BP/LAN**

1.4 EFFICIENCIES

1.4.1 BRAKE THERMAL EFFICIENCY

This tells us how much of the fuel power is converted into brake power.

$$\eta_{BTh} = B.P./F.P.$$

1.4.2 INDICATED THERMAL EFFICIENCY

This tells us how much of the fuel power is converted into brake power.

$$\eta_{ITh} = I.P./F.P.$$

1.4.3 MECHANICAL EFFICIENCY

This tells us how much of the indicated power is converted into brake power. The difference between them is due to frictional losses between the moving parts and the energy taken to run the auxiliary equipment such as the fuel pump, water pump, oil pump and alternator.

$$H_{mech} = B.P./I.P.$$

WORKED EXAMPLE No.2

A 4 cylinder, 4 stroke engine gave the following results on a test bed.

Shaft Speed $N = 2\ 500$ rev/min
Torque arm $R = 0.4$ m
Net Brake Load $F = 200$ N
Fuel consumption $m_f = 2$ g/s
Calorific value = 42 MJ/kg
Area of indicator diagram $A_d = 300\ mm^2$
Pressure scale $S_p = 80$ kPa/mm
Stroke $L = 100$ mm
Bore $D = 100$ mm
Base length of diagram $Y = 60$ mm.

Calculate the B.P., F.P., I.P., MEP, η_{BTh}, η_{ITh}, and η_{mech},

SOLUTION

$BP = 2\pi NT = 2\pi \times (2500/60) \times (200 \times 0.4) = 20.94$ kW

FP = mass/s x C.V. = 0.002 kg/s x 42 000 kJ/kg = 84 kW

$IP = pLAN$

$p = MEP = A_d/Y \times S_p = (300/60) \times 80 = 400$ kPa

$IP = 400 \times 0.1 \times (\pi \times 0.1^2/4) \times (2500/60)/2$ per cylinder

$IP = 6.54$ kW per cylinder.

For 4 cylinders $IP = 6.54 \times 4 = 26.18$ kW

$\eta_{BTh} = 20.94/84 = 24.9\%$

$\eta_{ITh} = 26.18/84 = 31.1\ \%$

$\eta_{mech} = 20.94/26.18 = 80\%$

SELF ASSESSMENT EXERCISE No.2

1. A 4 stroke spark ignition engine gave the following results during a test.

Number of cylinders	6
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5000 rev/min
Fuel consumption rate	0.3 dm^3/min
Fuel density	750 kg/m^3
Calorific value	44 MJ/kg
Net brake load	180 N
Torque arm	0.5 m
Net indicated area	720 mm^2
Base length of indicator diagram	60 mm
Pressure scale	40 kPa/mm

Calculate the following.

i) The Brake Power. (47.12 kW)
ii) The Mean effective Pressure. (480 kPa).
iii) The Indicated Power. (61 kW).
iv) The Mechanical Efficiency. (77.2%).
v) The Brake Thermal efficiency. (28.6 %).

2. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	100 mm
Stroke	100 mm
Speed	2000 rev/min
Fuel consumption rate	5 g/s
Calorific value	46 MJ/kg
Net brake load	500 N
Torque arm	0.5 m
Net indicated area	1 500 mm^2
Base length of indicator diagram	66 mm
Pressure scale	25 kPa/mm

Calculate the following.

i) The Indicated thermal efficiency. (26.3 %)
ii) The Mechanical Efficiency. (87%).
iii) The Brake Thermal efficiency. (22.8%).

3. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	80 mm
Stroke	80 mm
Speed	2 200 rev/min
Fuel consumption rate	1.6 cm^3/s
Fuel density	750 kg/m^3
Calorific value	60 MJ/kg
Net brake load	195 N
Torque arm	0.4 m
Net indicated area	300 mm^2
Base length of indicator diagram	40.2 mm
Pressure scale	50 kPa/mm

Calculate the following.

i) The Indicated thermal efficiency. (30.5 %)
ii) The Mechanical Efficiency. (81.7%).
iii) The Brake Thermal efficiency. (25%).

