

APPLIED THERMODYNAMICS

TUTORIAL 2

GAS COMPRESSORS

In order to complete this tutorial you should be familiar with gas laws and polytropic gas processes.

You will study the principles of reciprocating compressors in detail and some principles of rotary compressors. On completion you should be able to the following.

- Describe the working principles of reciprocating compressors.
- Define and calculate swept volume.
- Define and calculate volumetric efficiency.
- Define and calculate isothermal efficiency.
- Define and calculate indicated power.
- Dstate the benefits of cooling.
- Calculate the heat rejected through cooling.
- Define and calculate the interstage pressures for multiple compressors.
- Define polytropic efficiency.

Let's start by considering the general use of compressed air.

1. COMPRESSED AIR

1.1 TYPES

Air is an expansive substance and dangerous when used at high pressures. For this reason, most applications are confined to things requiring low pressures (10 bar or lower) but there are industrial uses for high pressure air up to 100 bar.

The common source of the air is the compressor. There are many types of compressors with different working principles and working conditions. These are examples.

- Reciprocating.
- Sliding vane compressors.
- Lobe compressors.
- Helical screws.
- Centrifugal.
- Axial turbine compressors.

The function of all of them is to draw in air from the atmosphere and produce air at pressures substantially higher. Usually a storage vessel or receiver is used with the compressor. The same principles are applied to the compression of other gasses. This tutorial is mainly about reciprocating compressors. The other types are covered briefly. The reciprocating compressor is probably the most versatile of all the types and is only out performed by rotary types when large volumes at low pressures are required. For high pressures, the reciprocating compressor is almost universal.

1.2 ATMOSPHERIC VAPOUR

Air and vapour mixtures are covered in detail in a later tutorial. We should note, however, the effects it has on the performance of an air compressor. Atmospheric air contains *WATER VAPOUR* mixed with the other gases. When the air is cooled to the dew point, the vapour condenses into water and we see rain or fog. The ratio of the mass of water vapour in the air to the mass of the air is called the *ABSOLUTE HUMIDITY*. The quantity of water that can be absorbed into the air at a given pressure depends upon the temperature. The hotter the air, the more water it can evaporate. When the air contains the maximum possible amount of vapour it is at its dew point and rain or fog will appear. The air is then said to have 100% humidity. When the air contains no water vapour at all (dry air), it has 0% humidity. This refers to *RELATIVE HUMIDITY*. For example if the air has 40% humidity it means that it contains 40% of the maximum that it could contain. There are various ways to determine the humidity of air and instruments for doing this are called *HYGROMETERS*.

The importance of humidity to air compressors is as follows. When air is sucked into the compressor, it brings with it water vapour. When the air is compressed the pressure and the temperature of the air goes up and the result is that the compressed air will have a relative humidity of about 100% and it will be warm. When the air leaves the compressor it will cool down and the water vapour will condense. Water will then clog the compressor, the receiver and the pipes.

Water causes damage to air tools, ruins paint sprays and corrodes pipes and equipment. For this reason the water must be removed and the best way is to use a well designed compressor installation and distribution network.

1.3 TYPICAL COMPRESSOR LAYOUT

The diagram below shows the layout of a two stage reciprocating compressor typically for supplying a workshop.

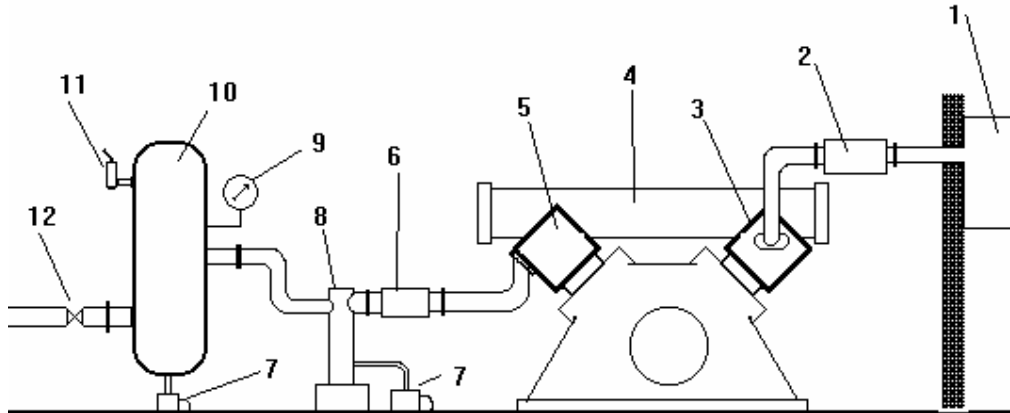


Figure 1

1. Induction box and silencer on outside of building with coarse screen.
2. Induction filter.
3. Low pressure stage.
4. Intercooler.
5. High pressure stage.
6. Silencer.
7. Drain trap.
8. After cooler
9. Pressure gauge.
10. Air receiver.
11. Safety pressure relief valve.
12. Stop valve

1.4 FREE AIR DELIVERY

When a gas such as air flows in a pipe, the mass of the air depends upon the pressure and temperature. It would be meaningless to talk about the volume of the air unless the pressure and temperature are considered. For this reason the volume of air is usually stated as FREE AIR DELIVERY or FAD.

