UNIT 61: ENGINEERING THERMODYNAMICS

Unit code: D/601/1410

QCF level: 5

Credit value: 15

OUTCOME 2
INTERNAL COMBUSTION ENGINE PERFORMANCE

TUTORIAL No. 4 – HEAT ENGINE CYCLES

2 **Be able to evaluate the performance of internal combustion engines**

Second law of thermodynamics: statement of law; schematic representation of a heat engine to show heat and work flow

Heat engine cycles: Carnot cycle; Otto cycle; Diesel cycle; dual combustion cycle; Joule cycle; property diagrams; Carnot efficiency; air-standard efficiency

Performance characteristics: engine trials; indicated and brake mean effective pressure; indicated and brake power; indicated and brake thermal efficiency; mechanical efficiency; relative efficiency; specific fuel consumption; heat balance

Improvements: turbocharging; turbocharging and intercooling; cooling system and exhaust gas heat recovery systems

- Define AIR STANDARD CYCLES.
- Identify the ideal cycle for a given type of engine.
- Explain and solve problems for the OTTO cycle
- Explain and solve problems for the DIESEL cycle
- Explain and solve problems for the Dual Combustion cycle
- Explain and solve problems for the JOULE cycle

1. <u>THEORETICAL CYCLES FOR ENGINES</u>

Internal combustion engines fall into two groups, those that use a sparking plug to ignite the fuel (spark ignition engines) and those that use the natural temperature of the compressed air to ignite the fuel (compression ignition engines).

Another way to group engines is into those that use non-flow processes and those that use flow processes. For example, non-flow processes are used in piston engines. Flow processes are used in gas turbine engines.

Theoretical cycles are made up of ideal thermodynamic processes to resemble those that occur in a real engine as closely as possible. Many of these cycles are based on air as the working fluid and are called ***AIR STANDARD CYCLES***. Before looking at air standard cycles, we should briefly revise the Carnot Cycle from tutorial 3.

1. THE CARNOT CYCLE

The most efficient way of transferring heat into or out of a fluid is at constant temperature. All the heat transfer in the Carnot cycle is at constant temperature so it follows that the Carnot cycle is the most efficient cycle possible. The heat transfer into the cycle occurs at a hot temperature T_{hot} and the heat transfer out of the cycle occurs at a colder temperature T_{cold}. The thermodynamic efficiency was sown to be given as follows.

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$$

None of the following cycles can have an efficiency greater than this when operating between the same temperatures limits.

2 SPARK IGNITION ENGINE

2.1 THE OTTO CYCLE

The ideal cycle is named after Count N.A.Otto. It represents the ideal cycle for a spark ignition engine. In an ideal spark ignition engine, there are four processes as follows.

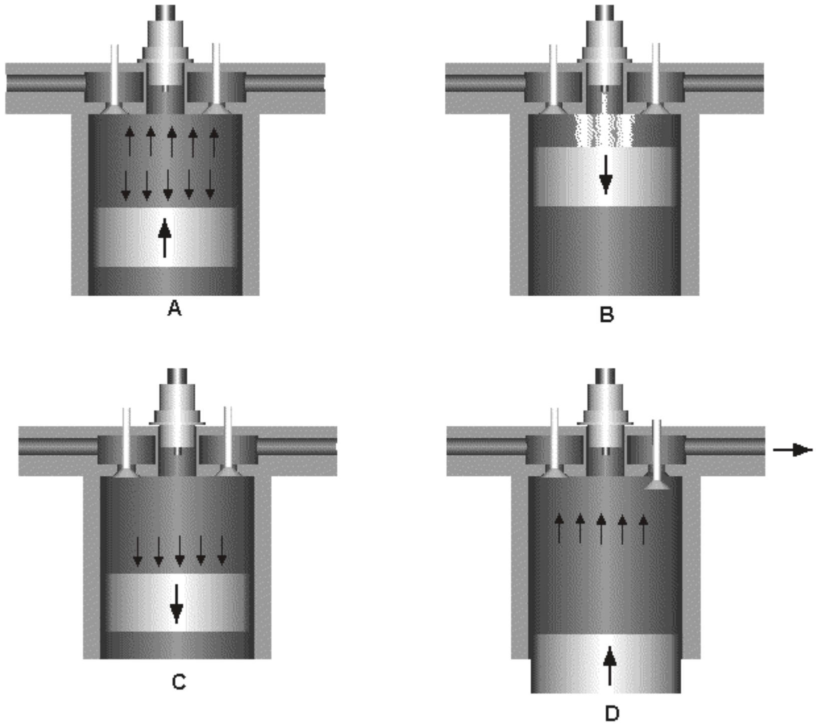

Fig.1

COMPRESSION STROKE
Air and fuel are mixed and compressed so rapidly that there is no time for heat to be lost. (Figure A) In other words the compression is adiabatic. Work must be done to compress the gas.

