EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 1

TUTORIAL No. 2 – THERMODYNAMIC SYSTEMS

Thermodynamic systems

Polytropic processes: general equation $pv^n=c$, relationships between index `n' and heat transfer during a process; constant pressure and reversible isothermal and adiabatic processes; expressions for work flow

Thermodynamic systems and their properties: closed systems; open systems; application of first law to derive system energy equations; properties; intensive; extensive; two-property rule

Relationships: $R = c_p - c_v$. and $\gamma = c_p/c_v$

When you have completed this tutorial you should be able to do the following.

- Explain and use the First Law of Thermodynamics.
- Solve problems involving various kinds of thermodynamic systems.
- Explain and use polytropic expansion and compression processes.

1. ENERGY TRANSFER

There are two ways to transfer energy in and out of a system, by means of work and by means of heat. Both may be regarded as a quantity of energy transferred in Joules or energy transfer per second in Watts.

When you complete section one you should be able to explain and calculate the following.

- Heat transfer.
- Heat transfer rate.
- Work transfer
- Work transfer rate (Power)

1.1. HEAT TRANSFER

Heat transfer occurs because one place is hotter than another. Under normal circumstances, heat will only flow from a hot body to a cold body by virtue of the temperature difference. There are 3 mechanisms for this, *Conduction, convection and radiation.* You do not need to study the laws governing conduction, convection and radiation in this module.

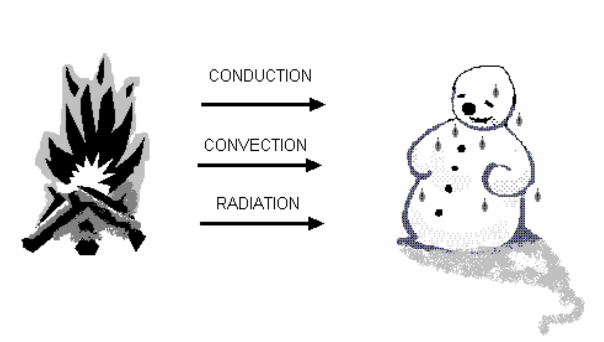

Fig.1

A quantity of energy transferred as heat is given the symbol Q and it's basic unit is the Joule. The quantity transferred in one second is the heat transfer rate and this has the symbol Φ and the unit is the Watt.

An example of this is when heat passes from the furnace of a steam boiler through the walls separating the combustion chamber from the water and steam. In this case, conduction, convection and radiation all occur together.

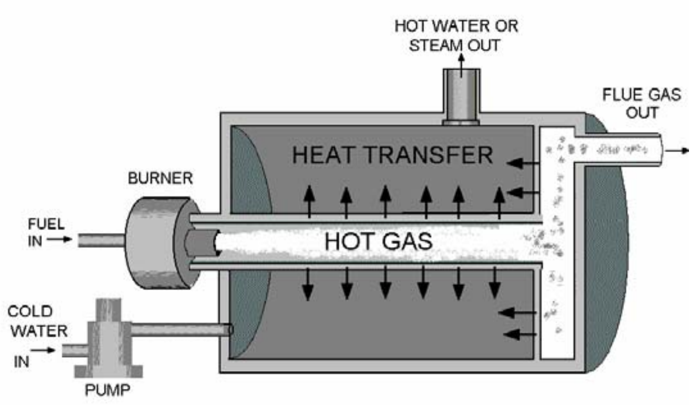

Fig.2

SELF ASSESSMENT EXERCISE No.1

1. 1 kg/s of steam flows in a pipe 40 mm bore at 200 bar pressure and 400^{o}C.

i. Look up the specific volume of the steam and determine the mean velocity in the pipe.
(7.91 m/s)

ii. Determine the kinetic energy being transported per second.
(31.3 W)

iii. Determine the enthalpy being transported per second.
(2819 W)

1.2. WORK TRANSFER

Energy may be transported from one place to another mechanically. An example of this is when the output shaft of a car engine transfers energy to the wheels. A quantity of energy transferred as work is 'W' Joules but the work transferred in one second is the Power 'P' Watts.

An example of power transfer is the shaft of a steam turbine used to transfer energy from the steam to the generator in an electric power station.

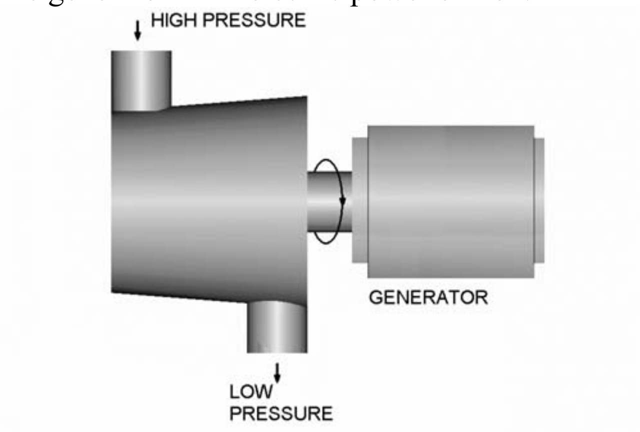

Fig.3

It is useful to remember that the power transmitted by a shaft depends upon the torque and angular velocity.

The formulae used are $P = \omega T$ or $P = 2\pi NT$

ω is the angular velocity in radian per second and N is the angular velocity in revolutions per second.

WORKED EXAMPLE No. 1

A duct has a cross section of 0.2 m x 0.4 m. Steam flows through it at a rate of 3 kg/s with a pressure of 2 bar. The steam has a dryness fraction of 0.98. Calculate all the individual forms of energy being transported.

SOLUTION

Cross sectional area = 0.2 x 0.4 = 0.08 m^2.

Volume flow rate = mxv_g at 2 bar

Volume flow rate = 3 x 0.98 x 0.8856 = 2.6 m^3/s.

velocity = c = Volume/area = 2.6/0.08 = 32.5 m/s.

