THERMODYNAMICS
TUTORIAL 4 – POLYTROPIC PROCESSES

This is set at level QCF 3 and 4

On completion of this tutorial you should be able to apply thermodynamic principles to the expansion and compression of gases and vapours.

You should be able to explain and solve problems involving:

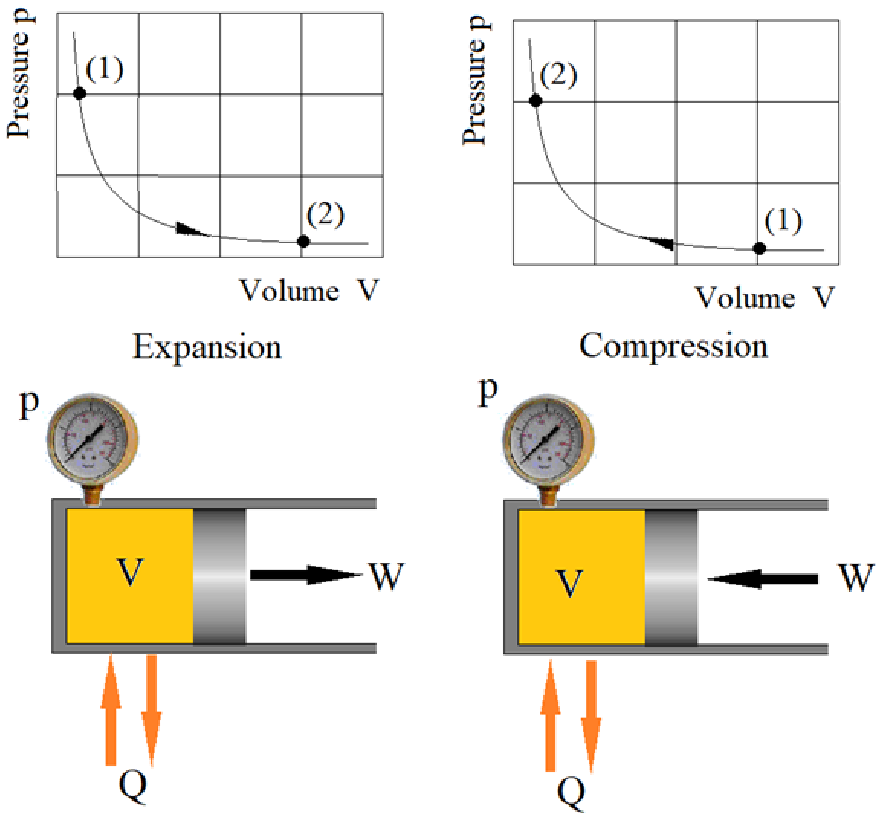
- Isobaric processes
- Isothermal processes
- Adiabatic processes

Contents

1. Polytropic Expansions and Compressions

Gas is expanded under many circumstances such as when released from a container but in this unit we are concerned with gas that expands in a controlled manner such as in an engine or turbine. Gas compressions are the reverse of this and conducted in compressors of many types. Let's consider a gas that is trapped in a cylinder and it may be compressed by moving the piston reducing the volume or expanding it by letting the volume get bigger.

When the volume of a compressible fluid changes, the pressure and temperature may also change. The resulting pressure depends upon the final temperature. The final temperature depends on whether the fluid is cooled or heated during the process. It is normal to show these changes on a graph of pressure plotted against volume. (p - V graphs). A typical graph for a compression and an expansion process is shown.



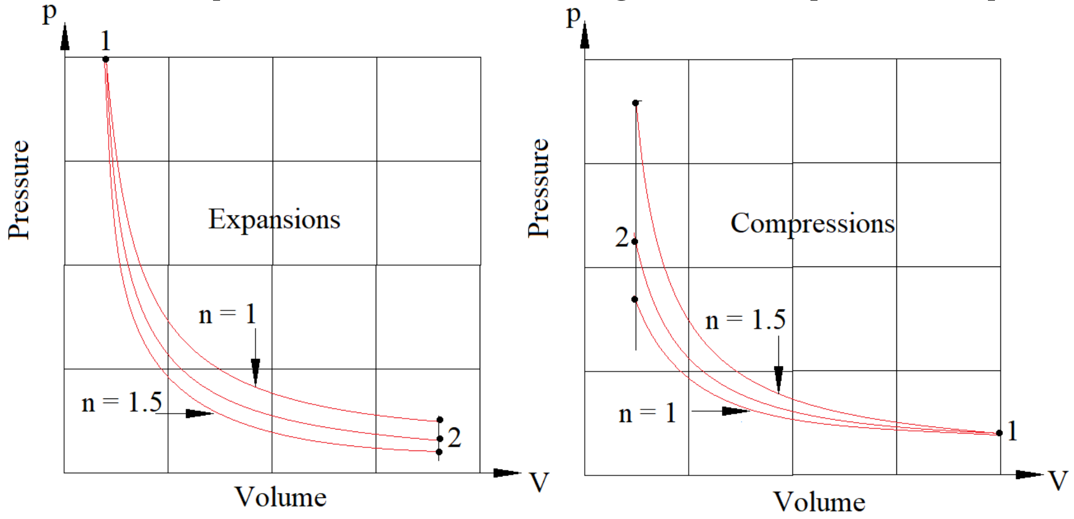
Depending on whether the fluid is heated or cooled as it is expanded or compressed, a family of such curves is obtained as shown next. It has been discovered that these curves follows the mathematical law

$$\mathbf{pV^n = constant}$$

Each graph has a different value of n and n is called the index of expansion or compression.