4. A four stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5 000 rev/min
Fuel consumption rate	0.09 kg/min
Calorific value	44 MJ/kg
Net brake load	60 N
Torque arm	0.5 m
MEP	280 kPa

Calculate the following.

i) The Mechanical Efficiency. (66.1%).
ii) The Brake Thermal efficiency. (23.8%).

5. Define Indicated Mean Effective Pressure and Brake Mean Effective Pressure.

The BMEP for a 4 cylinder, 4 stroke spark ignition engine is 8.4 bar. The total capacity is 1.3 dm^3 (litres). The engine is run at 4 200 rev/min.

Calculate the Brake Power. (38.22 kW)

There are 10 kW of mechanical losses in the engine.

Calculate the Indicated Mean effective Pressure. (10.6 bar).

The Volumetric Efficiency is 85% and the Brake Thermal Efficiency of the engine is 28%. The air drawn in to the engine is at 5^oC and 1.01 bar. The fuel has a calorific value of 43.5 MJ/kg.

Calculate the air/fuel ratio. (Answer 12.3/1).

Let's now have a look at the theoretical cycle for spark ignition engines.

2. SPARK IGNITION ENGINES

THE OTTO CYCLE

The Otto cycle represents the ideal cycle for a spark ignition engine. In an ideal spark ignition engine, there are four processes as follows.

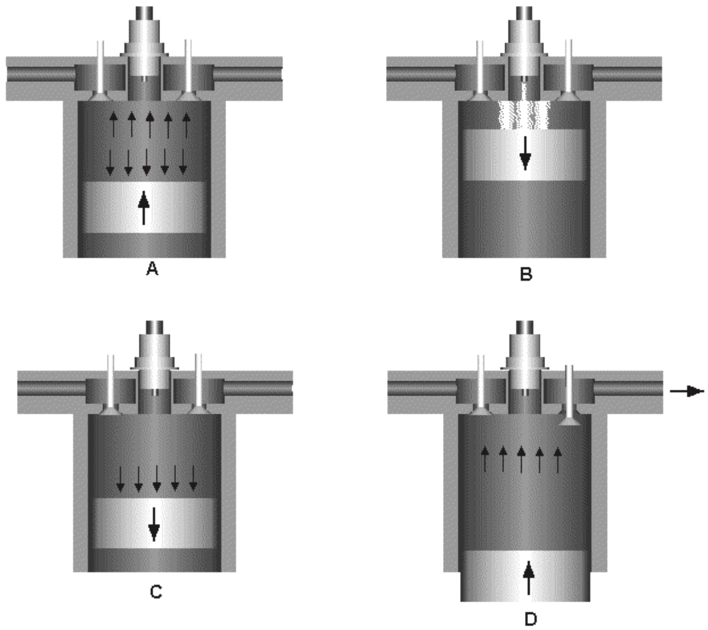


Fig.4

COMPRESSION STROKE

Air and fuel are mixed and compressed so rapidly that there is no time for heat to be lost. (Figure A) In other words the compression is adiabatic. Work must be done to compress the gas.

IGNITION

Just before the point of maximum compression, the air is hot and a spark ignites the mixture causing an explosion (Figure B). This produces a rapid rise in the pressure and temperature. The process is idealised as a constant volume process in the Otto cycle.

EXPANSION OR WORKING STROKE

The explosion is followed by an adiabatic expansion pushing the piston and giving out work. (Figure C)

EXHAUST

At the end of the working stroke, there is still some pressure in the cylinder. This is released suddenly by the opening of an exhaust valve. (Figure D) This is idealised by a constant volume drop in pressure in the Otto cycle. In 4 stroke engines a second cycle is performed to push out the products of combustion and draw in fresh air and fuel. It is only the power cycle that we are concerned with.

The four ideal processes that make up the Otto cycle are as follows.

1 to 2. The air is compressed reversibly and adiabatically. Work is put in and no heat transfer occurs.