FAD refers to the volume the air would have if let out of the pipe and returned to atmospheric pressure at the same temperature.

The FAD is the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at a higher pressure. Suppose atmospheric conditions are $p_a T_a$ and V_a (the FAD) and the compressed conditions are p, V and T .

Applying the gas law we have

$$\frac{pV}{T} = \frac{p_a V_a}{T_a}$$

$$V_a = \frac{pVT_a}{Tp_a} = F.A.D.$$

2. CYCLE FOR RECIPROCATING COMPRESSOR

2.1 THEORETICAL CYCLE

The diagram shows the basic design of a reciprocating compressor. The piston reciprocates drawing in gas, compressing it and expelling it when the pressure inside the cylinder reaches the same level as the pressure in the delivery pipe.

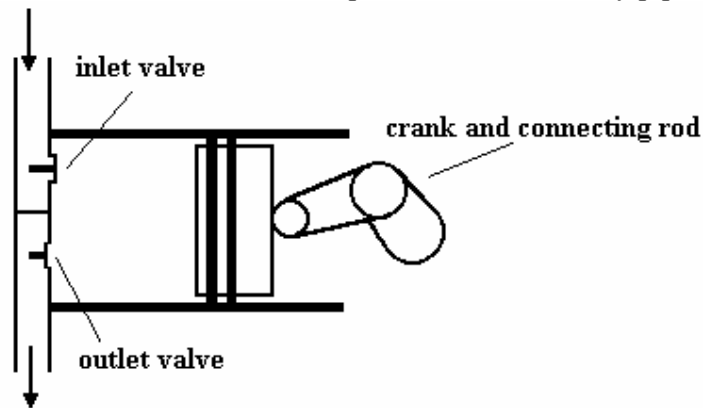


Figure 2

If the piston expels all the air and there is no restriction at the valves, the pressure - volume cycle is as shown below.

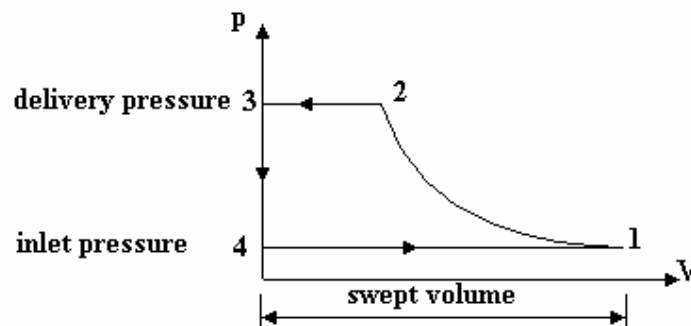


Figure 3

Gas is induced from 4 to 1 at the inlet pressure. It is then trapped inside the cylinder and compressed according to the law $pV^n = C$. At point 2 the pressure reaches the same level as that in the delivery pipe and the outlet valve pops open. Air is then expelled at the delivery pressure. The delivery pressure might rise very slightly during expulsion if the gas is being compacted into a fixed storage volume. This is how pressure builds up from switch on.

2.2 VOLUMETRIC EFFICIENCY

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.

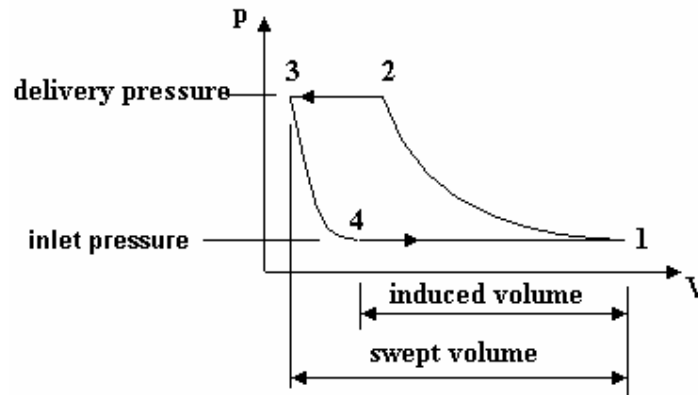


Figure 4

The volumetric efficiency is defined as

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

This efficiency is made worse if leaks occur past the valves or piston.