The diagrams are called p -V graphs and they are very useful for explaining things about the processes.

Consider the process taking place between a single starting point (1) and different finishing point (2). Cooling produces a lower final pressure and heating produces a higher final pressure so lower values of n (e.g. 1) indicates cooling and higher values of n (e.g. 1.5) indicate heating. Somewhere in between there is a value of n that represents neither heat nor cooling and this is a special case explained later.



The following derivation is not strictly needed at this level but you would do well to study it. We will start by studying the expansion of a fluid inside a cylinder against a piston which may do work against the surroundings.

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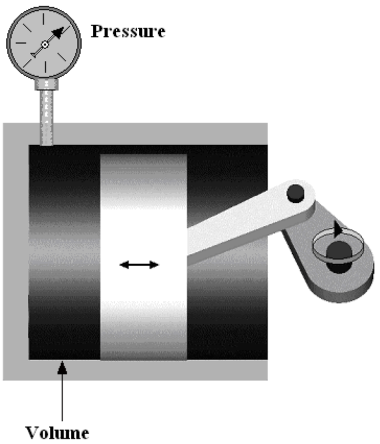


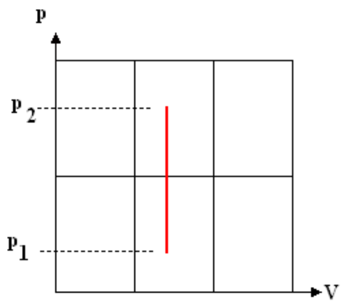
At any moment the pressure and volume are p and V.

We have covered the gas law PV/T = constant and the gas will obey this law.

The way the pressure and volume change depends on what happens to the temperature of the gas. If we compress a gas it naturally tends to get hotter and if we expand it, it tends to get cooler. We can affect what happens by either heating or cooling the gas at the same time or doing nothing. Such processes are called ***Polytropic*** which means various degrees of heat transfer into or out of the gas. The process is best explained with a ***pressure - volume graph***.

1.1 Constant Volume also known as Isochoric

A vertical graph is a constant volume process and so it is neither a compression nor expansion. Since no movement of the piston occurs no work is done. Nevertheless, it still fits the law with n having a value of infinity.



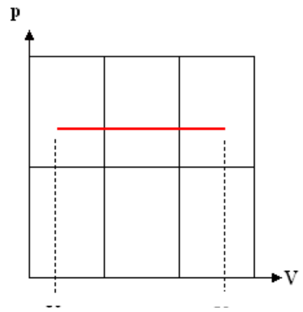
$$p_1V_1^n = p_2V_2^n \qquad V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$$

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\infty}} = V_1\left(\frac{p_1}{p_2}\right)^{0} = V_1$$

Note anything to the power of zero is 1.

1.2 Constant Pressure also known as Isobaric

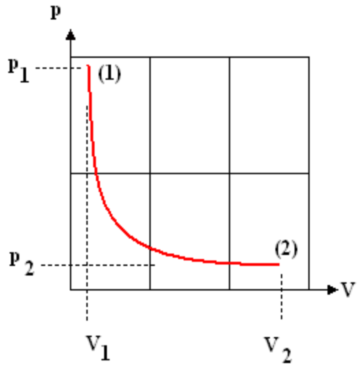
A horizontal graph represents a change in volume with no pressure change (constant pressure process). The value of n is zero in this case.



$$p_1V_1^n = p_2V_2^n \qquad p_1 = \frac{p_2V_2^n}{V_1^n} = \frac{p_2V_2^0}{V_1^0} = p_2\frac{1}{1} = p_2$$

1.3 Constant Temperature also known as Isothermal

All the graphs in between constant volume and constant pressure represent processes with a value of n between infinity and zero. One of these represents the case when the temperature is maintained constant by cooling or heating by just the right amount.



For a gas Boyle's Law applies.

pV = constant so it follows that n is 1

The graph is a hyperbola so the name hyperbolic process is also used.

1.4 Adiabatic Process

When the pressure and volume change in such a way that no heat is added nor lost from the gas (e.g. by using an insulated cylinder), the process is called ***adiabatic***. This is an important process and is the one that occurs when the change takes place so rapidly that there is no time for heat transfer to occur. This process represents a demarcation between those in which heat flows into the gas and those in which heat flows out of the gas. In order to show it is special, the symbol γ (gamma) is used instead of n and the law is:

$$\mathbf{pV^{\gamma} = C}$$

It will be found that each gas has a special value for γ (e.g. 1.4 for dry air). Remember that all the processes and all in between are examples of *Polytropic* expansion or compression.

WORKED EXAMPLE No. 1

A gas is compressed from 1 bar and 100 cm^3 to 20 cm^3 by the law $pV^{1.3}$ = constant. Calculate the final pressure.

SOLUTION

If $pV^{1.3} = C$ then $p_1V_1^{1.3} = C = p_2V_2^{1.3}$ hence $1 \times 100^{1.3} = p_2 \times 20^{1.3}$

$$p_2 = 1 \times \left(\frac{100}{20}\right)^{1.3} = 8.1 \text{ bar}$$

WORKED EXAMPLE No. 2

Gas at 10 bar and 30 cm^3 is expanded to 1 bar by the law $pV^{1.2} = C$. Find the final volume.

SOLUTION.

$p_1V_1^{1.2} = C = p_2V_2^{1.2}$

$10 \times 30^{1.2} = 1 \times V_2^{1.2}$ $\qquad V_2 = (592.3)^{1/1.2} = 204.4 \text{ cm}^3$

WORKED EXAMPLE No. 3

A gas is compressed from 200 kPa and 120 cm^3 to 30 cm^3 and the resulting pressure is 1 MPa. Calculate the index of compression n.