Kinetic Energy being transported = $mc^2/2$ = 3 x 32.5^2 /2 = 1 584 Watts.

Enthalpy being transported = $m(h_f + x\ h_{fg})$

H= 3(505 + 0.98 x 2202) = 7988.9kW

Flow energy being transported = pressure x volume

Flow Energy = $2x10^5$ x 2.6 = 520 000 Watts

Internal energy being transported = $m(u_f + x\ u_{fg})$

U = 3(505 + 0.98 x 2025)=7468.5 kW

Check flow energy = H - U = 7988.9 - 7468.5 = 520 kW

SELF ASSESSMENT EXERCISE No.2

1. The shaft of a steam turbine produces 600 Nm torque at 50 rev/s. Calculate the work transfer rate from the steam.
(188.5 W)

2. A car engine produces 30 kW of power at 3000 rev/min. Calculate the torque produced.
(95.5 Nm)

2. THE FIRST LAW OF THERMODYNAMICS

When you have completed section two, you should be able to explain and use the following terms.

- The First Law of Thermodynamics.
- Closed systems.
- The Non-Flow Energy Equation.
- Open systems.
- The Steady Flow Energy Equation.

2.1 THERMODYNAMIC SYSTEMS

In order to do energy calculations, we identify our system and draw a boundary around it to separate it from the surroundings. We can then keep account of all the energy crossing the boundary. The first law simply states that

The nett energy transfer = nett energy change of the system.

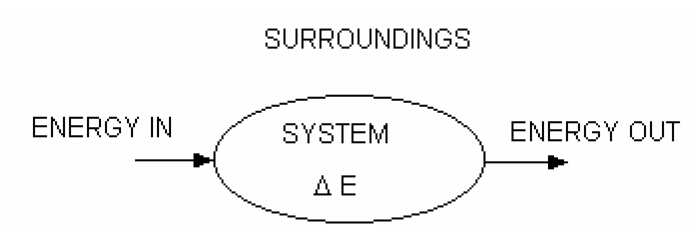

Fig. 4

Energy transfer into the system = E(in)

Energy transfer out of system = E (out)

Nett change of energy inside system = E(in) - E (out) = ΔE

This is the fundamental form of the first law.

Thermodynamic systems might contain only static fluid in which case they are called **NON-FLOW or CLOSED SYSTEMS.**

Alternatively, there may be a steady flow of fluid through the system in which case it is known as a **STEADY FLOW or OPEN SYSTEM.**

The energy equation is fundamentally different for each because most energy forms only apply to a fluid in motion. We will look at non-flow systems first.

2.2 NON-FLOW SYSTEMS

The rules governing non-flow systems are as follows.

- The volume of the boundary may change.
- No fluid crosses the boundary.
- Energy may be transferred across the boundary.

When the volume enlarges, work (-W) is transferred from the system to the surroundings. When the volume shrinks, work (+W) is transferred from the surroundings into the system. Energy may also be transferred into the system as heat (+Q) or out of the system (-Q). This is best shown with the example of a piston sliding inside a cylinder filled with a fluid such as gas.

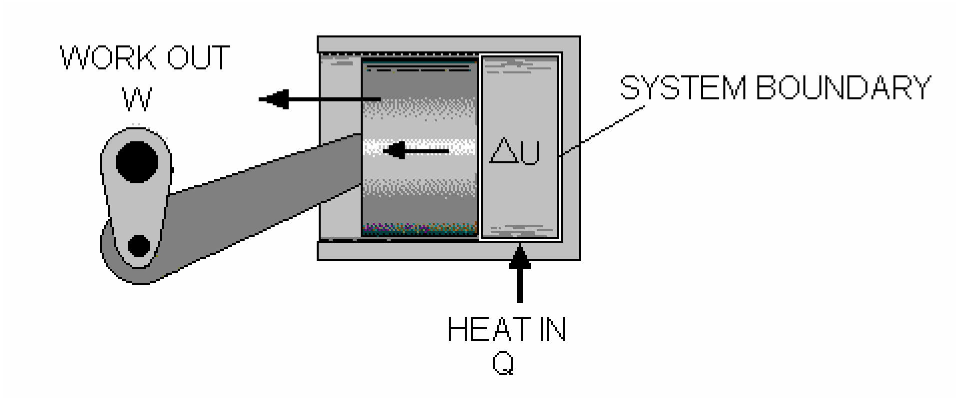

Fig.5

The only energy possessed by the fluid is internal energy (U) so the net change is Δ U. The energy equation becomes

$$\mathbf{Q + W = \Delta U}$$

This is known as the **NON-FLOW ENERGY EQUATION (N.F.E.E.)**

2.3 STEADY FLOW SYSTEMS

The laws governing this type of system are as follows.

- Fluid enters and leaves through the boundary at a steady rate.
- Energy may be transferred into or out of the system.

A good example of this system is a steam turbine. Energy may be transferred out as a rate of heat transfer Φ or as a rate of work transfer P.