The clearance ratio is defined as $c = \text{Clearance volume/Swept volume}$.

Ideally the process 2 to 3 and 4 to 1 are isothermal. That is to say, there is no temperature change during induction and expulsion.

WORKED EXAMPLE No.1

Gas is compressed in a reciprocating compressor from 1 bar to 6 bar. The FAD is 13 dm³/s. The clearance ratio is 0.05. The expansion part of the cycle follows the law $pV^{1.2} = C$. The crank speed is 360 rev/min. Calculate the swept volume and the volumetric efficiency.

SOLUTION

Swept Volume = V Clearance volume = 0.05 V

Consider the expansion from 3 to 4 on the p-V diagram.

$$p_4 = 1 \text{ bar} \quad p_3 = 6 \text{ bar.} \quad p_3 V_3^{1.2} = p_4 V_4^{1.2}$$

$$6(0.05V)^{1.2} = 1(V_4^{1.2})$$

$$V_4 = 0.222V \text{ or } 22.2\% \text{ of } V$$

$$\text{F.A.D.} = 0.013 \text{ m}^3/\text{s.}$$

$$\text{Induced volume} = V_1 - V_4 = 1.05V - 0.222V = 0.828V$$

$$\text{Induced volume} = 0.013$$

$$V = 0.013/0.828 = 0.0157 \text{ m}^3/\text{s}$$

$$\text{Crank speed} = 6 \text{ rev/s so the swept volume} = 0.0157/6 = 2.62 \text{ dm}^3.$$

$$V_1 = V + 0.05V = 1.05V$$

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

$$\eta_{vol} = \frac{0.828V}{V} = 82.8 \%$$

WORKED EXAMPLE No.2

Show that if the clearance ratio of an ideal single stage reciprocating compressor is c that the volumetric efficiency is given by

$$\eta_{vol} = 1 - c \left[\left(\frac{p_H}{p_L} \right)^{(1/n)} - 1 \right]$$

where p_L is the inlet pressure and p_H the outlet pressure.

SOLUTION

Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$

Clearance volume = V_3

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} \quad c = \frac{V_3}{V_1 - V_3}$$

$$V_1 - V_3 = \frac{V_3}{c} \quad \frac{V_1}{V_3} = \frac{(1+c)}{c}$$

$$\eta_{vol} = \frac{c(V_1 - V_3)}{V_3} = c \left\{ \left(\frac{V_1}{V_3} \right) - \left(\frac{V_4}{V_3} \right) \right\}$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4} \right)^{1/n} = \left(\frac{p_H}{p_L} \right)^{1/n}$$

$$\eta_{vol} = c \left[\left\{ \left(\frac{1+c}{c} \right) \right\} - \left(\frac{p_H}{p_L} \right)^{1/n} \right]$$

$$\eta_{vol} = 1 + c - c \left(\frac{p_H}{p_L} \right)^{1/n}$$

$$\eta_{vol} = 1 - c \left\{ \left(\frac{p_H}{p_L} \right)^{1/n} - 1 \right\}$$

In real compressors the warm cylinder causes a slight temperature rise over the induction from 4 to 1. The gas is restricted by the valves and p_1 is slightly less than p_4 . The valves also tend to move so the real cycle looks more like this.

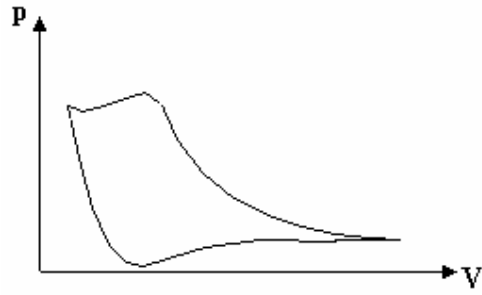


Figure 5

WORKED EXAMPLE No.3

A single stage reciprocating compressor produces a FAD of $2 \text{ dm}^3/\text{s}$ at 420 rev/min. The inlet conditions are 1 bar and 10°C . The polytropic index is 1.2 for the compression and expansion. The outlet pressure is 8 bar. The clearance volume is 10 cm^3 .

Due to the restriction of the inlet valve and the warming effect of the cylinder walls, the pressure at the start of compression is 0.97 bar and the temperature is 17°C .

Determine the volumetric efficiency.