IGNITION
Just before the point of maximum compression, the air is hot and a spark ignites the mixture causing an explosion (Figure B). This produces a rapid rise in the pressure and temperature. The process is idealised as a constant volume process in the Otto cycle.

EXPANSION OR WORKING STROKE
The explosion is followed by an adiabatic expansion pushing the piston and giving out work. (Figure C)

EXHAUST
At the end of the working stroke, there is still some pressure in the cylinder. This is released suddenly by the opening of an exhaust valve. (Figure D) This is idealised by a constant volume drop in pressure in the Otto cycle. In 4 stroke engines a second cycle is performed to push out the products of combustion and draw in fresh air and fuel. It is only the power cycle that we are concerned with.

The four ideal processes that make up the Otto cycle are as follows.

1 to 2 The air is compressed reversibly and adiabatically. Work is put in and no heat transfer occurs.

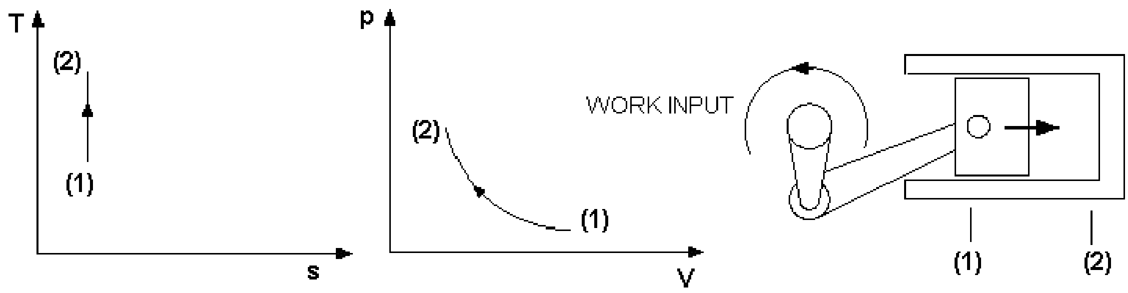

Fig.2

2 to 3 The air is heated at constant volume. No work is done. $Q_{in} = mc_V(T_3-T_2)$

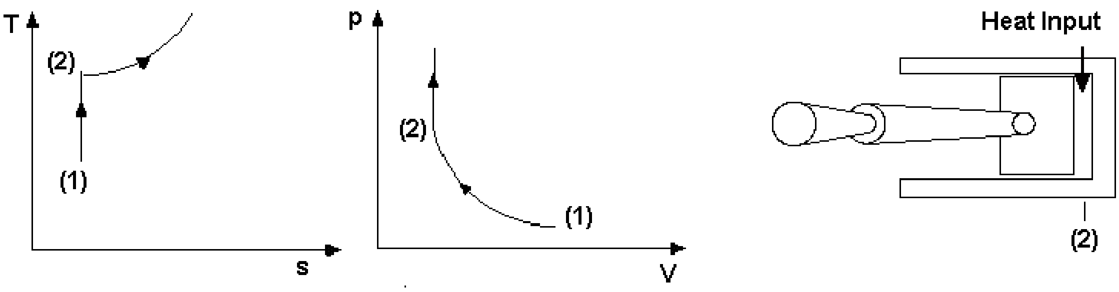

Fig.3

3 to 4 The air expands reversibly and adiabatically with no heat transfer back to its original volume. Work output is obtained.

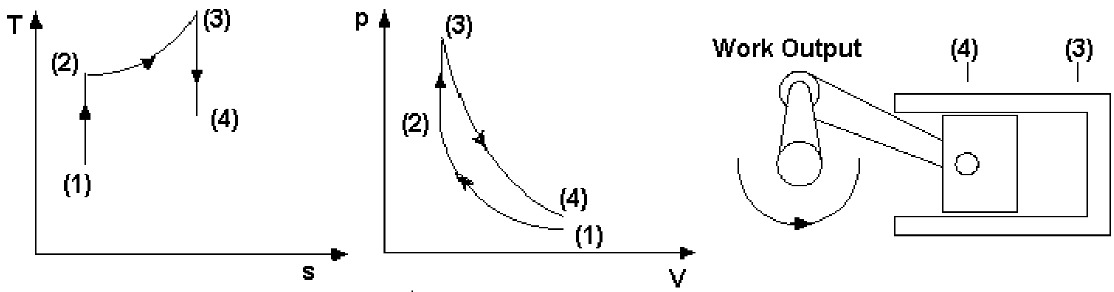

Fig.4

4 to 1 The air is cooled at constant volume back to its original pressure and temperature. No work is done $Q_{out} = mc_V(T_4-T_1)$