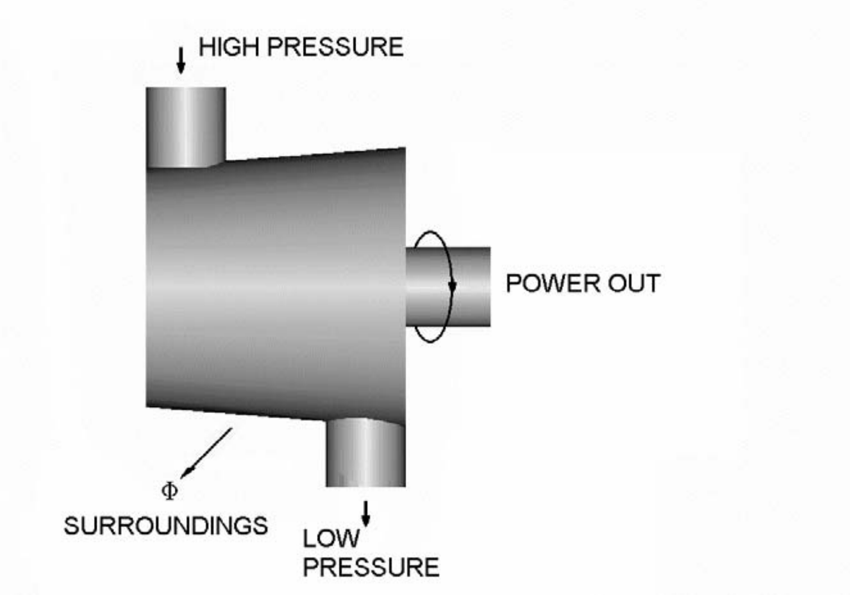

Fig.6.

The fluid entering and leaving has potential energy (PE), kinetic energy (KE) and enthalpy (H).

The first law becomes Φ + P = Nett change in energy of the fluid.

$$\mathbf{\Phi + P = \Delta(PE)/s + \Delta(KE)/s + \Delta(H)/s}$$

This is called the **STEADY FLOW ENERGY EQUATION (S.F.E.E.)**

Again, we will use the convention of positive for energy transferred into the system.

Note that the term Δ means '*change of*' and if the inlet is denoted point (1) and the outlet point (2). The change is the difference between the values at (2) and (1). For example ΔH means (H_2-H_1).

WORKED EXAMPLE No.3

A steam turbine is supplied with 30 kg/s of superheated steam at 80 bar and 400°C with negligible velocity. The turbine shaft produces 200 kNm of torque at 3000 rev/min. There is a heat loss of 1.2 MW from the casing. Determine the thermal power remaining in the exhaust steam.

SOLUTION

Shaft Power = $2\pi NT = 2\pi(3000/60) \times 200\,000 = 62.831 \times 10^6$ W = 62.831 MW

Thermal power supplied = H at 80 bar and 400°C

H = 30(3139) = 94170 kW = 94.17 MW

Total energy flow into turbine = 94.17 MW

Energy flow out of turbine = 94.17 MW = SP + Loss + Exhaust.

Thermal Power in exhaust = 94.17 -1.2 - 62.831 = **30.14 MW**

SELF ASSESSMENT EXERCISE No.3

1. A non-flow system receives 80 kJ of heat transfer and loses 20 kJ as work transfer. What is the change in the internal energy of the fluid?
(60 kJ)
2. A non-flow system receives 100 kJ of heat transfer and also 40 kJ of work is transferred to it. What is the change in the internal energy of the fluid?
(140 kJ)
3. A steady flow system receives 500 kW of heat and loses 200 kW of work. What is the net change in the energy of the fluid flowing through it?
(300 kW)
4. A steady flow system loses 2 kW of heat also loses 4 kW of work. What is the net change in the energy of the fluid flowing through it?
(-6 kW)
5. A steady flow system loses 3 kW of heat also loses 20 kW of work. The fluid flows through the system at a steady rate of 70 kg/s. The velocity at inlet is 20 m/s and at outlet it is 10 m/s. The inlet is 20 m above the outlet. Calculate the following.

   i. The change in K.E./s (-10.5 kW)
   ii. The change in P.E/s (-13.7 kW)
   iii. The change in enthalpy/s (1.23 kW)

3. MORE EXAMPLES OF THERMODYNAMIC SYSTEMS

When we examine a thermodynamic system, we must first decide whether it is a non-flow or a steady flow system. First, we will look at examples of non-flow systems.

3.1 PISTON IN A CYLINDER

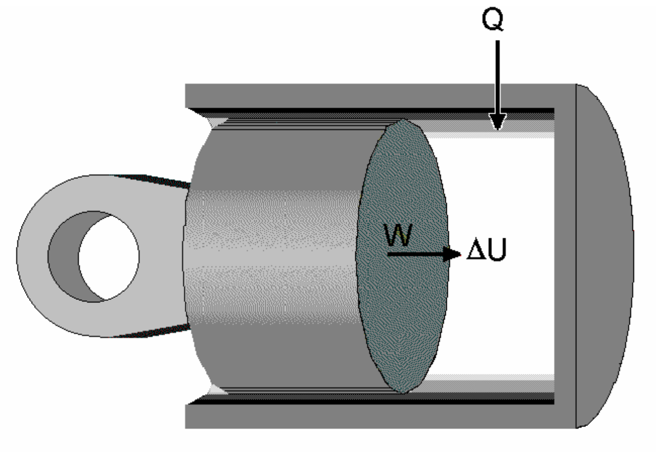

Fig. 7

There may be heat and work transfer. The N.F.E.E. is, $Q + W = \Delta U$

Sometimes there is no heat transfer (e.g. when the cylinder is insulated).

$Q = 0$ so $W = \Delta U$

If the piston does not move, the volume is fixed and no work transfer occurs. In this case

$$Q = \Delta U$$

For a GAS ONLY the change in internal energy is $\Delta U = mC_V\Delta T$.

3.2. SEALED EVAPORATOR OR CONDENSER.

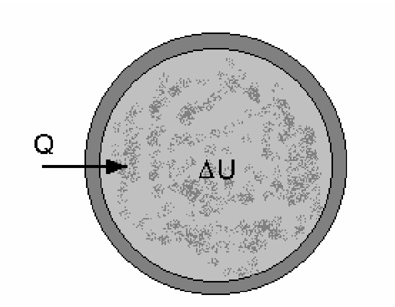

Fig. 8

Since no change in volume occurs, there is no work transfer so **$Q = \Delta U$**

WORKED EXAMPLE No.4

30 g of gas inside a cylinder fitted with a piston has a temperature of 15^oC. The piston is moved with a mean force of 200 N so that that it moves 60 mm and compresses the gas. The temperature rises to 21^oC as a result.