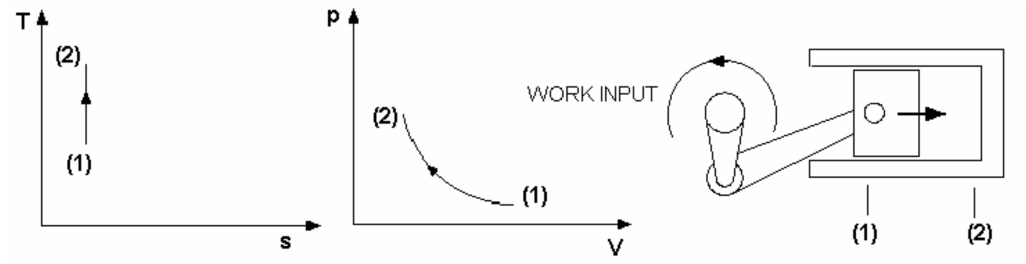


Fig.5

2 to 3. The air is heated at constant volume. No work is done. $Q_{in} = mc_V(T_3-T_2)$

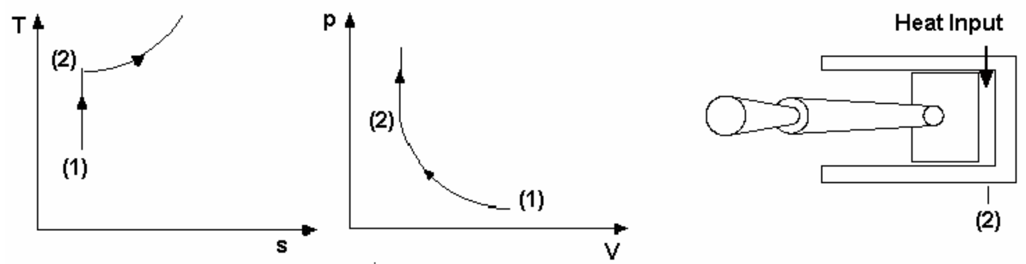
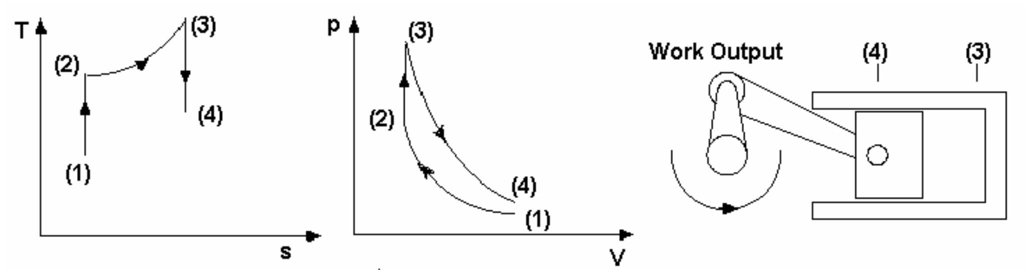


Fig.6

3 to 4. The air expands reversibly and adiabatically with no heat transfer back to its original volume. Work output is obtained.



FFig.7

4 to 1. The air is cooled at constant volume back to its original pressure and temperature. No work is done $Q_{out} = mc_V(T_4-T_1)$

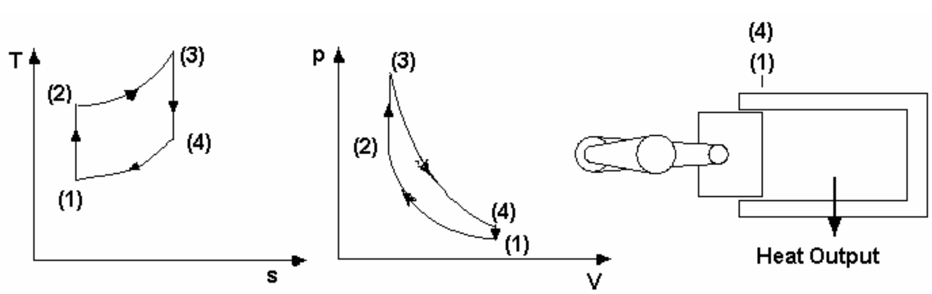


Fig.8

If the engine is successful, then W_{out} The area under the top curve) is larger than W_{in} (the area under the lower curve). The enclosed area represents the net work obtained from the cycle.