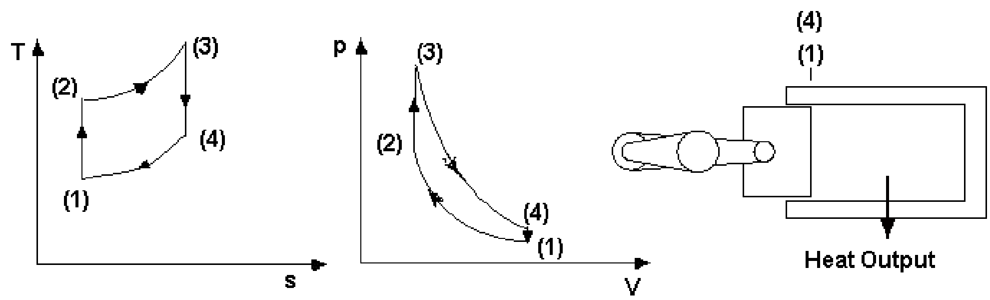

Fig.5

The total heat transfer into the system during one cycle is $Q_{nett} = Q_{in} - Q_{out}$

The total work output per cycle is W_{nett}

From the 1st. Law of thermodynamics $Q_{nett} = W_{nett}$

EFFICIENCY

$$\eta = \frac{W_{nett}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For the process (1) to (2) we may use the rule $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_v^{\gamma-1}$

For the process (3) to (4) we may similarly write $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r_v^{\gamma-1}$

where r_v is the volume compression ratio $r_v = \frac{V_1}{V_2} = \frac{V_4}{V_3}$

It follows that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ and $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

and that $\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{\frac{T_3 T_1}{T_2} - T_1}{\frac{T_2 T_4}{T_1} - T_2} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$

$\frac{T_4}{T_1} = \frac{T_3}{T_2}$ then $\frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - r_v^{1-\gamma}$$

Since this theoretical cycle is carried out on air for which $\gamma = 1.4$ then the efficiency of an Otto Cycle is given by $\eta = 1 - r_v^{0.4}$

This shows that the thermal efficiency depends only on the compression ratio. If the compression ratio is increased, the efficiency is improved. This in turn increases the temperature ratios between the two isentropic processes and explains why the efficiency is improved.

WORKED EXAMPLE No.1

An Otto cycle is conducted as follows. Air at 100 kPa and 20°C is compressed reversibly and adiabatically. The air is then heated at constant volume to 1500°C. The air then expands reversibly and adiabatically back to the original volume and is cooled at constant volume back to the original pressure and temperature. The volume compression ratio is 8. Calculate the following.

i. The thermal efficiency.

ii. The heat input per kg of air.

iii. The net work output per kg of air.

iv. The maximum cycle pressure.

c_V = 718 kJ/kg γ= 1.4. R = 287 J/kg K

SOLUTION

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

T_1=20 +273=293K T_3=1500+273=1773K r_v=8

$$\eta = 1 - r^{1-\gamma} = 1 - 8^{\ 0.4} = 0.565 \quad \text{or } 56.5\%$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293\left(8^{0.4}\right) = 673.1\,K$$

$$Q_{in} = mc_v(T_3 - T_2) = 1x718(1773 - 673.1) = 789700\ J/kg = 789.7\ kJ/kg$$

$$W_{nett} = \eta Q_{in} = 0.56x789.7 = 446.2\ kJ/kg$$

From the gas law we have

$$\mathrm{p_3} = \frac{\mathrm{p_1 V_1 T_3}}{\mathrm{T_1 V_3}} = \frac{100000\ \mathrm{x}\ \mathrm{V_1}\ \mathrm{x}\ 1773}{293\ \mathrm{x}\ \mathrm{V_3}}$$

$$\frac{\mathrm{V_1}}{\mathrm{V_3}} = 8$$

$$\mathrm{p_3} = \frac{100000\ \mathrm{x}\ 1773}{293}\ \mathrm{x}\ 8 = 4.84\ \mathrm{MPa}$$

If you have followed the principles used here you should be able to solve any cycle.

SELF ASSESSMENT EXERCISE No.1

Take Cv = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40°C respectively. Calculate the following.

   i. The thermal efficiency. (58.5%)
   ii. The maximum cycle temperature. (1450 K).
   iii. The maximum pressure. (4.17 MPa).
   iv. The net work output per kg of air. (293 kJ/kg).

2. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

   The pressure and temperature before compression are 105 kPa and 25°C respectively. The net work output is 500 kJ/kg). Calculate the following.

   i. The heat input. (833 kJ/kg).
   ii. The maximum temperature. (1 906 K)
   iii. The maximum pressure. (6.64 MPa).

3. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40°C respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

   i. The thermal efficiency. (59.4%).
   ii. The net work output. (4.1 kJ/cycle)
   iii. The net power output. (205 kW).

4. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300°C and 20°C respectively. (1.235 kg).

5. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air. (871 kJ/kg and 1129 kJ/kg).

Now let's move on to study engines with compression ignition.

3 COMPRESSION IGNITION ENGINES

The invention of compression ignition engines, commonly known as diesel engines, was credited to Rudolf Diesel, although many other people worked on similar engines. The basic principle is that when high compression ratios are used, the air becomes hot enough to make the fuel detonate without a spark. Diesel's first engine used coal dust blasted into the combustion chamber with compressed air. This developed into blasting in oil with compressed air. In modern engines the fuel oil is injected directly into the cylinder as fine droplets. There are two ideal cycles for these engines, the Diesel Cycle and the Dual Combustion Cycle.

3.1 DUAL COMBUSTION CYCLE

This is the air standard cycle for a modern fast running diesel engine. First the air is compressed isentropically making it hot. Fuel injection starts before the point of maximum compression. After a short delay in which fuel accumulates in the cylinder, the fuel warms up to the air temperature and detonates causing a sudden rise in pressure. This is ideally a constant volume heating process. Further injection keeps the fuel burning as the volume increases and produces a constant pressure heating process. After cut off, the hot air expands isentropically and then at the end of the stroke, the exhaust valve opens producing a sudden drop in pressure. This is ideally a constant volume cooling process. The ideal cycle is shown in figure 6.

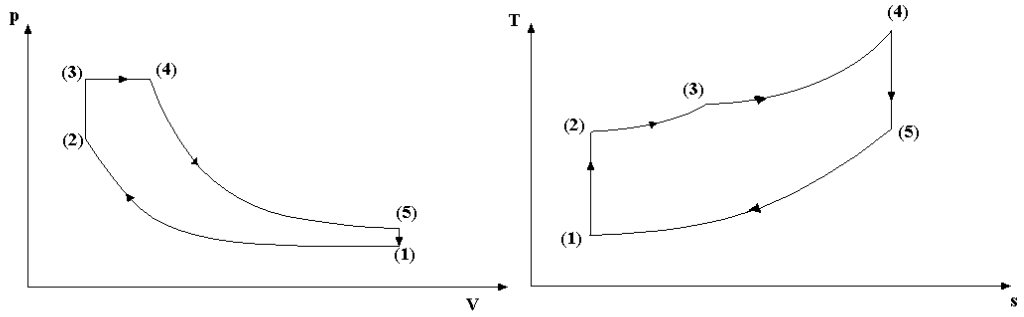

Fig. 6

The processes are as follows.

1 - 2 reversible adiabatic (isentropic) compression.

2 - 3 constant volume heating.

3 - 4 constant pressure heating.

4 - 5 reversible adiabatic (isentropic) expansion.

5 - 1 constant volume cooling.

The analysis of the cycle is as follows.

The heat is supplied in two stages hence $Q_{in} = mC_p(T_4 - T_3) + mC_V(T_3 - T_2)$

The heat rejected is $Q_{out} = mC_V(T_5 - T_1)$

The thermal efficiency may be found as follows.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

The formula can be further developed to show that

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}}$$

r_v is the VOLUME COMPRESSION RATIO. $r_v = V_1/V_2$
β is the CUT OFF RATIO. $\beta = V_4/V_3$
k is the ratio p_3/p_2.

Most students will find this adequate to solve problems concerning the dual combustion cycle. Generally, the method of solution involves finding all the temperatures by application of the gas laws.