SOLUTION.

$200 \times 120^n = 1\,000 \times 30^n$

$$\left(\frac{120}{30}\right)^n = \frac{1\,000}{200} = 5$$

$$(4)^n = 5$$

$$n \log 4 = \log 5$$

$$n = \frac{\log 5}{\log 4} = \frac{1.6094}{1.3863} = 1.161$$

Note this may be solved with natural or base 10 logs or directly on suitable calculators.

SELF ASSESSMENT EXERCISE No. 1

1. A vapour is expanded from 12 bar and 50 cm^3 to 150 cm^3 and the resulting pressure is 6 bar. Calculate the index of compression n.
(0.63)

2.a. A gas is compressed from 200 kPa and 300 cm^3 to 800 kPa by the law $pV^{1.4}$=C. Calculate the new volume. (111.4 cm^3)

2.b. The gas was at 50°C before compression. Calculate the new temperature using the gas law pV/T = C. (207°C)

3.a. A gas is expanded from 2 MPa and 50 cm^3 to 150 cm^3 by the law $pV^{1.25}$ = C. Calculate the new pressure. (506 kPa)

3.b. The temperature was 500°C before expansion. Calculate the final temperature.
(314°C)

2. Combining the Gas Law with the Polytropic Law.

For gases only, the general law may be combined with the law of expansion as follows.

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \text{ so } \frac{T_2}{T_1} = \frac{p_2V_2}{p_1V_1}$$

Since for an expansion or compression

$$p_1V_1^n = p_2V_2^n \text{ so } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

Substituting into the gas law we get

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

Further since

$$\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \frac{V_1}{V_2}$$

Substituting into the gas law gives

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

To summarise we have found that

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

In the case of an adiabatic process this is written as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$$

For an isothermal process n = 1 and the temperatures are the same.

WORKED EXAMPLE No. 4

A gas is compressed adiabatically with a volume compression ratio of 10. The initial temperature is 25°C. Calculate the final temperature given $\gamma = 1.4$

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298(10)^{1.4-1} = 748.5\text{ K or }475.5^{o}\text{C}$$

WORKED EXAMPLE No. 5

A gas is compressed polytropically by the law $pV^{1.2} = C$ from 1 bar and 20°C to 12 bar. calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} = 293(12)^{1-\frac{1}{1.2}}$$

$$T_2 = 293(12)^{0.167} = 293 \times 1.513 = 443.3\text{ K}$$

WORKED EXAMPLE No. 6

A gas is expanded from 900 kPa and 1 100°C to 100 kPa by the law $pV^{1.3} = C$.
Calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

$$T_2 = 1\,373\left(\frac{100}{900}\right)^{1-\frac{1}{1.3}} = 1\,373(0.111)^{0.2308} = 1\,373 \times 0.602 = 826.9\text{ K}$$

3. Link Between c_v c_p and R.

From the above work it is apparent that: $\Delta H = m\,c_p\,\Delta T = \Delta U + p\,\Delta V$

We have already defined

$$\Delta U = m\,c_v\,\Delta T$$

Furthermore for a gas only

$$p\,\Delta V = m\,R\,\Delta T$$

Hence

$$m\,c_p\,\Delta T = m\,c_v\,\Delta T + m\,R\,\Delta T$$

Hence

$$\mathbf{c_p = c_v + R} \text{ and } \mathbf{R = c_p - c_v} \text{ (the difference)}$$

SELF ASSESSMENT EXERCISE No. 2

1. A gas is expanded from 1 MPa and 1 000°C to 100 kPa. Calculate the final temperature when the process is

   i. Isothermal (n = 1) (1 000°C)

   ii Polytropic (n = 1.2) (594°C)

   iii. Adiabatic (γ = 1.4) (386°C)

   iv. Polytropic (n = 1.6) (264°C)

2. A gas is compressed from 120 kPa and 15°C to 800 kPa. Calculate the final temperature when the process is

   i. Isothermal (n = 1) (15°C)

   ii. Polytropic (n = 1.3) (173°C)

   iii Adiabatic (γ = 1.4) (222°C)

   iv. Polytropic (n = 1.5) (269°C)

3. A gas is compressed from 200 kPa and 20°C to 1.1 MPa by the law $pV^{1.3}$=C.
   Calculate

   i. The final temperature. (434 K)
   ii. The change in internal energy (2.027 kJ)
   iii. The change in enthalpy (2.838 kJ)

4. A gas is expanded from 900 kPa and 1200°C to 120 kPa by the law $pV^{1.4}$ = C.

   Calculate

   i. The final temperature. (828 K)
   ii. The change in internal energy (-7.25 kJ)
   iii. The change in enthalpy (-10.64 kJ)

4. *Examples Involving Vapour*

Problems involving vapour make use of the formulae $pV^n = C$ in the same way as those involving gas. You cannot apply gas laws, however, unless it is superheated into the gas region. You must make use of vapour tables so a good understanding of this is essential. This is best explained with worked examples.

WORKED EXAMPLE No. 7

A steam turbine expands steam from 20 bar and 300°C to 1 bar by the law $pV^{1.2} = C$.

Determine for each kg flowing:

a. the initial and final volume.
b. the dryness fraction after expansion.
c. the initial and final enthalpies.
d. the change in enthalpy.

SOLUTION

The system is a steady flow system in which expansion takes place as the fluid flows. The law of expansion applies in just the same way as in a closed system.