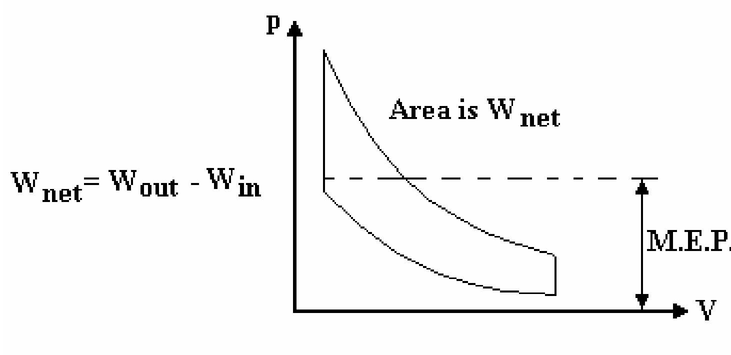


Figure 9

The Mean Effective Pressure (MEP) is the average pressure such that

W(net) = Enclosed Area = MEP x A x L

W(net) = MEP x Swept Volume

This is true for all cycles and for real engines.

A corresponding net amount of heat must have been transferred into the cycle of:

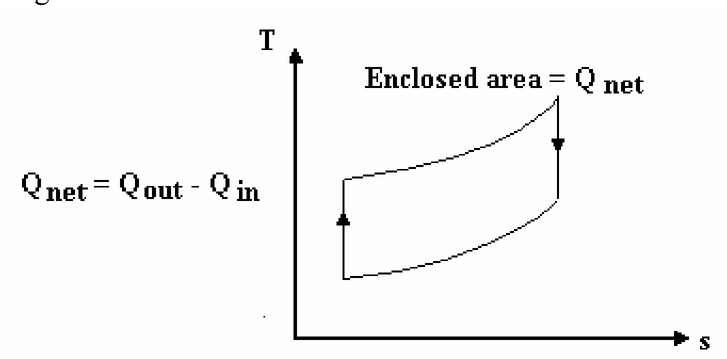


Figure 10

Applying the first law, it follows $\mathbf{Q_{net} = W_{net}}$

It also follows that since the heat transfer is equal to the area under a T - S graph, then the area enclosed by the cycle on the T - S diagram is equal to the Qnet and this is true for all cycles.

EFFICIENCY

$$\eta = \frac{W_{nett}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For the process (1) to (2) we may use the rule $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_v^{\gamma-1}$

For the process (3) to (4) we may similarly write $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r_v^{\gamma-1}$

where r_v is the volume compression ratio $r_v = \frac{V_1}{V_2} = \frac{V_4}{V_3}$

It follows that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ and $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

and that $\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{\frac{T_3 T_1}{T_2} - T_1}{\frac{T_2 T_4}{T_1} - T_2} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$

$\frac{T_4}{T_1} = \frac{T_3}{T_2}$ then $\frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - r_v^{1-\gamma}$$

Since this theoretical cycle is carried out on air for which $\gamma = 1.4$ then the efficiency of an Otto Cycle is given by $\eta = 1 - r_v^{0.4}$

This shows that the thermal efficiency depends only on the compression ratio. If the compression ratio is increased, the efficiency is improved. This in turn increases the temperature ratios between the two isentropic processes and explains why the efficiency is improved.

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WORKED EXAMPLE No.3

An Otto cycle is conducted as follows. Air at 100 kPa and 20°C is compressed reversibly and adiabatically. The air is then heated at constant volume to 1500°C. The air then expands reversibly and adiabatically back to the original volume and is cooled at constant volume back to the original pressure and temperature. The volume compression ratio is 8. Calculate the following.

i. The thermal efficiency.

ii. The heat input per kg of air.

iii. The net work output per kg of air.

iv. The maximum cycle pressure.

c_V = 718 kJ/kg γ= 1.4. R = 287 J/kg K

SOLUTION

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

T_1=20 +273=293K T_3=1500+273=1773K r_v=8

$$\eta = 1 - r^{1-\gamma} = 1 - 8^{\;0.4} = 0.565 \quad \text{or } 56.5\%$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293\left(8^{0.4}\right) = 673.1\,K$$

$$Q_{in} = mc_v(T_3 - T_2) = 1x718(1773 - 673.1) = 789700\,J/kg = 789.7\,kJ/kg$$

$$W_{nett} = \eta Q_{in} = 0.56x789.7 = 446.2\,kJ/kg$$

From the gas law we have

$$p_3 = \frac{p_1 V_1 T_3}{T_1 V_3} = \frac{100000 \text{ x } V_1 \text{ x } 1773}{293 \text{ x } V_3}$$

$$\frac{V_1}{V_3} = 8$$

$$p_3 = \frac{100000 \text{ x } 1773}{293} \text{ x } 8 = 4.84 \text{ MPa}$$

If you have followed the principles used here you should be able to solve any cycle.