Those requiring a detailed analysis of the cycle should study the following derivation.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Obtain all the temperatures in terms of T_2

Isentropic compression 1 to 2

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_2}{r_v^{\gamma-1}}$$

Constant volume heating 2 to 3 note $V_3 = V_2$

$$T_3 = \frac{p_3 V_3 T_2}{p_2 V_2} = \frac{p_3 T_2}{p_2} = kT_2$$

Constant pressure heating 3 to 4 note $p_3 = p_4$

$$T_4 = \frac{p_4 V_4 T_3}{p_3 V_3} = \frac{V_4 T_3}{V_3} = \beta T_3 = \beta k T_2$$

Isentropic expansion 4 to 5

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = T_4\left(\frac{V_4 V_2}{V_5 V_2}\right)^{\gamma-1} = T_4\left(\frac{\beta}{r_v}\right)^{\gamma-1} = \frac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}}$$

Substitute for all temperatures in the efficiency formula.

$$\eta = 1 - \frac{\dfrac{k\beta^{\gamma} T_2}{r_r^{\gamma\ 1}} - \dfrac{T_2}{r_v^{\gamma\ 1}}}{(kT_2 - T_2) + \gamma(\beta k T_2 - kT_2)} = 1 - \frac{\dfrac{k\beta^{\gamma}}{r_r^{\gamma-1}} - \dfrac{1}{r_v^{\gamma-1}}}{(k-1) + \gamma(\beta k - k)}$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}}$$

Note that if β=1, the cycle becomes an Otto cycle and the efficiency formulae becomes the same as for an Otto cycle.

WORKED EXAMPLE No. 2

In a dual combustion cycle, the compression starts from 1 bar and 20^{o}C. The compression ratio is 18/1 and the cut off ratio is 1.15. The maximum cycle pressure is 1360 K. The total heat input is 1 kJ per cycle. Calculate the following.

i. The thermal efficiency of the cycle.

ii. The net work output per cycle.

Check that the efficiency does not contravene the Carnot principle.

SOLUTION

Known data.

T_1 = 20 +273 = 293 K The hottest temperature is T_4 = 1360K.

β=1.15 r_v = 18 γ=1.4

$$T_2 = T_1 r_v^{\gamma-1} = 293 \text{ x } 18^{0.4} = 931 \text{ K}$$

$$T_3 = \frac{V_3 T_4}{V_4} = \frac{T_4}{\beta} = \frac{1360}{1.15} = 1183 \text{ K}$$

$$\frac{p_3}{p_2} = k = \frac{T_3}{T_2} = 1.27$$

$$\eta = 1 - \frac{k\beta^\gamma - 1}{[(k-1)+\gamma k(\beta-1)]r_v^{\gamma-1}} = 1 - \frac{1.27 \text{ x } 1.15^{1.4} - 1}{[(1.27-1)+(1.4 \text{ x } 1.27 \text{ x } (1.15-1))] \text{ x } 18^{0.4}}$$

$$\eta = 0.68 \text{ or } 68\%$$

$W_{nett} = \eta$ x Q_{in} = 0.68 x 1 = 0.68 kJ per cycle.

The Carnot efficiency should be higher.

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{293}{1360} = 0.785$$

The figure of 0.68 is lower so the Carnot principle has not been contravened.

WORKED EXAMPLE No.3

A dual combustion cycle has a compression ratio of 18/1. The maximum pressure in the cycle is 9 MPa and the maximum temperature is 2000°C. The pressure and temperature before compression is 115 kPa and 25°C respectively. Calculate the following.

i. the cut off ratio.
ii. the cycle efficiency.
iii. the net work output per kg of air.

Assume γ =1.4 C_p = 1.005 kJ/kgK C_v = 0.718 kJ/kg K.

SOLUTION

Known data.

T_1 = 298 K T_4 = 2273 K $p_3 = p_4$ = 9 MPa p_1 = 115 kPa

$V_1/V_2 = V_1/V_3 = 18$ $V_2 = V_3$

$T_2 = 298 \text{ x } 18^{(\gamma-1)} = 947 \text{ K}$

$$T_3 = \frac{p_3 T_1 V_3}{p_1 V_1} = \frac{9 \text{ x } 10^6 \text{ x } 298}{115 \text{ x } 10^3} \text{ x } \frac{V_3}{V_1} = \frac{9 \text{ x } 10^6 \text{ x } 298}{115 \text{ x } 10^3} \text{ x } \frac{1}{18} = 1296\text{K}$$

$$\text{Cut off ratio} = \beta = \frac{V_4}{V_3} = \frac{p_3 T_4}{p_4 T_3} \quad \text{but } p_4 = p_3 \text{ so } \beta = \frac{T_4}{T_3}$$

$$\beta = \frac{2273}{1296} = 1.75$$

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma\text{-1}} \quad \text{but } \frac{V_4}{V_5} = \frac{V_4}{V_3} \text{ x } \frac{V_3}{V_5} = \frac{1.75}{18} = 0.0974$$