Calculate the heat transfer given c_V = 718 J/kg K.

SOLUTION

This is a non flow system so the law applying is $Q + W = \Delta U$
The change in internal energy is $\Delta U = mc_V \Delta T = 0.03 \times 718 \times (21 - 15)$
$\Delta U = 129.24$ J

The work is transferred into the system because the volume shrinks.

W = force x distance moved = 200 x 0.06 = 12 J

$Q = \Delta U - W = 117.24$ J

Now we will look at examples of steady flow systems.

3.3. PUMPS AND FLUID MOTORS

The diagram shows graphical symbols for hydraulic pumps and motors.

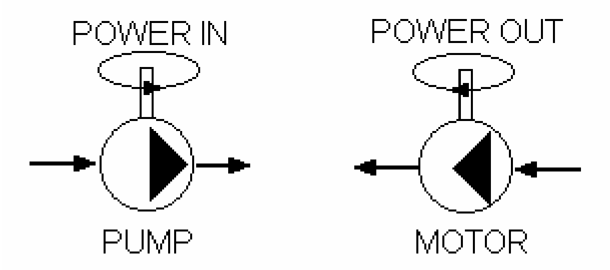

Fig.9

The S.F.E.E. states,

$$\Phi + P = \Delta KE/s + \Delta PE/s + \Delta H/s$$

In this case, especially if the fluid is a liquid, the velocity is the same at inlet and outlet and the kinetic energy is ignored. If the inlet and outlet are at the same height, the PE is also neglected. Heat transfer does not usually occur in pumps and motors so Φ is zero.

The S.F.E.E. simplifies to $\mathbf{P = \Delta H/s}$

Remember that enthalpy is the sum of internal energy and flow energy. The enthalpy of gases, vapours and liquids may be found. In the case of liquids, the change of internal energy is small and so the change in enthalpy is equal to the change in flow energy only.

The equation simplifies further to $P = \Delta FE/s$

Since FE = pV and V is constant for a liquid, this becomes $\mathbf{P = V\Delta p}$

WORKED EXAMPLE No.5

A pump delivers 20 kg/s of oil of density 780 kg/m^3 from atmospheric pressure at inlet to 800 kPa gauge pressure at outlet. The inlet and outlet pipes are the same size and at the same level. Calculate the theoretical power input.

SOLUTION

Since the pipes are the same size, the velocities are equal and the change in kinetic energy is zero. Since they are at the same level, the change in potential energy is also zero. Neglect heat transfer and internal energy.

$P = V\ \Delta p$

$V = m/\rho = 20/780 = 0.0256\ m^3/s$

$\Delta p = 800 - 0 = 800$ kPa

P = 0.0256 x 800000 = 20 480 W or 20.48 kW

WORKED EXAMPLE No.6

A feed pump on a power station pumps 20 kg/s of water. At inlet the water is at 1 bar and 120°C. At outlet it is at 200 bar and 140°C. Assuming that there is no heat transfer and that PE and KE are negligible, calculate the theoretical power input.

In this case the internal energy has increased due to frictional heating.

The SFEE reduces to $P = \Delta H/s = m(h_2 - h_1)$

The h values may be found from tables.

$h_1 = 504$ kJ/kg

This is near enough the value of h_f at 120°C bar in steam tables.

$h_2 = 602$ kJ/kg

P = 20 (602 - 504) = 1969 kW or 1.969 MW

If water tables are not to hand the problem may be solved as follows.
$\Delta h = \Delta u + \Delta f.e.$

$\Delta u = c\,\Delta T$ where c = 4.18 kJ/kg K for water

$\Delta u = 4.18 (140 - 120) = 83.6$ kJ/kg

$\Delta f.e. = V\Delta p$

The volume of water is normally around 0.001 m^3/kg.

$\Delta f.e. = 0.001 \times (200 - 1) \times 10^5 = 19\,900$ J/kg or 19.9 kJ/kg

hence $\Delta h = \Delta u + \Delta fe = 83.6 + 19.9 = 103.5$ kJ/kg

P = mΔh = 20 x 103.5 = 2070 kW or 2.07 MW

The discrepancies between the answers are slight and due to the fact the value of the specific heat and of the specific volume are not accurate at 200 bar.

3.4. GAS COMPRESSORS AND TURBINES.

Figure 10 shows the basic construction of an axial flow compressor and turbine. These have rows of aerofoil blades on the rotor and in the casing. The turbine passes high pressure hot gas or steam from left to right making the rotor rotate. The compressor draws in gas and compresses it in stages.