The initial volume is found from steam tables. At 20 bar and 300°C it is superheated and from the tables we find $v = 0.1255 \text{ m}^3/\text{kg}$

Next apply the law $pV^{1.2} = C \quad p_1V_1^{1.2} = p_2V_2^{1.2} \qquad 20 \times 0.1255^{1.2} = 1 \times V_2^{1.2}$

Hence $V_2 = 1.523 \text{ m}^3/\text{kg}$

Next, find the dryness fraction as follows.
Final volume = $1.523 \text{ m}^3/\text{kg} = xv_g$ at 1 bar.
From the tables we find v_g is $1.694 \text{ m}^3/\text{kg}$

hence $1.523 = 1.694x \qquad x = 0.899$

We may now find the enthalpies in the usual way.

h_1 at 20 bar and 300°C is 3 025 kJ/kg
$h_2 = h_f + xh_{fg}$ at 1 bar (wet steam)
$h_2 = 417 + (0.899)(2\,258) = 2\,447$ kJ/kg

The change in enthalpy is $h_2 - h_1 = -578$ kJ/kg

SELF ASSESSMENT EXERCISE No. 3

1. 3 kg/s of steam is expanded in a turbine from 10 bar and 200°C to 1.5 bar by the law $pV^{1.2} = C$. Determine the following.

   i. The initial and final volumes. (0.618 m^3 and 3 m^3)

   ii. The dryness fraction after expansion. (0.863)

   iii. The initial and final enthalpies. (2 829 kJ/kg and 2 388 kJ/kg)

   iv. The change in enthalpy. -1 324 kW)

2. 1.5 kg/s of steam is expanded from 70 bar and 450°C to 0.05 bar by the law $pV^{1.3} = C$. Determine the following.

   i. The initial and final volumes. (0.066 m^3/kg and 17.4 m^3/kg)

   ii. The dryness fraction after expansion. (0.411)

   iii. The initial and final enthalpies. (3 287 kJ/kg and 1 135 kJ/kg)

   iv. The change in enthalpy. (-3 228 kW)

1. A horizontal cylindrical vessel is divided into two sections each 1m^3 volume, by a non-conducting piston. One section contains steam of dryness fraction 0.3 at a pressure of 1 bar, while the other contains air at the same pressure and temperature as the steam. Heat is transferred to the steam very slowly until its pressure reaches 2 bar.

   Assume that the compression of the air is adiabatic (γ=1.4) and neglect the effect of friction between the piston and cylinder. Calculate the following.

   i. The final volume of the steam. (1.39 m^3)

   ii. The mass of the steam. (1.97 kg)

   iii. The initial internal energy of the steam. (2 053 kJ)

   iv The final dryness fraction of the steam. (0.798)

   v. The final internal energy of the steam. (4 172 kJ)

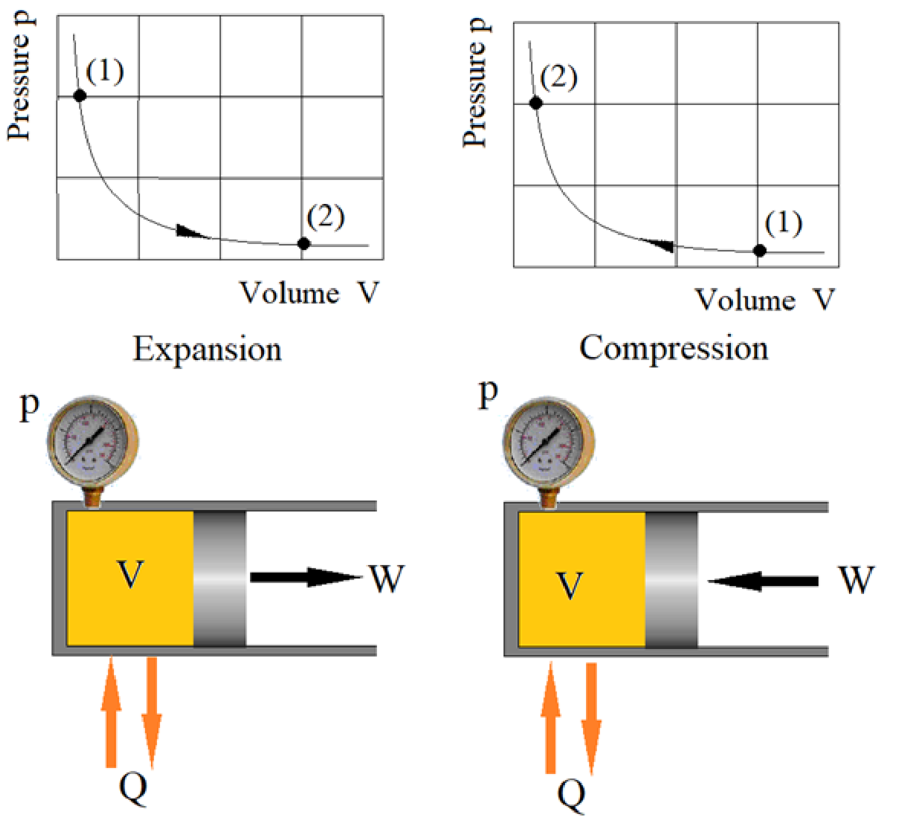
   vi. The heat added to the steam. (2 119 kJ)

5. *Work Done by Expansion and Compression of a Gas against a Piston*

In this section you will learn how to calculate the work done when a gas changes volume and acts against a piston in a cylinder. It will be shown that:

$$\mathbf{W = -\int_{V_1}^{V_2} pdV}$$

When the volume of a compressible fluid changes, the pressure and temperature may also change. The resulting pressure depends upon the final temperature. The final temperature depends on whether the fluid is cooled or heated during the process. It is normal to show these changes on a graph of pressure plotted against volume. (p - V graphs). A typical graph for a compression and an expansion process is shown.