SELF ASSESSMENT EXERCISE No.3

Take c_v = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. In an Otto cycle air is drawn in at 20°C. The maximum cycle temperature is 1500°C. The volume compression ratio is 8/1. Calculate the following.

   i. The thermal efficiency. (56.5%)
   ii. The heat input per kg of air. (789 kJ/kg).
   iii. The net work output per kg of air. (446 kJ/kg).

2. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40°C respectively. Calculate the following.

   i. The thermal efficiency. (58.5%)
   ii. The maximum cycle temperature. (1450 K).
   iii. The maximum pressure. (4.17 MPa).
   iv. The net work output per kg of air. (293 kJ/kg).

3. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

   The pressure and temperature before compression are 105 kPa and 25°C respectively. The net work output is 500 kJ/kg). Calculate the following.

   i. The heat input. (833 kJ/kg).
   ii. The maximum temperature. (1 906 K)
   iii. The maximum pressure. (6.64 MPa).

4. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40°C respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

   i. The thermal efficiency. (59.4%).
   ii. The net work output. (4.1 kJ/cycle)
   iii. The net power output. (205 kW).

5. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300°C and 20°C respectively.
   (Answer 1.235 kg).

6. The air standard cycle appropriate to the reciprocating spark ignition engine internal-combustion engine is the Otto. Using this, find the efficiency and output of a 2 litre (dm^3), 4 stroke engine with a compression ratio of 9 running at 3000 rev/min. The fuel is supplied with a gross calorific value of 46.8 MJ/kg and an air fuel ratio of 12.8.

   Calculate the answers for two cases.

   a. The engine running at full throttle with the air entering the cylinder at atmospheric conditions of 1.01 bar and 10°C with an efficiency ratio of 0.49.
   (Answers 58.5% and 65 kW)

   b. The engine running at part throttle with the air entering the cylinder at 0.48 bar and efficiency ratio 0.38.
   (Answers 58.5% and 24 kW).

7. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air.
   (Answers 871 kJ/kg and 1129 kJ/kg).

Now let's move on to study engines with compression ignition.

3. COMPRESSION IGNITION ENGINES

Compression ignition is achieved by compressing the air until it is so hot that fuel sprayed into it will ignite spontaneously.

On modern engines the fuel is injected as a spray of droplets. Since it takes a finite time for the droplets to warm up to the air temperature, there is a time delay between injection and explosion. The accumulation of droplets in this time cause an initial sharp detonation (diesel knock) and a rapid rise in pressure and temperature inside the cylinder. This is a constant volume rise in pressure similar to the spark ignition and the Otto cycle. The big difference is that with fuel injection, fuel may continue to be injected after the explosion and controlled burning of the fuel in the air may take place as the piston moves away from the cylinder head. This ideally maintains the pressure constant as the volume increases. When the fuel is cut off, a natural expansion occurs and the rest of the cycle is similar to the Otto cycle. The air standard cycle for this engine is the DUAL COMBUSTION CYCLE.

The man most credited with the invention of this engine is Rudolf Diesel but many others worked on similar ideas. Diesel's first engine used coal dust blasted into the combustion chamber with compressed air. This developed into blasting in oil with compressed air. The air standard cycle for these old fashioned engines was deemed to be as described above but with no constant volume process. This cycle is called the DIESEL CYCLE. The Diesel cycle may have been a reasonable approximation of what happened in older slow running engines but it is not representative of a modern engine.