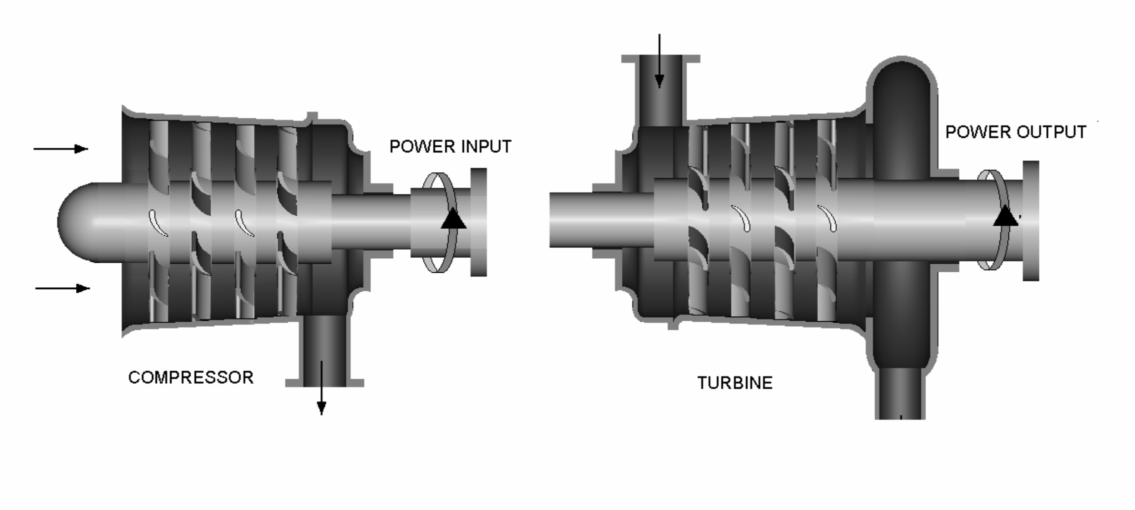

Fig. 10

Compressing a gas normally makes it hotter but expanding it makes it colder. This is because gas is compressible and unlike the cases for liquids already covered, the volumes change dramatically with pressure. This might cause a change in velocity and hence kinetic energy. Often both kinetic and potential energy are negligible. The internal energy change is not negligible. Figure 11 shows graphical symbols for turbines and compressors. Note the narrow end is always the high pressure end.

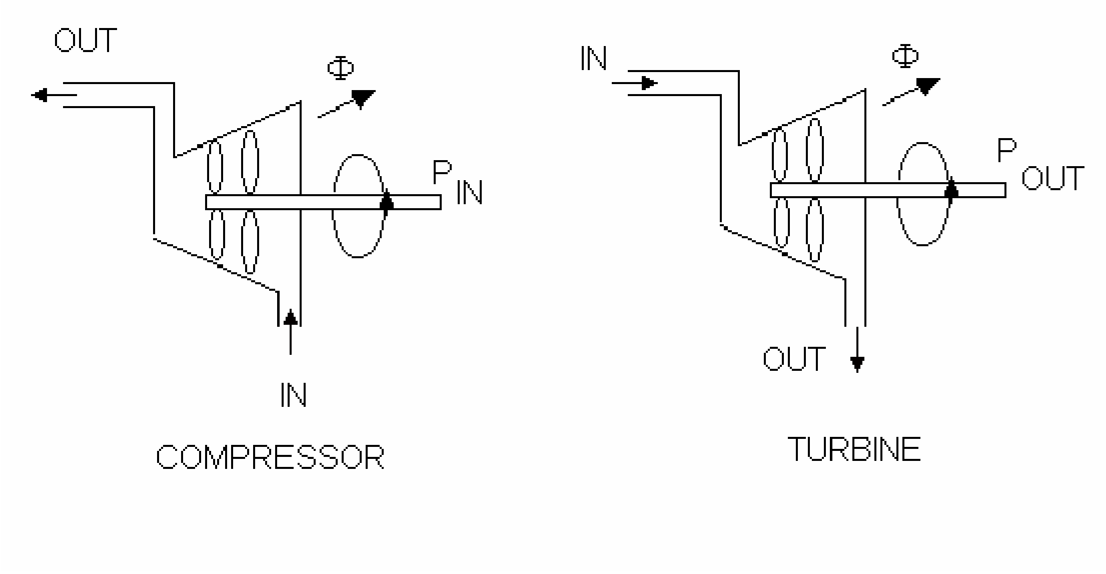

Fig.11

WORKED EXAMPLE No.7

A gas turbine uses 5 kg/s of hot air. It takes it in at 6 bar and 900°C and exhausts it at 450°C. The turbine loses 20 kW of heat from the casing. Calculate the theoretical power output given that c_p = 1005 J/kg K.

First identify this as a steady flow system for which the equation is

$\Phi + P = \Delta K.E./s + \Delta P.E./s + \Delta H/s$

For lack of further information we assume K.E. and PE to be negligible. The heat transfer rate is -20 kW.

The enthalpy change for a gas is $\Delta H = mC_p\Delta T$

$\Delta H = 5 \times 1005 \times (450 - 900) = -2261000$ W or -2.261 MW

$P = \Delta H - \Phi = -2261 - (-20) = -2241$ kW

The minus sign indicates that the power is leaving the turbine. Note that if this was a steam turbine, you would look up the h values in the steam tables.

3.5 STEADY FLOW EVAPORATORS AND CONDENSERS

A refrigerator is a good example of a thermodynamic system. In particular, it has a heat exchanger inside that absorbs heat at a cold temperature and evaporates the liquid into a gas. The gas is compressed and becomes hot. The gas is then cooled and condensed on the outside in another heat exchanger.

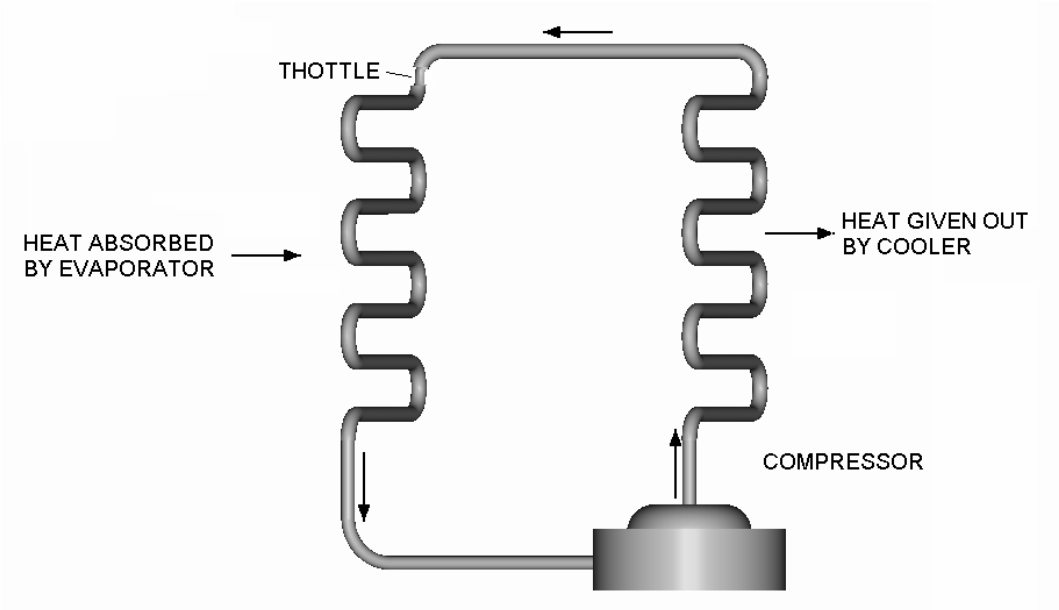

Fig. 2.12

For both the evaporator and condenser, there is no work transferred in or out. K.E. and P.E. are not normally a feature of such systems so the S.F.E.E. reduces to