SOLUTION

Because the induction stroke is neither at constant pressure nor constant temperature, we must solve the swept volume by using the expulsion stroke, which is assumed to be at constant pressure and temperature. The numbers of the cycle points are as before.

$$T_2 = 290 \left(\frac{8}{0.97} \right)^{\frac{0.2}{1.2}} = 412.2 \text{ K}$$

$$F.A.D. \text{ per stroke} = \frac{2 \times 60}{420} = 0.2857 \text{ dm}^3 \text{ per stroke}$$

$$\text{The compressed volume expelled} = \frac{0.2857 \times p_a T_H}{T_a p_H}$$

$$\text{Expulsion volume} = V_2 - V_3 = \frac{0.2857 \times 1 \times 412.2}{283 \times 8} = 0.052 \text{ dm}^3$$

$$p_3 V_3^n = p_4 V_4^n$$

$$8 \times 0.01^{1.2} = 1 \times V_4^{1.2} \quad \text{hence } V_4 = 0.0566 \text{ dm}^3$$

$$V_2 = 0.01 + 0.052 = 0.062 \text{ dm}^3$$

$$p_1 V_1^n = p_2 V_2^n$$

$$0.96 V_1^{1.2} = 8 \times 0.062^{1.2}$$

$$V_1 = 0.363 \text{ dm}^3$$

$$\text{Induced volume} = V_1 - V_4 = 0.306 \text{ dm}^3$$

$$\text{Swept volume} = V_1 - V_3 = 0.353 \text{ dm}^3$$

$$\eta_{vol} = \frac{0.306}{0.353} = 86.7\%$$

2.3 INDICATED POWER

The indicated work per cycle is the area enclosed by the p - V diagram. The easiest way to find this is by integrating with respect to the pressure axis.

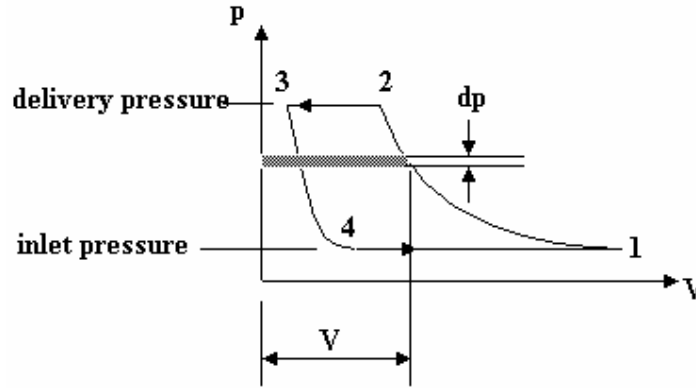


Figure 6

The processes 1 to 2 and 3 to 4 are polytropic $pV^n = C$.

$$V = C^{1/n} p^{-1/n} \quad C = Vp^{1/n}$$

Consider the expression

$$\int V dp = C^{1/n} \int p^{-1/n} dp = \left[\frac{C^{1/n} p^{1-1/n}}{(1-1/n)} \right] = \left[\frac{nVp^{1/n} p^{1-1/n}}{(n-1)} \right] = \left[\frac{npV}{(n-1)} \right]$$

Between the limits of p_2 and p_1 this becomes $\frac{n[p_2V_2 - p_1V_1]}{(n-1)}$

Between the limits p_4 and p_3 this becomes $\frac{n[p_4V_4 - p_3V_3]}{(n-1)}$

The indicated work (input) is then $W = \frac{n[p_2V_2 - p_1V_1]}{(n-1)} - \frac{n[p_3V_3 - p_4V_4]}{(n-1)}$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ \left(\frac{p_2 V_2}{p_1 V_1} \right) - 1 \right\} - \left(\frac{n}{n-1} \right) p_4 V_4 \left\{ \left(\frac{p_3 V_3}{p_4 V_4} \right) - 1 \right\}$$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} - \left(\frac{n}{n-1} \right) p_4 V_4 \left\{ \left(\frac{p_3}{p_4} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ r^{\frac{n-1}{n}} - 1 \right\} - \left(\frac{n}{n-1} \right) p_4 V_4 \left\{ r^{\frac{n-1}{n}} - 1 \right\}$$

Where r is the pressure ratio. Since $p_1 = p_4$ this reduces to

$$W = \left(\frac{n}{n-1} \right) p_1 \left\{ r^{\frac{n-1}{n}} - 1 \right\} [V_1 - V_4]. \quad \text{Where } V_1 - V_4 \text{ is the swept volume.}$$

If the clearance volume is neglected this becomes

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ r^{\frac{n-1}{n}} - 1 \right\}$$

Since $pV = mRT$

$$W = mRT_1 \left(\frac{n}{n-1} \right) \left\{ r^{\frac{n-1}{n}} - 1 \right\}$$

Note that if the process was isothermal and $n=1$ then the integration would yield $W = mRT_1 \ln(p_2 / p_1)$

m is the mass compressed each cycle and W is the indicated work per cycle. The indicated power is found by multiplying W by the strokes per second.

$$\text{I.P.} = W \times N \quad \text{where } N \text{ is the shaft speed in Rev/s}$$

If the clearance volume is neglected, the mass compressed is the mass expelled. In this case the actual mass flow rate delivered may be used for m and W becomes the indicated power.