3.1 DUAL COMBUSTION CYCLE

This is the air standard cycle for a modern fast running diesel engine. First the air is compressed isentropically making it hot. Fuel injection starts before the point of maximum compression. After a short delay in which fuel accumulates in the cylinder, the fuel warms up to the air temperature and detonates causing a sudden rise in pressure. This is ideally a constant volume heating process. Further injection keeps the fuel burning as the volume increases and produces a constant pressure heating process. After cut off, the hot air expands isentropically and then at the end of the stroke, the exhaust valve opens producing a sudden drop in pressure. This is ideally a constant volume cooling process. The ideal cycle is shown in figure 11

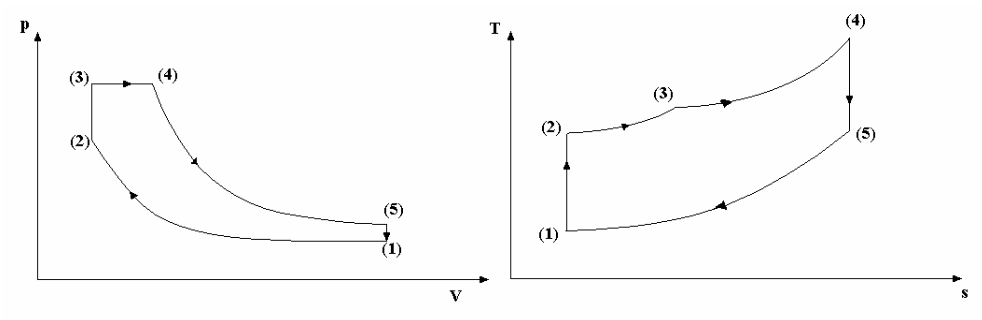

Fig. 11

The processes are as follows.

1 - 2	reversible adiabatic (isentropic) compression.
2 - 3	constant volume heating.
3 - 4	constant pressure heating.
4 – 5	reversible adiabatic (isentropic) expansion.
5 - 1	constant volume cooling.

The analysis of the cycle is as follows.

The heat is supplied in two stages hence $Q_{in} = mc_p(T_4 - T_3) + mc_v(T_3 - T_2)$

The heat rejected is $Q_{out} = mc_v(T_5 - T_1)$

The thermal efficiency may be found as follows.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

The formula can be further developed to show that $\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}}$

r_v is the VOLUME COMPRESSION RATIO. $r_v = V_1/V_2$

β is the CUT OFF RATIO.

$\beta = V_4/V_3$

k is the ratio p_3/p_2.

Most students will find this adequate to solve problems concerning the dual combustion cycle. Generally, the method of solution involves finding all the temperatures by application of the gas laws. Those requiring a detailed analysis of the cycle should study the following derivation.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{\left(T_5 - T_1\right)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Obtain all the temperatures in terms of T_2

Isentropic compression 1 to 2

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_2}{r_v^{\gamma-1}}$$

Constant volume heating 2 to 3 note $V_3 = V_2$

$$T_3 = \frac{p_3 V_3 T_2}{p_2 V_2} = \frac{p_3 T_2}{p_2} = kT_2$$

Constant pressure heating 3 to 4 note $p_3 = p_4$

$$T_4 = \frac{p_4 V_4 T_3}{p_3 V_3} = \frac{V_4 T_3}{V_3} = \beta T_3 = \beta k T_2$$

Isentropic expansion 4 to 5

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = T_4\left(\frac{V_4 V_2}{V_5 V_2}\right)^{\gamma-1} = T_4\left(\frac{\beta}{r_v}\right)^{\gamma-1} = \frac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}}$$

Substitute for all temperatures in the efficiency formula.

$$\eta = 1 - \frac{\dfrac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}} - \dfrac{T_2}{r_v^{\gamma-1}}}{\left(kT_2 - T_2\right) + \gamma\left(\beta k T_2 - kT_2\right)} = 1 - \frac{\dfrac{k\beta^{\gamma}}{r_r^{\gamma-1}} - \dfrac{1}{r_v^{\gamma-1}}}{\left(k-1\right) + \gamma\left(\beta k - k\right)}$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{\left[\left(k-1\right) + \gamma k\left(\beta - 1\right)\right] r_v^{\gamma-1}}$$

Note that if β=1, the cycle becomes an Otto cycle and the efficiency formulae becomes the same as for an Otto cycle.