$$\mathbf{\Phi = \Delta H/s}$$

On steam power plant, boilers are used to raise steam and these are examples of large evaporators working at high pressures and temperatures. Steam condensers are also found on power stations. The energy equation is the same, whatever the application.

WORKED EXAMPLE No.8

A steam condenser takes in wet steam at 8 kg/s and dryness fraction 0.82. This is condensed into saturated water at outlet. The working pressure is 0.05 bar. Calculate the heat transfer rate.

SOLUTION

$\Phi = \Delta H/s = m(h_2 - h_1)$

$h_1 = h_f + x\,h_{fg}$ at 0.05 bar

from the steam tables we find that

$h_1 = 138 + 0.82(2423) = 2125$ kJ/kg

$h_2 = h_f$ at 0.05 bar = 138 kJ/kg

hence **$\Phi = 8(138 - 2125) = -15896$ kW**

The negative sign indicates heat transferred from the system to the surroundings.

SELF ASSESSMENT EXERCISE No.4

1. Gas is contained inside a cylinder fitted with a piston. The gas is at 20°C and has a mass of 20 g. The gas is compressed with a mean force of 80 N which moves the piston 50 mm. At the same time 5 Joules of heat transfer occurs out of the gas. Calculate the following.

   i. The work done.(4 J)

   ii. The change in internal energy. (-1 J)

   iii. The final temperature. (19.9°C)

   Take c_V as 718 J/kg K

2. A steady flow air compressor draws in air at 20°C and compresses it to 120°C at outlet. The mass flow rate is 0.7 kg/s. At the same time, 5 kW of heat is transferred into the system. Calculate the following.

   i. The change in enthalpy per second. (70.35 kW)

   ii. The work transfer rate. (65.35 kW)

   Take c_p as 1005 J/kg K.

3. A steady flow boiler is supplied with water at 15 kg/s, 100 bar pressure and 200°C. The water is heated and turned into steam. This leaves at 15 kg/s, 100 bar and 500°C. Using your steam tables, find the following.

   i. The specific enthalpy of the water entering. (856 kJ/kg)

   ii. The specific enthalpy of the steam leaving. (3373 kJ/kg)

   iii. The heat transfer rate. (37.75 kW)

4. A pump delivers 50 dm^3/min of water from an inlet pressure of 100 kPa to an outlet pressure of 3 MPa. There is no measurable rise in temperature. Ignoring K.E. and P.E, calculate the work transfer rate. (2.42 kW)

5. A water pump delivers 130 dm^3/minute (0.13 m^3/min) drawing it in at 100 kPa and delivering it at 500 kPa. Assuming that only flow energy changes occur, calculate the power supplied to the pump. (860 W)

6. A steam condenser is supplied with 2 kg/s of steam at 0.07 bar and dryness fraction 0.9. The steam is condensed into saturated water at outlet. Determine the following.

   i. The specific enthalpies at inlet and outlet. (2331 kJ/kg and 163 kJ/kg)

   ii. The heat transfer rate. (4336 kW)

7. 0.2 kg/s of gas is heated at constant pressure in a steady flow system from 10^{o}C to 180^{o}C. Calculate the heat transfer rate Φ. (37.4 kW)

   C_p = 1.1 kJ/kg K

8. 0.3 kg of gas is cooled from 120^{o}C to 50^{o}C at constant volume in a closed system. Calculate the heat transfer. (-16.8 kJ)

   C_V = 0.8 kJ/kg.

4. POLYTROPIC PROCESSES.

When you complete section four you should be able to do the following.

- Use the laws governing the expansion and compression of a fluid.
- State the names of standard processes.
- Derive and use the work laws for closed system expansions and compressions.
- Solve problems involving gas and vapour processes in closed systems.

We will start by examining expansion and compression processes.

4.1 COMPRESSION AND EXPANSION PROCESSES.

A compressible fluid (gas or vapour) may be compressed by reducing its volume or expanded by increasing its volume. This may be done inside a cylinder by moving a piston or by allowing the pressure to change as it flows through a system such as a turbine. For ease of understanding, let us consider the change as occurring inside a cylinder. The process is best explained with a pressure - volume graph.

When the volume changes, the pressure and temperature may also change. The resulting pressure depends upon the final temperature. The final temperature depends on whether the fluid is cooled or heated during the process. It is normal to show these changes on a graph of pressure plotted against volume. (p-V graphs). A typical graph for a compression and an expansion process is shown in fig.13.