2.4 ISOTHERMAL EFFICIENCY

The minimum indicated power is obtained when the index n is a minimum. The ideal compression is hence isothermal with $n=1$. The isothermal efficiency is defined as

$$\eta(\text{iso}) = \text{Isothermal work} / \text{Actual work} = \frac{(n-1)T_1 \ln(p_2 / p_1)}{n(T_2 - T_1)}$$

Note that in the ideal case, T_1 and T_2 are the inlet and outlet temperatures.

SELF ASSESSMENT EXERCISE No. 1

Show how the volumetric efficiency of an ideal single stage reciprocating air compressor may be represented by the equation

$$\eta_{vol} = 1 - c \left[\left(\frac{p_H}{p_L} \right)^{(1/n)} - 1 \right]$$

Where c is the clearance ratio, p_H the delivery pressure and p_L the induction pressure.

A reciprocating air compressor following the ideal cycle has a free air delivery of $60 \text{ dm}^3/\text{s}$. The clearance ratio is 0.05. The inlet is at atmospheric pressure of 1 bar. The delivery pressure is 7 bar and the compression is polytropic with an index of 1.3.

Calculate the following.

- i. The ideal volumetric efficiency. (82.7%)
- ii. The ideal indicated power. (14.7 kW)

3. MULTIPLE COMPRESSOR STAGES

3.1 THE EFFECT OF INTERCOOLING

The advantage of compressing the fluid in stages is that intercoolers may be used and the overall compression is nearer to being isothermal. Consider the p - V diagram for a two stage compressor.

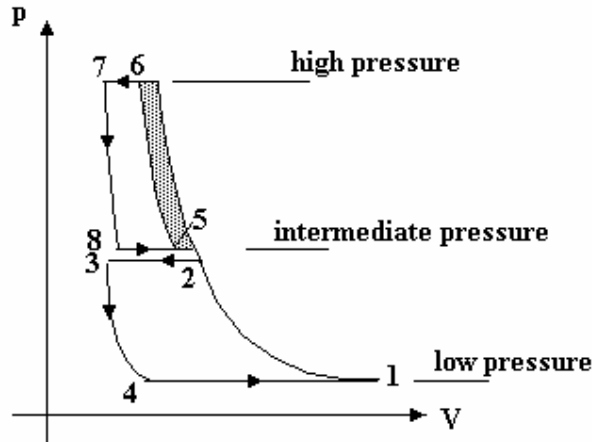


Figure 7

The cycle 1 to 4 is a normal cycle conducted between p_L and p_M . The air is expelled during process 3 to 4 at p_M and constant temperature. The air is then cooled at the intermediate pressure and this causes a contraction in the volume so that the volume entering the high pressure stage is V_5 and not V_2 . The high pressure cycle is then a normal cycle conducted between p_M and p_H .

The shaded area of the diagram represents the work saved by using the intercooler. The optimal saving is obtained by choosing the correct intermediate pressure. This may be found as follows.

3.2 OPTIMAL INTERSTAGE PRESSURE

$W = W_1 + W_2$ where W_1 is the work done in the low pressure stage and W_2 is the work done in the high pressure stage.

$$W = \frac{mRn(T_2 - T_1)}{(n-1)} + \frac{mRn(T_6 - T_5)}{(n-1)}$$

$$\text{Since } T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(1-1/n)} \text{ and } T_6 = T_5 \left(\frac{p_6}{p_5} \right)^{(1-1/n)}$$

then assuming the same value of n for each stage

$$W = mR \left[\left\{ \frac{nT_1}{(n-1)} \right\} \left\{ \frac{p_2}{p_1} \right\}^{1-(1/n)} - 1 \right] + mR \left[\left\{ \frac{nT_5}{(n-1)} \right\} \left\{ \frac{p_6}{p_5} \right\}^{1-(1/n)} - 1 \right]$$

Since $p_2 = p_5 = p_m$ and $p_6 = p_H$ and $p_1 = p_L$

$$W = mR \left[\left\{ \frac{nT_1}{(n-1)} \right\} \left\{ \frac{p_M}{p_L} \right\}^{1-(1/n)} - 1 \right] + mR \left[\left\{ \frac{nT_6}{(n-1)} \right\} \left\{ \frac{p_H}{p_M} \right\}^{1-(1/n)} - 1 \right]$$

For a minimum value of W we differentiate with respect to p_M and equate to zero.

$$\frac{dW}{dp_M} = mRT_1 p_L^{(1-n)/n} p_M^{-1/n} - mRT_5 p_H^{(n-1)/n} p_M^{(1-2n)/n}$$

If the intercooler returns the air to the original inlet temperature so that $T_1 = T_5$, then equating to zero reveals that for minimum work

$$p_M = (p_L p_H)^{1/2}$$

It can further be shown that when this is the case, the work done by both stages are equal.