WORKED EXAMPLE No. 4

In a dual combustion cycle, the compression starts from 1 bar and 20°C. The compression ratio is 18/1 and the cut off ratio is 1.15. The maximum cycle pressure is 1360 K. The total heat input is 1 kJ per cycle. Calculate the following.

i. The thermal efficiency of the cycle.

ii. The net work output per cycle.

Check that the efficiency does not contravene the Carnot principle.

SOLUTION

Known data.

T_1 = 20 +273 = 293 K The hottest temperature is T_4 = 1360K.

$\beta=1.15 \quad r_v = 18 \quad \gamma=1.4$

$$T_2 = T_1 r_v^{\gamma-1} = 293 \text{ x } 18^{0.4} = 931\text{ K}$$

$$T_3 = \frac{V_3 T_4}{V_4} = \frac{T_4}{\beta} = \frac{1360}{1.15} = 1183\text{ K}$$

$$\frac{p_3}{p_2} = k = \frac{T_3}{T_2} = 1.27$$

$$\eta = 1 - \frac{k\beta^\gamma - 1}{[(k-1)+\gamma k(\beta-1)]r_v^{\gamma-1}} = 1 - \frac{1.27 \text{ x } 1.15^{1.4} - 1}{[(1.27-1)+(1.4 \text{ x } 1.27 \text{ x } (1.15-1))] \text{ x } 18^{0.4}}$$

$$\eta = 0.68 \text{ or } 68\%$$

$W_{nett} = \eta$ x Q_{in} = 0.68 x 1 = 0.68 kJ per cycle.

The Carnot efficiency should be higher.

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{293}{1360} = 0.785$$

The figure of 0.68 is lower so the Carnot principle has not been contravened.

WORKED EXAMPLE No.5

A dual combustion cycle has a compression ratio of 18/1. The maximum pressure in the cycle is 9 MPa and the maximum temperature is 2000°C. The pressure and temperature before compression is 115 kPa and 25°C respectively. Calculate the following.

i. The cut off ratio.
ii. The cycle efficiency.
iii. The nett work output per kg of air.

Assume $\gamma = 1.4$ $c_p = 1.005$ kJ/kgK $c_v = 0.718$ kJ/kg K.

SOLUTION

Known data.

$T_1 = 298$ K $\quad T_4 = 2273$ K $\quad p_3 = p_4 = 9$ MPa $\quad p_1 = 115$ kPa

$V_1/V_2 = V_1/V_3 = 18 \quad V_2 = V_3$

$T_2 = 298 \text{ x } 18^{(\gamma-1)} = 947$ K

$$T_3 = \frac{p_3 T_1 V_3}{p_1 V_1} = \frac{9 \text{ x } 10^6 \text{ x } 298}{115 \text{ x } 10^3} \text{ x } \frac{V_3}{V_1} = \frac{9 \text{ x } 10^6 \text{ x } 298}{115 \text{ x } 10^3} \text{ x } \frac{1}{18} = 1296\text{K}$$

$$\text{Cut off ratio} = \beta = \frac{V_4}{V_3} = \frac{p_3 T_4}{p_4 T_3} \text{ but } p_4 = p_3 \text{ so } \beta = \frac{T_4}{T_3}$$

$$\beta = \frac{2273}{1296} = 1.75$$

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma-1} \text{ but } \frac{V_4}{V_5} = \frac{V_4}{V_3} \text{ x } \frac{V_3}{V_5} = \frac{1.75}{18} = 0.0974$$

$$T_5 = 2273 \text{ x } 0.0974^{0.4} = 895.6 \text{ K}$$

$Q_{in} = mc_p(T_4 - T_3) + mc_v (T_3 - T_2)$ $\quad$ m= 1 kg

$Q_{in} = 1.005(2274 - 1296) + 0.718(1296 - 947) = 1232.5$ kJ/kg

$Q_{out} = mc_v (T_5 - T_1)$

$Q_{out} = 0.718(895.6 - 298) = 429$ kJ/g

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{429}{1232} = 0.65 \text{ or } 65\%$$

$$W_{nett} = Q_{in} \quad Q_{out} = 1232 \quad 428.6 = 803.5 \text{ kJ/kg}$$

3.2 THE DIESEL CYCLE

The Diesel Cycle proceeded the dual combustion cycle. The Diesel cycle is a reasonable approximation of what happens in slow running engines such as large marine diesels. The initial accumulation of fuel and sharp detonation does not occur and the heat input is idealised as a constant pressure process only.