$$T_5 = 2273 \text{ x } 0.0974^{0.4} = 895.6 \text{ K}$$

$Q_{in} = mC_p(T_4 - T_3) + mC_v(T_3 - T_2)$ m= 1 kg

Q_{in} =1.005(2274 -1296) + 0.718(1296-947) = 1232.5 kJ/kg

$Q_{out} = mC_v(T_5 - T_1)$

Q_{out} = 0.718(895.6 - 298) = 429 kJ/g

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{429}{1232} = 0.65 \text{ or } 65\%$$

$$W_{nett} = Q_{in} \quad Q_{out} = 1232 \quad 428.6 = 803.5 \text{ kJ/kg}$$

3.2 THE DIESEL CYCLE

The Diesel Cycle precedes the dual combustion cycle. The Diesel cycle is a reasonable approximation of what happens in slow running engines such as large marine diesels. The initial accumulation of fuel and sharp detonation does not occur and the heat input is idealised as a constant pressure process only.

Again consider this cycle as being carried out inside a cylinder fitted with a piston. The p-V and T-s cycles diagrams are shown in figure 7

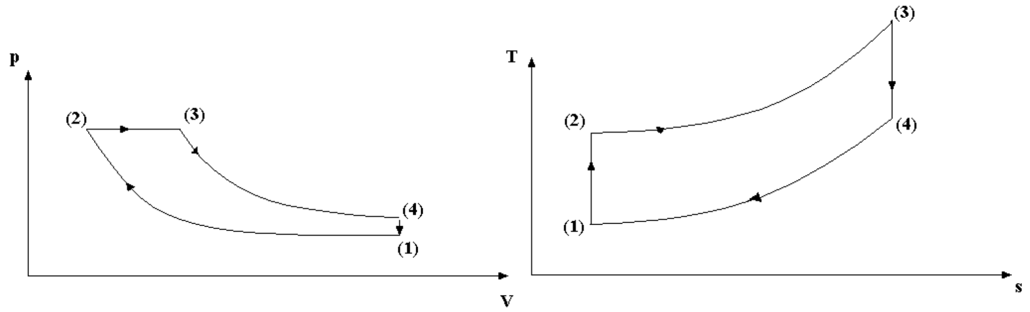

Fig. 7

1 - 2 reversible adiabatic (isentropic) compression.

2 - 3 constant pressure heating.

3 - 4 reversible adiabatic (isentropic) expansion.

4 - 1 constant volume cooling.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

The cycle is the same as the dual combustion cycle without the constant volume heating process. In this case since k=1 the efficiency is given by the following formula.

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma - 1}}$$

WORKED EXAMPLE No.4

An engine using the Diesel Cycle has a compression ratio of 20/1 and a cut off ratio of 2. At the start of the compression stroke the air is at 1 bar and 15°C. Calculate the following.

i. The air standard efficiency of the cycle.
ii. The maximum temperature in the cycle.
iii. The heat input.
iv. The net work output.

SOLUTION

Initial data.

$\beta=2$ $r_v=20$ $\gamma=1.4$ c_v = 718 J/kg K for air T_1=288 K p_1=1 bar.

The maximum temperature is T_3 and the maximum pressure is p_3 and p_2.

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

$$\eta = 1 - \frac{2^{1.4} - 1}{(2-1) \text{ x } 1.4 \text{ x } 20^{0.4}}$$

$$\eta = 1 - \frac{1.639}{1 \text{ x } 1.4 \text{ x } 3.314} = 0.647 \text{ or } 64.7\%$$

$$T_2 = T_1 r_v^{\gamma-1} = 288 \text{ x } 20^{0.4} = 954.5 \text{ K}$$

$$T_3 = \frac{V_2}{V_3} T_2 = \beta T_2 = 954.3 \text{ x } 2 = 1909 \text{ K}$$

$$Q_{in} = mc_p(T_3 - T_2)$$

$$Q_{in} = 1.005(1909 - 954.5) = 959.3 \text{ kJ}$$

$$\eta = \frac{W_{nett}}{Q_{in}}$$

$$W_{nett} = \eta Q_{in} = 0.647 \text{ x } 959.3 = 620.6 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No.2

Use c_v = 0.718 kJ/kg K , c_p = 1.005 kJ/kg K and γ=1.4 throughout.

1. Draw a p - V and T - s diagram for a Diesel Cycle.

The performance of a compression ignition engine is to be compared to the Diesel cycle. The compression ratio is 16. The pressure and temperature at the beginning of compression are 1 bar and 15°C respectively. The maximum temperature in the cycle is 1200 K.

Calculate the following.

i. The cut off ratio.(1.374)

ii. The air standard efficiency. (66%)

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

i. The net work output per cycle. (680 kJ/kg).
ii. The thermal efficiency. (57.6 %).

3. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30°C and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

i. The maximum cycle temperature. (2424 K).
ii. The net work output per cycle. (864 kJ/kg).
iii. The thermal efficiency. (67.5 %).

4 GAS TURBINES

A gas turbine engine normally burns fuel in the air that it uses as the working fluid. From this point of view it is an internal combustion engine that uses steady flow processes. Figure 8 shows a basic design.

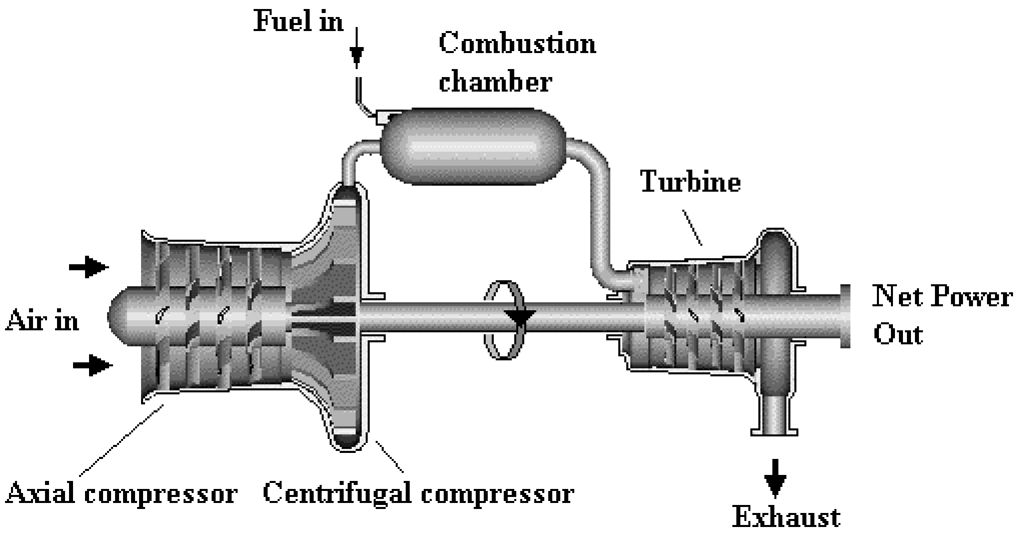

Fig.8

The air is drawn in from atmosphere and compressed. This makes it hotter. The compressed air is blown into a combustion chamber and fuel is burned in it making it even hotter. This makes the volume increase. The hot air expands out of the chamber through a turbine forcing it to revolve and produce power. The air becomes colder as it expands and eventually exhausts to atmosphere. The temperature drop over the turbine is larger than the temperature increase over the compressor. The turbine produces more power than is needed to drive the compressor. Net power output is the result. In the basic system, the turbine is coupled directly to the compressor and the power output is taken from the same shaft. The ideal air standard cycle is the Joule Cycle.

4.1 THE JOULE CYCLE

The Joule Cycle is also known as the constant pressure cycle because the heating and cooling processes are conducted at constant pressure. The cycle is that used by a gas turbine engine but could conceivably be used in a closed system.

We may draw the layout in block diagram form as shown in figure 9

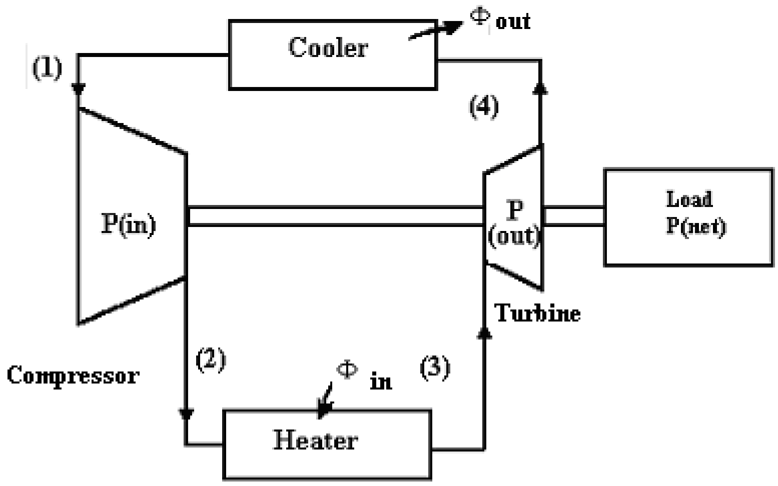

Figure 9

There are 4 ideal processes in the cycle.