When K stages are used, the same process reveals that the minimum work is done when the pressure ratio for each stage is $(p_L/p_H)^{1/K}$

WORKED EXAMPLE No.5

A single acting reciprocating compressor runs at 360 rev/min and takes in air at 1 bar and 15°C and compresses it in 3 stages to 64 bar. The free air delivery is 0.0566 m³/s. There is an intercooler between each stage, which returns the air to 15°C. Each stage has one piston with a stroke of 100 mm. Calculate the following.

- The ideal interstage pressure.
- The ideal indicated power per stage.
- The heat rejected from each cylinder.
- The heat rejected from each intercooler.
- The isothermal efficiency.
- The swept volume of each stage.
- The bore of each cylinder.

Ignore leakage and the effect of the clearance volume. The index of compression is 1.3 for all stages.

SOLUTION

$$\text{Pressure ratio for each stage} = (64/1)^{1/3} = 4$$

Hence the pressure after stage 1 is 1 x 4 = 4 bar.

The pressure after the second stage is 4 x 4 = 16 bar

The final pressure is 16 x 4 = 64 bar.

$$T_1 = 288 \text{ K. } m = p_1 V / RT_1 = 1 \times 10^5 \times 0.0566 / (287 \times 288) = 0.06847 \text{ kg/s}$$

$$T_2 = 288(4)^{0.3/1.3} = 396.5 \text{ K}$$

The indicated power for each stage is the same so it will be calculated for the 1st. stage.

$$I.P. = \frac{mRnT_1}{n-1} \times \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} - 1 \right\} \text{ since } m \text{ is the mass compressed.}$$

$$I.P. = \frac{0.06847 \times 287 \times 1.3 \times 288}{1.3 - 1} \left\{ 4^{\frac{0.3}{1.3}} - 1 \right\} = 9246 \text{ Watts}$$

CYLINDER COOLING

Consider the energy balance over the first stage.

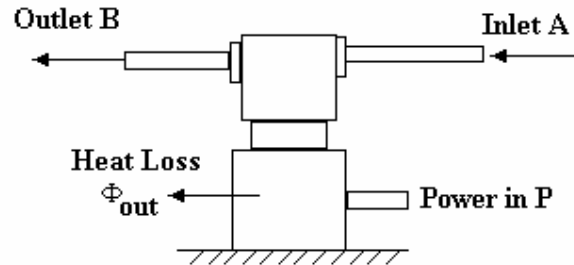


Figure 8

Balancing the energy we have

$$H_A + P(\text{in}) = H_B + \Phi(\text{out})$$

$$\Phi(\text{out}) = P(\text{in}) - mC_p(T_B - T_A)$$

$$\Phi(\text{out}) = 9.246 - 0.06847 \times 1.005 (396.5 - 288)$$

$$\Phi(\text{out}) = 1.78 \text{ kW (rejected from each cylinder)}$$

INTERCOOLER

Now consider the Intercooler. No work is done and the temperature is cooled from T_2 to T_5 .

$$\Phi(\text{out}) = mC_p(T_2 - T_5) = 0.0687 \times 1.005 (396.5 - 288) = 7.49 \text{ kW}$$

ISOTHERMAL EFFICIENCY

The ideal isothermal power = $mRT_1 \ln(p_1/p_2)$ per stage.

$$P(\text{isothermal}) = 0.06847 \times 287 \times 288 \ln 4 = 7.846 \text{ kW}$$

$$\eta(\text{iso}) = 7.846/9.246 = 84.9 \%$$

SWEPT VOLUMES

Consider the first stage.

The F.A.D. is $0.0566 \text{ m}^3/\text{s}$. In the ideal case where the air is drawn in at constant temperature and pressure from the atmosphere, the FAD is given by

$$\text{FAD} = \text{Swept Volume} \times \text{Speed} \quad \text{and the speed is } 6 \text{ rev/s}$$

$$\text{Hence S.V. (1st. Stage)} = 0.0566/6 = 0.00943 \text{ m}^3$$

S.V. = Bore Area x Stroke

$$0.00943 = \pi D^2/4 \times 0.1 \quad D_1 = 0.347 \text{ m.}$$

Now consider the second stage. The air is returned to atmospheric pressure at inlet with a pressure of 4 bar. The volume drawn is hence 1/4 of the original FAD.