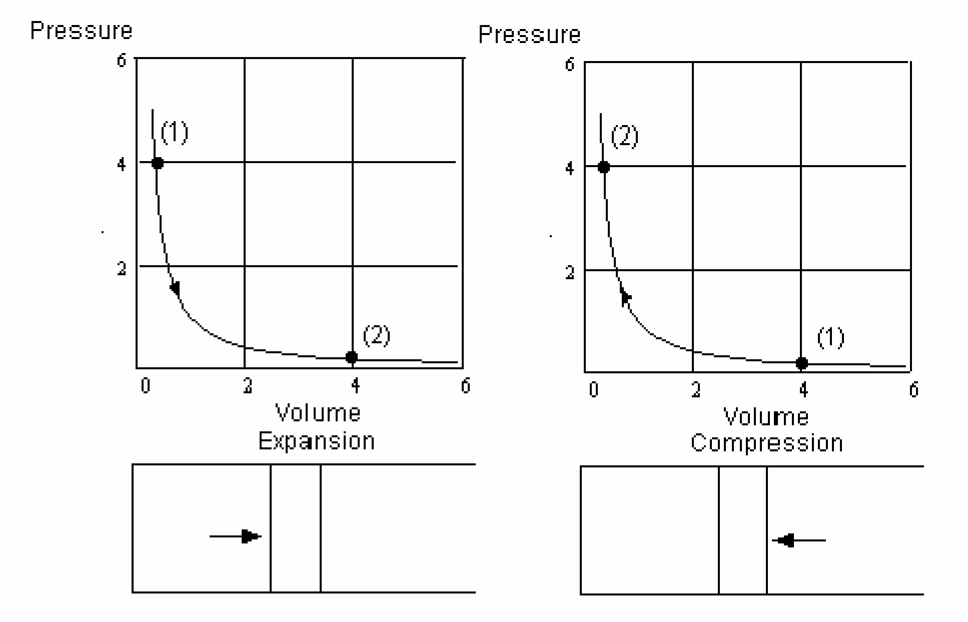

Fig. 13

It has been discovered that the resulting curves follows the mathematical law

$$pV^n = \text{constant.}$$

Depending on whether the fluid is heated or cooled, a family of such curves is obtained as shown (fig.14). Each graph has a different value of n and n is called the index of expansion or compression.

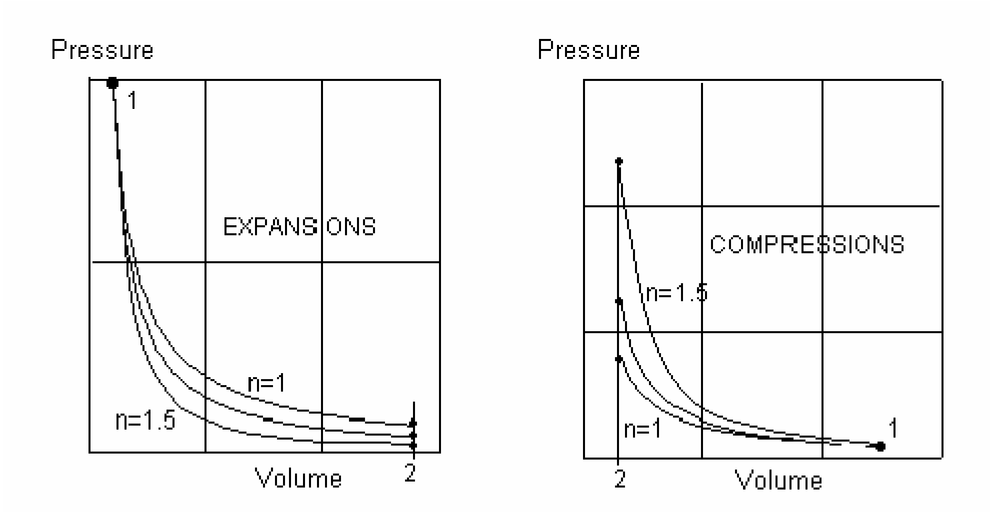

Fig.14

The most common processes are as follows.

CONSTANT VOLUME also known as ISOCHORIC

A vertical graph is a constant volume process and so it is not a compression nor expansion. Since no movement of the piston occurs no work transfer has taken place. Nevertheless, it still fits the law with n having a value of infinity.

CONSTANT PRESSURE also known as ISOBARIC

A horizontal graph represents a change in volume with no pressure change (constant pressure process). The value of n is zero in this case.

CONSTANT TEMPERATURE also known as ISOTHERMAL

All the graphs in between constant volume and constant pressure, represent processes with a value of n between infinity and zero. One of these represents the case when the temperature is maintained constant by cooling or heating by just the right amount.

When the fluid is a gas, the law coincides with Boyle's Law pV= constant so it follows that n is 1.

When the fluid is a vapour, the gas law is not accurate and the value of n is close to but not equal to 1.

ADIABATIC PROCESS

When the pressure and volume change in such a way that no heat is added nor lost from the fluid (e.g. by using an insulated cylinder), the process is called adiabatic. This is an important process and is the one that occurs when the change takes place so rapidly that there is no time for heat transfer to occur. This process represents a demarcation between those in which heat flows into the fluid and those in which heat flows out of the fluid. In order to show it is special, the symbol γ is used instead of n and the law is

$$pV^{\gamma} = C$$

It will be found that each gas has a special value for γ (e.g. 1.4 for dry air).

POLYTROPIC PROCESS

All the other curves represent changes with some degree of heat transfer either into or out of the fluid. These are generally known as polytropic processes.

HYPERBOLIC PROCESS

The process with n=1 is a hyperbola so it is called a hyperbolic process. This is also isothermal for gas but not for vapour. It is usually used in the context of a steam expansion.

WORKED EXAMPLE No.9

A gas is compressed from 1 bar and 100 cm^3 to 20 cm^3 by the law $pV^{1.3}$=constant. Calculate the final pressure.

SOLUTION.

If $pV^{1.3} = C$ then $p_1V_1^{1.3} = C = p_2V_2^{1.3}$

hence $1 \times 100^{1.3} = p_2 \times 20^{1.3}$

$1 \times (100/20)^{1.3} = p_2 = 8.1$ bar

WORKED EXAMPLE No.10

Vapour at 10 bar and 30 cm^3 is expanded to 1 bar by the law $pV^{1.2} = C$. Find the final volume.

SOLUTION.