Again consider this cycle as being carried out inside a cylinder fitted with a piston. The p-V and T-s cycles diagrams are shown in figure 12

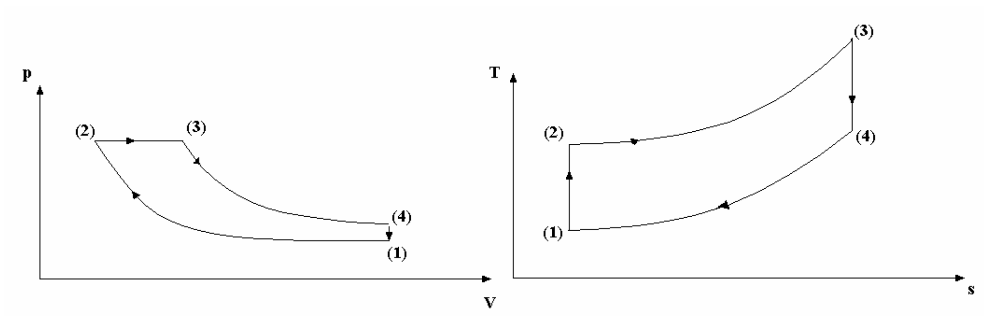

Fig.12

1 - 2 reversible adiabatic (isentropic) compression.

2 - 3 constant pressure heating.

3 - 4 reversible adiabatic (isentropic) expansion.

4 - 1 constant volume cooling.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

The cycle is the same as the dual combustion cycle without the constant volume heating process. In this case since k=1 the efficiency is given by the following formula.

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma - 1}}$$

WORKED EXAMPLE No.6

An engine using the Diesel Cycle has a compression ratio of 20/1 and a cut off ratio of 2. At the start of the compression stroke the air is at 1 bar and 15°C. Calculate the following.

i. The air standard efficiency of the cycle.
ii. The maximum temperature in the cycle.
iii. The heat input.
iv. The net work output.

SOLUTION

Initial data.

β=2 r_v=20 γ=1.4 c_v = 718 J/kg K for air T_1=288 K p_1=1 bar.

The maximum temperature is T_3 and the maximum pressure is p_3 and p_2.

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

$$\eta = 1 - \frac{2^{1.4} - 1}{(2-1) \text{ x } 1.4 \text{ x } 20^{0.4}}$$

$$\eta = 1 - \frac{1.639}{1 \text{ x } 1.4 \text{ x } 3.314} = 0.647 \text{ or } 64.7\%$$

$$T_2 = T_1 r_v^{\gamma-1} = 288 \text{ x } 20^{0.4} = 954.5 \text{ K}$$

$$T_3 = \frac{V_2}{V_3} T_2 = \beta T_2 = 954.3 \text{ x } 2 = 1909 \text{ K}$$

$$Q_{in} = mc_p(T_3 - T_2)$$

$$Q_{in} = 1.005(1909 - 954.5) = 959.3 \text{ kJ}$$

$$\eta = \frac{W_{nett}}{Q_{in}}$$

$$W_{nett} = \eta Q_{in} = 0.647 \text{ x } 959.3 = 620.6 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No.4

1. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30°C and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

   i. The maximum cycle temperature. (2424 K).
   ii. The net work output per cycle. (864 kJ/kg).
   iii. The thermal efficiency. (67.5 %).

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

   i. The net work output per cycle. (680 kJ/kg).
   ii. The thermal efficiency. (57.6 %).

3. Draw a p - V and T - s diagram for the Dual Combustion Cycle.

   A reciprocating engine operates on the Dual Combustion Cycle. The pressure and temperature at the beginning of compression are 1 bar and 15°C respectively. The compression ratio is 16. The heat input is 1800 kJ/kg and the maximum pressure is 80 bar. Calculate the following.

   i. The pressure, volume and specific volume at all points in the cycle.
   ii. The cycle efficiency. (62.8 %).
   iii. The mean effective pressure. (14.52 bar).