The swept volume of the second stage is hence $0.00943/4 = 0.00236 \text{ m}^3$.

$$0.00236 = \pi D^2/4 \times 0.1 \quad \text{hence } D_2 = 0.173 \text{ m}$$

By the same reasoning the swept volume of the third stage is

$$\text{SV(3rd stage)} = 0.00943/16 = 0.000589 \text{ m}^3.$$

$$0.000589 = \pi D^2/4 \times 0.1 \quad D_3 = 0.0866 \text{ m}$$

SELF ASSESSMENT EXERCISE No. 2

1. A single acting 2 stage compressor draws in $8.5 \text{ m}^3/\text{min}$ of free air and compresses it to 40 bar. The compressor runs at 300 rev/min. The atmospheric conditions are 1.013 bar and 15°C . There is an intercooler between stages which cools the air back to 15°C . The polytropic index for all compressions is 1.3. The volumetric efficiency is 90% for the low pressure stage and 85% for the high pressure stage. Ignore the effect of the clearance volume. Calculate the following.

The intermediate pressure for minimum indicated work. (6.365 bar)

The theoretical indicated power for each stage. (32.85 kW)

The heat rejected in each cylinder. (6.31 kW)

The heat rejected by the intercooler. (26.53 kW)

The swept volumes of both stages. (31.4 dm^3 and 5.3 dm^3)

What advantage is there in using an after-cooler?

State the effect on your answers of not ignoring the clearance volume and leakages.

2. A single acting 2 stage compressor draws in free air and compresses it to 8.5 bar. The compressor runs at 600 rev/min. The atmospheric conditions are 1.013 bar and 15°C .

The interstage pressure is 3 bar and the intercooler cools the air back to 30°C . The polytropic index for all compressions is 1.28.

Due to the effect of warming from the cylinder walls and the pressure loss in the inlet valve, the pressure and temperature at the start of the low pressure compression stroke is 0.96 bar and 25°C . The high pressure cycle may be taken as ideal.

The clearance volume for each stages is 4% of the swept volume of that stage. The low pressure cylinder is 300 mm diameter and the stroke for both stages is 160 mm.

Calculate the following.

The free air delivery. ($5.858 \text{ m}^3/\text{min}$)

The volumetric efficiency of the low pressure stage. (86.3 %)

The diameter of the high pressure cylinder. (171 mm)

The indicated power for each stage. (14.6 kW and 13.4 kW)

3. A 2 stage reciprocating air compressor has an intercooler between stages. The induction and expulsion for both stages are at constant pressure and temperature. All the compressions and expansions are polytropic.

Neglecting the effect of the clearance volume show that the intermediate pressure, which gives minimum, indicated work is

$$p_M = (p_L p_H)^{1/2}$$

Explain with the aid of a sketch how the delivery temperature from both cylinders varies with the intermediate pressure as it changes from p_L to p_H .

- 4.a. Prove that the ideal volumetric efficiency of a single stage reciprocating compressor is

$$\eta_{vol} = 1 - c(r^{1/n} - 1)$$

r is the pressure ratio, n is the polytropic index and c the clearance ratio.

Sketch curves of η_{vol} against r for typical values of n and c .

- b. A two stage reciprocating air compressor works between pressure limits of 1 and 20 bar. The inlet temperature is 15°C and the polytropic index is 1.3. Intercooling between stages reduces the air temperature back to 15°C .

Find the free air delivery and mass of air that can be compressed per kW h of work input.

$$(10.06 \text{ m}^3/\text{kW h} \quad 12.17 \text{ kg/kW h})$$

Find the ratio of the cylinder diameters if the piston have the same stroke. Neglect the effect of the clearance volume.

$$(d/D = 0.473)$$

4. POLYTROPIC or SMALL STAGE EFFICIENCY

This is an alternative way of approaching isentropic efficiency. In this method, the compression is supposed to be made up of many stages, each raising the pressure a small amount. The theory applies to any type of compressor.

For an adiabatic gas compression the law of compression and the gas law may be combined.

$$dT = C \left(\frac{\gamma - 1}{\gamma} \right) p^{-\frac{1}{\gamma}} dp$$

Divide by p

$$\frac{dT}{p} = C \left(\frac{\gamma - 1}{\gamma} \right) p^{-\frac{1}{\gamma}} \frac{dp}{p}$$

$$\frac{dT}{p^{1-\frac{1}{\gamma}}} = C \left(\frac{\gamma - 1}{\gamma} \right) \frac{dp}{p}$$

Since $\frac{T}{C} = p^{1-\frac{1}{\gamma}}$ differentiate and $\frac{dT}{T} = \left(\frac{\gamma - 1}{\gamma} \right) \frac{dp}{p}$

For an isentropic compression, let the final temperature be designated T' and the change in temperature be $\Delta T'$.