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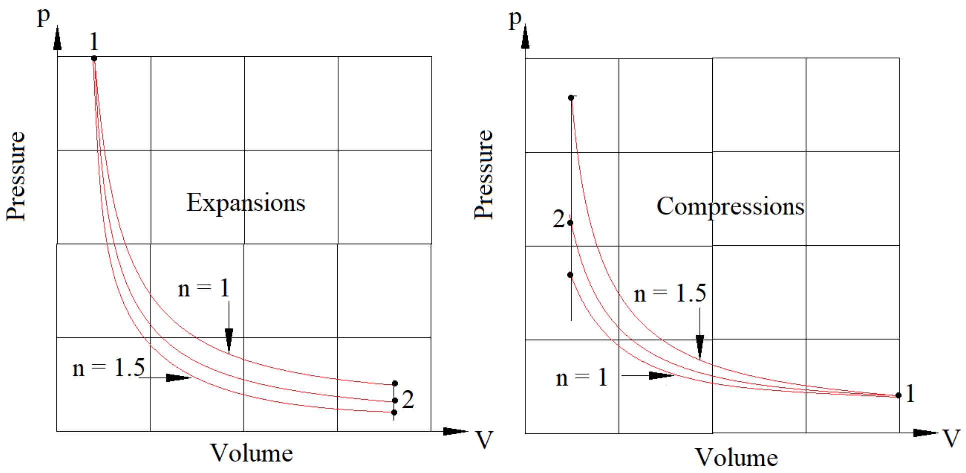
The diagrams are called p -V graphs and they are very useful for explaining things about the processes.

Consider the process taking place between a single starting point (1) and different finishing point (2). Cooling produces a lower final pressure and heating produces a higher final pressure so lower values of n (e.g. 1) indicates cooling and higher values of n (e.g. 1.5) indicate heating. Somewhere in between there is a value of n that represents neither heat nor cooling and this is a special case explained later.



The following derivation is not strictly needed at this level but you would do well to study it. We will start by studying the expansion of a fluid inside a cylinder against a piston which may do work against the surroundings.

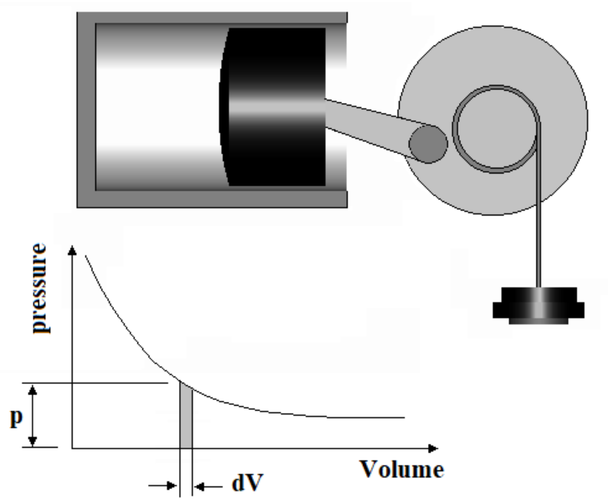
5.1 *Derivation of Work Law*

A fluid may expand in two ways.

a) It may expand rapidly and uncontrollably doing no useful work. In such a case the pressure could not be plotted against volume during the process. This is called an ***Unresisted Expansion***

b) It may expand moving the piston. The movement is resisted by external forces (such as the weights on the pulley shown below) so the gas pressure changes in order to overcome the external force and move the piston. In this case the movement is controlled and the variation of pressure with volume may be recorded and plotted on a p-V graph. Work is done against the surroundings. This process is called a ***Resisted Expansion.***

Consider the arrangement shown below. Movement of the piston left to right raises the weights on the pulley. Movement the other way lowers them.



Assume that there is no pressure outside. If the string holding the weight was cut, the gas pressure would slam the piston back and the energy would be dissipated first by acceleration of the moving parts and eventually as friction. The expansion would be unresisted.

If the weights were gradually reduced, the gas would push the piston and raise the remaining weights. In this way, work would be done against the surroundings (it ends up as potential energy in the weights). The process may be repeated in many small steps, with the change in volume each time being dV. The pressure although changing, is p at any time.

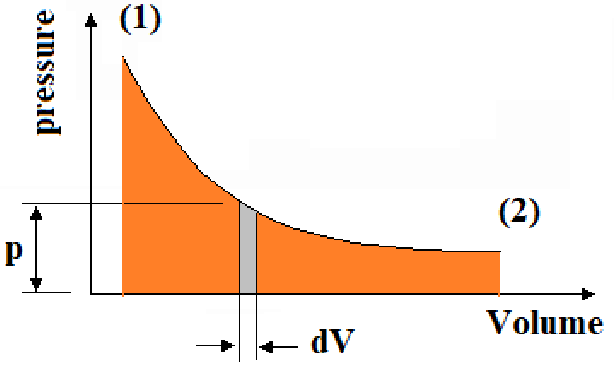
This process is characterised by two important factors.

- The process may be reversed at any time by adding weights and the potential energy is transferred back from the surroundings as work is done on the system. The fluid may be returned to its original pressure, volume, temperature and energy.

- The fluid force on one side of the piston is always exactly balanced by the external force (in this case due to the weights) on the other side of the piston.