1 - 2 Reversible adiabatic (isentropic) compression requiring power input.
$P_{in} = \Delta H/s = mC_p(T_2-T_1)$

2 - 3 Constant pressure heating requiring heat input.
$\Phi_{in} = \Delta H/s = mC_p(T_3-T_2)$

3 - 4 Reversible adiabatic (isentropic) expansion producing power output.
$P_{out} = \Delta H/s = mC_p(T_3-T_4)$

4 - 1 Constant pressure cooling back to the original state requiring heat removal.
$\Phi_{out} = \Delta H/s = mC_p(T_4-T_1)$

The pressure – volume, pressure - enthalpy and temperature-entropy diagrams are shown in figure 10

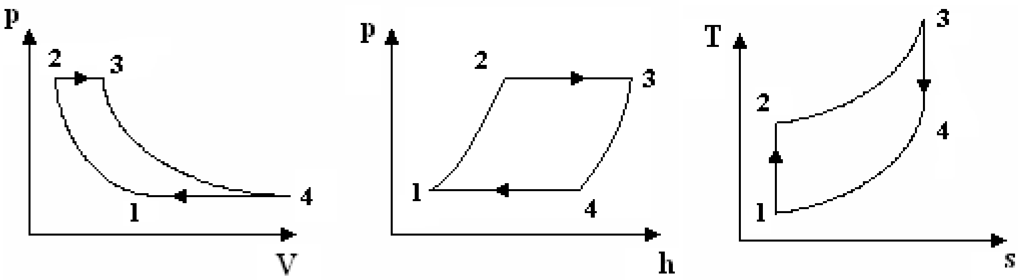

Fig. 10

The efficiency is found by applying the first law of thermodynamics.

$$\Phi_{nett} = P_{nett}$$

$$\Phi_{in} - \Phi_{out} = P_{out} - P_{in}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

It assumed that the mass and the specific heats are the same for the heater and cooler.

It is easy to show that the temperature ratio for the turbine and compressor are the same.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = r_p^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{1-\frac{1}{\gamma}} = r_p^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \frac{T_2}{T_1}$$

r_p is the pressure compression ratio for the turbine and compressor.

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\left(\frac{T_3 T_1}{T_2} - T_1\right)}{\left(\frac{T_2 T_4}{T_1} - T_2\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \qquad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_p^{1-\frac{1}{\gamma}}} = 1 - r_p^{-0.286} \quad \text{since } \gamma = 1.4$$

This shows that the efficiency depends only on the pressure ratio which in turn affects the hottest temperature in the cycle.

WORKED EXAMPLE No. 5

A gas turbine uses the Joule cycle. The pressure ratio is 6/1. The inlet temperature to the compressor is 10°C. The flow rate of air is 0.2 kg/s. The temperature at inlet to the turbine is 950°C. Calculate the following.

i. The cycle efficiency.

ii. The heat transfer into the heater.

iii. The net power output.

$\gamma = 1.4$ $C_p = 1.005$ kJ/kg K

SOLUTION

$$\eta_{th} = 1 - r_p^{-0.286} = 1 - 6^{-0.286} = 0.4 \text{ or } 40\%$$

$$T_2 = T_1 r_p^{0.286} = 283 \text{ x } 6^{0.286} = 472.4K$$

$$\Phi_{in} = mc_p(T_3 - T_2) = 0.2 \text{ x } 1.005 \text{ x } (1223 - 472.4) = 150.8 \text{ kW}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}}$$

$$P_{nett} = 0.4 \text{ x } 150.8 = 60.3 \text{ kW}$$

SELF ASSESSMENT EXERCISE No.3

γ= 1.4 and C_p = 1.005 kJ/kg K throughout.

1. A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10^{o}C. After constant pressure heating, the pressure and temperature are 7 bar and 700^{o}C respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

   i. The cycle efficiency.

   ii. The heat transfer into the heater.

   iii. The net power output.

   (Answers 42.7 % , 206.7 kW and 88.26 kW)

2. A gas turbine expands draws in 3 kg/s of air from atmosphere at 1 bar and 20^{o}C. The combustion chamber pressure and temperature are 10 bar and 920^{o}C respectively. Calculate the following.

   i. The Joule efficiency.

   ii. The exhaust temperature.

   iii. The net power output.

   (Answers 48.2 % , 617.5 K and 911 kW)

3. A gas turbine draws in 7 kg/s of air from atmosphere at 1 bar and 15^{o}C. The combustion chamber pressure and temperature are 9 bar and 850^{o}C respectively. Calculate the following.

   i. The Joule efficiency.

   ii. The exhaust temperature.

   iii. The net power output.

   (Answers 46.7 % , 599 K and 1.916 MW)