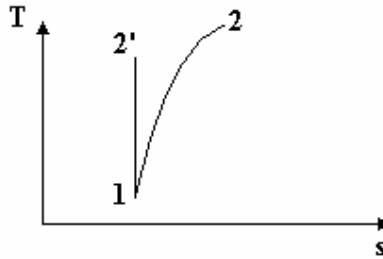


Figure 9

$$\frac{dT'}{T} = \left(\frac{\gamma - 1}{\gamma} \right) \frac{dp}{p} \dots\dots\dots(1)$$

$$\text{Isentropic efficiency} = \frac{T_{2'} - T_1}{T_2 - T_1}$$

Let the change be infinitesimally small such that

$$T_2 = T_1 + dT \quad \text{and} \quad T_{2'} = T_1 + dT'$$

The ratio $\frac{dT'}{dT}$ is designated η_∞

η_∞ is called the POLYTROPIC EFFICIENCY.

If we think of the compression as being made up of many tiny steps each with the same value of η_{∞} .

$$dT' = \eta_{\infty} dT$$

$$\frac{dT'}{T} = \eta_{\infty} \frac{dT}{T}$$

$$\left(\frac{\gamma-1}{\gamma}\right) \frac{dp}{p} = \eta_{\infty} \frac{dT}{T}$$

Integrate

$$\left(\frac{\gamma-1}{\gamma}\right) \int_{p_1}^{p_2} \frac{dp}{p} = \eta_{\infty} \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\left(\frac{\gamma-1}{\gamma}\right) \ln \frac{p_2}{p_1} = \eta_{\infty} \ln \frac{T_2}{T_1}$$

$$\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{T_2}{T_1}\right)^{\eta_{\infty}}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma\eta_{\infty}}\right)}$$

T_1 is the starting temperature and T_2 is the final temperature.

The overall efficiency is $\eta_0 = \frac{T_2' - T_1}{T_2 - T_1}$

ADIABATIC PROCESS $\frac{T_2'}{T_1} = r^{\frac{\gamma-1}{\gamma}} \quad T_2' = T_1 r^{\frac{\gamma-1}{\gamma}}$

POLYTROPIC PROCESS $\frac{T_2'}{T_1} = r^{\frac{n-1}{n}} = r^{\frac{\gamma-1}{\gamma\eta_{\infty}}} \quad T_2' = T_1 r^{\frac{\gamma-1}{\gamma\eta_{\infty}}}$

SUBSTITUTE

$$\eta_0 = \frac{T_1 r^{\frac{\gamma-1}{\gamma}} - T_1}{T_1 r^{\frac{\gamma-1}{\gamma\eta_{\infty}}} - T_1} = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_{\infty}}} - 1}$$

Comparing $\frac{T_2'}{T_1} = r^{\frac{n-1}{n}}$ with $\frac{T_2}{T_1} = r^{\frac{\gamma-1}{\gamma\eta_{\infty}}}$ $\eta_{\infty} = 1$ as expected.

This theory may be applied to expansions as well as compressions. It may also be applied to expansions in nozzles. In steam work, it is more usual to use the REHEAT FACTOR, which is based on the same principle.

WORKED EXAMPLE 6

A compressor draws in air at 15°C and 0.3 bar. The air is compressed to 1.6 bar with a polytropic efficiency of 0.86. Determine the temperature and the isentropic efficiency. Take $\gamma = 1.4$

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\eta_{\infty}}}$$

$$T_2 = 288 \left(\frac{1.6}{0.3} \right)^{\frac{1.4-1}{1.4(0.86)}} = 288(5.33)^{0.332} = 502K$$

$$T_2' = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 288(5.33)^{0.286} = 464.5K$$

$$\eta_{is} = \frac{464.5 - 288}{502 - 288} = 0.825$$

SELF ASSESSMENT EXERCISE No. 3

1. Show that for any compression process the overall efficiency is given by

$$\eta_o = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\eta_{\infty}} - 1}$$

where η_{∞} is the polytropic efficiency.

Determine the index of compression for a gas with an adiabatic index of 1.4 and a polytropic efficiency of 0.9. (1.465)

Determine the overall efficiency when the pressure compression ratio is 4/1 and 8/1. (0.879 and 0.866)

2. A compressor draws in air at 223.3 K temperature and 0.265 bar pressure. The compression ratio is 6. The polytropic efficiency is 0.86. Determine the temperature after compression. Take $\gamma = 1.4$
(405 K)