The expansion or compression done in this manner is said to be ***Reversible and Carried Out In Equilibrium.***

5.2 Work as the Area under the p - V Diagram



If the expansion is carried out in equilibrium, the force of the fluid must be equal to the external force F. It follows that F = p A

When the piston moves a small distance dx, the work done is dW

$$dW = - F\,dx = - p\,A\,dx = - p\,dV.$$

The minus sign is because the work is leaving the system.

For an expansion from points (1) to (2) it follows that the total work done is given by

$$\mathbf{W = -\int_{V_1}^{V_2} pdV}$$

This applies to a gas expanding or compressing. We must remember at this stage that our sign convention was that work leaving the system is negative so we expect an expansion to give work out (negative) and a compression requires work input (positive).

The result is that for a fully resisted expansion or compression in a closed system the area between the p – V graph and the volume axis is the work transfer and this provides a practical way to measure work in real systems by measuring the pressure and volume in real time.

Let's now see how to evaluate the work for various processes and values of n.

6. Work Laws for Closed Systems

The work laws for a closed system are obtained by solving the expression:

$$W = -\int_{V_1}^{V_2} p\,dV$$

The solution depends upon the relationship between p and V. The formulae now derived apply equally well to a compression process and an expansion process.

6.1 Constant Pressure

$$W = -\int_{V_1}^{V_2} p\,dV = -p\int_{V_1}^{V_2} dV = -p(V_2 - V_1)$$

The work done is the pressure $\times$ the change in volume

6.2 Constant Volume

If V is constant then dV = 0 **W = 0.**

There is no work done when the volume does not change.

6.3 Hyperbolic (Isothermal Gas Expansion)

This is an expansion which follows the law $pV^1 = C$ and is ***Isothermal*** when it is a gas.
Substituting $p = CV^{-1}$ the expression becomes:

$$W = -\int_{V_1}^{V_2} p\,dV = -C\int_{V_1}^{V_2} V^{-1}dV = -C\ln\left[\frac{V_2}{V_1}\right]$$

Since pV = C then

$$W = -C\ln\left[\frac{V_2}{V_1}\right] \text{ but since } \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

$$W = -pV\ln\left[\frac{p_1}{p_2}\right]$$

In the case of gas we can substitute pV = mRT and so

$$W = -mRT\ln\left[\frac{V_2}{V_1}\right] = -mRT\ln\left[\frac{p_1}{p_2}\right]$$

6.4 Polytropic

In this case the expansion follows the law $pV^n = C$. The solution is as follows.

$$W = -\int_{V_1}^{V_2} pdV \text{ but } p = CV^{-n}$$

$$W = -C\int_{V_1}^{V_2} V^{-n}\,dV = -C\int_{V_1}^{V_2} \frac{[V_2^{-n+1} - V_1^{-n+1}]}{-n+1}$$

$C = p_1V_1$ or p_2V_2

$$W = \frac{[p_2V_2 - p_1V_1]}{n-1}$$

For gas only we may substitute pV = mRT and so:

$$W = mR\frac{[T_2 - T_1]}{n-1}$$

6.5 Adiabatic

Since an adiabatic case is the special case of a polytropic expansion with no heat transfer, the derivation is identical but the symbol γ is used instead of n.

$$W = \frac{[p_2V_2 - p_1V_1]}{\gamma-1}$$

For gas only we may substitute pV = mRT and so:

$$W = mR\frac{[T_2 - T_1]}{\gamma-1}$$

This is the special case of the polytropic process in which Q = 0.

$$W = \frac{mR\Delta T}{\gamma-1}$$

Substituting for Q and ΔU in the NFEE we find:

$$Q + W = \Delta U \quad 0 + \frac{mR\Delta T}{\gamma-1} = mC_v\Delta T$$

$$C_v = \frac{R}{\gamma-1} \quad R = C_p - C_v = C_v(\gamma-1)$$

$$\boldsymbol{\frac{C_p}{C_v} = \gamma}$$

The Adiabatic Index is the ratio of the principal specific heats

WORKED EXAMPLE No. 8

Air at a pressure of 500 kPa and volume 50 cm^3 is expanded reversibly in a closed system to 800 cm^3 by the law $pV^{1.3} = C$. Calculate the following.

a. The final pressure.

b. The work done.

SOLUTION

p_1 = 500 kPa $V_1 = 50 \times 10^{-6}\ m^3$ $V_2 = 800 \times 10^{-6}\ m^3$

$$p_1 V_1^{1.3} = p_2 V_2^{1.3}$$

$$500 \times 10^3 \times (50 \times 10^{-6})^{1.3} = p_2 \times (800 \times 10^{-6})^{1.3}$$

$$p_2 = \frac{500 \times 10^3 \times (50 \times 10^{-6})^{1.3}}{(800 \times 10^{-6})^{1.3}} = 13.6 \times 10^3 \text{ Pa or } 13.6 \text{ kPa}$$

$$W = \frac{[p_2 V_2 - p_1 V_1]}{n-1} = \frac{[13.6 \times 10^3 \times 800 \times 10^{-6} - 500 \times 10^3 \times 50 \times 10^{-6}]}{1.3 - 1}$$

W = -47 Joules

WORKED EXAMPLE No. 9

Gas at 6 bar pressure and volume 100 cm^3 is expanded reversibly in a closed system to 2 dm^3 by the law $pV^{1.2} = C$. Calculate the work done.

SOLUTION

p_1 = 6 bar $V_1 = 100 \times 10^{-6}\ m^3$ $V_2 = 2 \times 10^{-3}\ m^3$

$$p_1 V_1^{1.2} = p_2 V_2^{1.2} \quad p_2 = p_1 \frac{V_1^{1.2}}{V_2^{1.2}} = 6\left(\frac{100 \times 10^{-6}}{2 \times 10^{-3}}\right)^{1.2} = 0.1648 \text{ bar}$$

$$W = \frac{[p_2 V_2 - p_1 V_1]}{n-1} = \frac{[0.1648 \times 10^5 \times 2 \times 10^{-3} - 6 \times 10^5 \times 100 \times 10^{-6}]}{1.2 - 1}$$

W = -135.2 Joules

SELF ASSESSMENT EXERCISE No. 4

1. 10 g of gas at 10 bar and 350°C expands reversibly in a closed system to 2 bar by the law $pV^{1.3} = C$. Taking R = 452.5 J/kg K, calculate the following.

   i. The initial volume. (0.00282 m^3)

   ii. The final volume. (0.00974 m^3)

   iii. The work done. (-2.92 kJ)

2. 20 g of gas at 20°C and 1 bar pressure is compressed to 9 bar by the law $pV^{1.4} = C$. Taking the gas constant R = 287 J/kg K calculate the work done. (Note that for a compression process the work will turn out to be positive if you correctly identify the initial and final conditions). (3.67 kJ)

3. Gas at 600 kPa and 0.05 dm^3 is expanded reversibly to 100 kPa by the law $pV^{1.35} = C$. Calculate the work done. (-31.8 kJ)

4. 15 g of gas is compressed isothermally from 100 kPa and 20°C to 1 MPa pressure. The gas constant is 287 J/kg K. Calculate the work done. (2.9 kJ)

5. Gas at 10 bar with a volume of 80 cm^3 is expanded reversibly and isothermally to 1 bar by the law pV=C. Calculate the work done. (-184.2 kJ)

6. Gas fills a cylinder fitted with a frictionless piston. The initial pressure and volume are 40 MPa and 0.05 dm^3 respectively. The gas expands reversibly and polytropically to 0.5 MPa and 1 dm^3 respectively. Calculate the index of expansion and the work done. (1.463 and -3.24 kJ)

7. An air compressor commences compression when the cylinder contains 12 g at a pressure is 1.01 bar and the temperature is 20°C. The compression is completed when the pressure is 7 bar and the temperature 90°C. (1.124 and 1 944 J)

   The characteristic gas constant R is 287 J/kg K. Assuming the process is reversible and polytropic, calculate the index of compression and the work done.

WORKED EXAMPLE No. 10

0.2 kg of gas at 100 ^oC is expanded isothermally and reversibly from 1 MPa pressure to 100 kPa. Take C_V = 718 J/kg K and R = 287 J/kg K.
Calculate

i. The work transfer.
ii. The change in internal energy.
iii. The heat transfer.

SOLUTION

$$W = -pV\ln\left(\frac{V_2}{V_1}\right) = -mRT\ln\left(\frac{V_2}{V_1}\right) = -mRT\ln\left(\frac{p_1}{p_2}\right)$$

$$W = -0.2 \times 287 \times 373\ln\left(\frac{1 \times 10^6}{100 \times 10^3}\right) = -49\ 300\ \text{J or} -49.3\ \text{kJ}$$

The work is leaving the system so it is a negative work transfer.

Since T is constant $\Delta U = 0$ Q - 49.3 = 0 Q = 49.3 kJ

Note that 49.3 kJ of heat is transferred into the gas and 49.3 kJ of work is transferred out of the gas leaving the internal energy unchanged.

WORKED EXAMPLE No. 11

Repeat worked example 10 but for an adiabatic process with $\gamma = 1.4$
Calculate

SOLUTION

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = 373 \times \left(\frac{100 \times 10^3}{1 \times 10^6}\right)^{1-\frac{1}{1.4}} = 193\ \text{K}$$

$$W = -mR\frac{[T_2 - T_1]}{\gamma - 1} = -0.2 \times 287 \times \frac{[193 - 373]}{1.4 - 1} = -25\ 830\ \text{J or} -25.83\ \text{kJ}$$

For an Adibatic process Q = 0

$$Q + W = \Delta U = -25.83\ \text{kJ}$$

Check the answer

$$\Delta U = m\,c_v\,\Delta T = 0.2 \times 718(193 - 373) = -25\ 848\ \text{J}$$

WORKED EXAMPLE No. 12

Repeat worked example 11 but for a polytropic process with n = 1.25

SOLUTION

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = 373 \times \left(\frac{100 \times 10^3}{1 \times 10^6}\right)^{1-\frac{1}{1.25}} = 235.3\ \text{K}$$

$$W = -mR\frac{[T_2 - T_1]}{\gamma - 1} = -0.2 \times 287 \times \frac{[235.3 - 373]}{1.25 - 1} = -31\ 605\ \text{J or} - 31.6\ \text{kJ}$$

$\Delta U = m\, c_v\, \Delta T = 0.2 \times 718(235.3 - 373) = -19\ 773.7$ J

$Q = \Delta U - W$

$Q = -19\ 773.7 - (-31\ 605) = 11\ 831.3$ J

SELF ASSESSMENT EXERCISE No. 5

Take C_V = 718 J/kg K and R = 287 J/kg K throughout.

1. 1 dm^3 of gas at 100 kPa and 20°C is compressed to 1.2 MPa reversibly by the law $pV^{1.2}$ = C. Calculate the following.

   i. The final volume. (0.126 dm^3)
   ii. The work transfer. (257 J)
   iii. The final temperature. (170°C)
   iv. The mass. (1.189 g)
   v. The change in internal energy. (128 J)
   vi. The heat transfer. (-128 J)

2. 0.05 kg of gas at 20 bar and 1100°C is expanded reversibly to 2 bar by the law $pV^{1.3}$ =C in a closed system. Calculate the following.

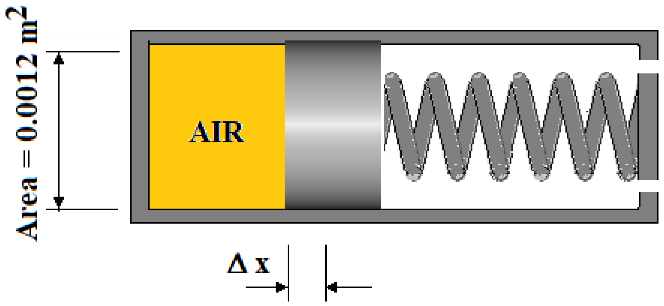
   i. The initial volume. (9.85 dm^3)
   ii. The final volume. (58 dm^3)
   iii. The work transfer. (-27 kJ)
   iv. The change in internal energy. (-20.3 kJ)
   v. The heat transfer. (6.7 kJ)

3. 0.08 kg of air at 700 kPa and 800°C is expanded adiabatically to 100 kPa in a closed system. Taking γ = 1.4 calculate the following.

   i. The final temperature. (615.4 K)
   ii. The work transfer. (26.3 kJ)
   iii. The change in internal energy. (-26.3 J)

This problem requires a lot of thought and might well serve as an assignment.

4. A horizontal cylinder is fitted with a frictionless piston and its movement is restrained by a spring as shown.



a. The spring force is directly proportional to movement such that $\Delta F/\Delta x = k$

Show that the change in pressure is directly proportional to the change in volume such that

$$\frac{\Delta p}{\Delta V} = \frac{k}{A^2}$$

a. The air is initially at a pressure and temperature of 100 kPa and 300 K respectively.

b. Given k = 28 800 N/m, calculate the initial volume such that when the air is heated, the pressure – volume graph is a straight line that extends to the origin. (0.5 dm^3)

c. The air is heated making the volume three times the original value. Calculate the following.

i. The mass. (0.58 g)
ii. The final pressure. (300 kPa)
iii. The final temperature. (2700 K)
iv. The work done. (-200 kJ)
v. The change in internal energy. (917 J)
vi. The heat transfer. (1.12 kJ)

6.6. Closed System Problems Involving Vapour

The solution of problems involving steam and other vapours is done in the same way as for gases with the important proviso that gas laws must not be used. Volumes and internal energy values should be obtained from tables and property charts. This is best illustrated with a worked example.

WORKED EXAMPLE No. 13

1kg of steam occupies a volume of 0.2 m^3 at 9 bar in a closed system. The steam is heated at constant pressure until the volume is 0.3144 m^3. Calculate the following.

i. The initial dryness fraction.
ii. The final condition.
iii. The work transfer.
iv. The change in internal energy.
v. The heat transfer.

SOLUTION

First find the initial dryness fraction.

$$V_1 = 0.2 = mx_1v_g \text{ at 9 bar}$$

$$x_1 = \frac{0.2}{1 \times 0.2149}$$

x_1= 0.931 (initial dryness fraction).

Now determine the specific volume after expansion.

p_2 = 9 bar (constant pressure) V_2 = 0.3144 m^3.

$V_2 = mv_2$

$$V_2 = \frac{0.3144}{1} = 0.3144 \text{ m}^3/\text{kg}$$

First, look in the superheat tables to see if this value exists for superheat steam. We find that at 9 bar and 350^{o}C, the specific volume is indeed 0.3144 m^3/kg.

The final condition is superheated to 350^{o}C .

Note that if v_2 was less than v_g at 9 bar the steam would be wet and x_2 would have to be found.

Next find the work.

$W = -p(V_2 - V_1) = -9 \times 10^5 (0.3144 - 0.2) = -102\ 950$ J

W = -102.95 kJ (Energy leaving the system)

Next determine the internal energy from steam tables.

$U_1 = m\,u_1$ and $u_1 = u_f + x_1 u_{fg}$ at 9 bar

u_{fg} at 9 bar = $u_g - u_f$ = 2 581 - 742 = 1 839 kJ/kg

$U_1 = 1\{742 + 0.931(1\,839)\} = 2\,454$ kJ

$U_2 = m\,u_2$ and u_2 = u at 9 bar and 350 ^oC = 2877 kJ/kg

$U_2 = m\,u_2 = 1(2\,877) = 2\,877$ kJ.

The change in internal energy = $U_2 - U_1$ = 423 kJ (increased)

Finally deduce the heat transfer from the NFEE

$$Q + W = \Delta U$$

Hence

$$Q = \Delta U - W = 423 - (-102.95)$$

Q = 526 kJ (energy entering the system)

SELF ASSESSMENT EXERCISE No. 6

1. 0.2 kg of dry saturated steam at 10 bar pressure is expanded reversibly in a closed system to 1 bar by the law $pV^{1.2}$ =C. Calculate the following.

i. The initial volume. (38.9 dm^3)

ii. The final volume. (264 dm^3)

iii. The work transfer. (-62 kJ)

iv. The dryness fraction. (0.779)

v. The change in internal energy. (-108 kJ)

vi. The heat transfer. (-46 kJ)

2. Steam at 15 bar and 250^oC is expanded reversibly in a closed system to 5 bar. At this pressure the steam is just dry saturated. For a mass of 1 kg calculate the following.

i. The final volume. (0.375 m^3)
ii. The change in internal energy. (-165 kJ)
iii. The work done. (-187 kJ)
iv. The heat transfer. (22.1 kJ)