$p_1V_1^{1.2} = C = p_2V_2^{1.2}$

$10 \times 30^{1.2} = 1 \times V_2^{1.2}$ $\qquad V_2 = (592.3)^{1/1.2} = 204.4\ cm^3$

WORKED EXAMPLE No.11

A gas is compressed from 200 kPa and 120 cm^3 to 30 cm^3 and the resulting pressure is 1 MPa. Calculate the index of compression n.

SOLUTION.

$200 \times 120^n = 1000 \times 30^n$

$(120/30)^n = 1000/200 = 5$

$4^n = 5$

$n\log 4 = \log 5$

$n = \log 5/\log 4 = 1.6094/1.3863 = 1.161$

Note this may be solved with natural or base 10 logs or directly on suitable calculators.

SELF ASSESSMENT EXERCISE No. 5

1. A vapour is expanded from 12 bar and 50 cm^3 to 150 cm^3 and the resulting pressure is 6 bar. Calculate the index of compression n.
(0.63)

2.a. A gas is compressed from 200 kPa and 300 cm^3 to 800 kPa by the law $pV^{1.4}$=C. Calculate the new volume. (111.4 cm^3)

2.b. The gas was at 50°C before compression. Calculate the new temperature using the gas law pV/T = C. (207°C)

3.a. A gas is expanded from 2 MPa and 50 cm^3 to 150 cm^3 by the law $pV^{1.25}$ = C. Calculate the new pressure. (506 kPa)

3.b. The temperature was 500°C before expansion. Calculate the final temperature.
(314°C)

4.2. COMBINING THE GAS LAW WITH THE POLYTROPIC LAW.

For gases only, the general law may be combined with the law of expansion as follows. $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ and so $\frac{T_2}{T_1} = \frac{p_2V_2}{p_1V_1}$

Since for an expansion or compression

$$p_1V_1^n = p_2V_2^n$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

Substituting into the gas law we get $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$

and further since $\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \frac{V_2}{V_1}$

substituting into the gas law gives $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$

To summarise we have found that $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$

In the case of an adiabatic process this is written as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$$

For an isothermal process n = 1 and the temperatures are the same.

WORKED EXAMPLE No.12

A gas is compressed adiabatically with a volume compression ratio of 10. The initial temperature is 25^{o}C. Calculate the final temperature given $\gamma = 1.4$

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298(10)^{1.4-1} = 748.5K \text{ or } 475.5^o C$$

WORKED EXAMPLE No.13

A gas is compressed polytropically by the law $pV^{1.2} = C$ from 1 bar and 20°C to 12 bar. calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \qquad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} = 293(12)^{1-\frac{1}{1.2}}$$

$$T_2 = 293(12)^{0.167} = 293(1.513) = 443.3K$$

WORKED EXAMPLE No.14

A gas is expanded from 900 kPa and 1100°C to 100 kPa by the law $pV^{1.3} = C$. Calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \qquad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

$$T_2 = 1373\left(\frac{100}{900}\right)^{1-\frac{1}{1.3}} = 1373(0.111)^{0.2308} = 1373(0.602) = 826.9K$$

SELF ASSESSMENT EXERCISE No. 6

1. A gas is expanded from 1 MPa and 1000°C to 100 kPa. Calculate the final temperature when the process is

   i. Isothermal (n=1) (1000°C)

   ii Polytropic (n=1.2) (594°C)

   iii. Adiabatic (γ =1.4) (386°C)

   iv. Polytropic (n= 1.6) (264°C)

2. A gas is compressed from 120 kPa and 15°C to 800 kPa. Calculate the final temperature when the process is

   i. Isothermal (n=1) (15°C)

   ii. Polytropic (n=1.3) (173°C)

   iii Adiabatic (γ=1.4) (222°C)

   iv. Polytropic (n= 1.5) (269°C)

3. A gas is compressed from 200 kPa and 20°C to 1.1 MPa by the law $pV^{1.3}$=C. The mass is 0.02 kg. c_p=1005 J/kg K. c_v = 718 J/kg K. Calculate the following.

   i. The final temperature. (434 K)

   ii. The change in internal energy (2.03 kJ)

   iii. The change in enthalpy (2.84 kJ)

4. A gas is expanded from 900 kPa and 1200°C to 120 kPa by the law $pV^{1.4}$ = C. The mass is 0.015 kg. c_p=1100 J/kg K c_v = 750 J/kg K Calculate the following.

   i. The final temperature. (828 K)

   ii. The change in internal energy (-7.25 kJ)

   iii. The change in enthalpy (-10.72 kJ)

4.3. EXAMPLES INVOLVING VAPOUR

Problems involving vapour make use of the formulae $pV^n = C$ in the same way as those involving gas. You cannot apply gas laws, however, unless it is superheated into the gas region. You must make use of vapour tables so a good understanding of this is essential. This is best explained with worked examples.

WORKED EXAMPLE No.15

A steam turbine expands steam from 20 bar and 300°C to 1 bar by the law $pV^{1.2}= C$.

Determine for each kg flowing:

a. the initial and final volume.
b. the dryness fraction after expansion.
c. the initial and final enthalpies.
d. the change in enthalpy.

SOLUTION

The system is a steady flow system in which expansion takes place as the fluid flows. The law of expansion applies in just the same way as in a closed system.

The initial volume is found from steam tables. At 20 bar and 300°C it is superheated and from the tables we find $v= 0.1255\ m^3/kg$

Next apply the law $pV^{1.2} = C$ $\quad p_1V_1^{1.2} = p_2V_2^{1.2}$ $\quad 20 \times 0.1255^{1.2} = 1 \times V_2^{1.2}$

Hence $\quad V_2= 1.523 m^3/kg$

Next, find the dryness fraction as follows.
Final volume = $1.523\ m^3/kg = xv_g$ at 1 bar.
From the tables we find v_g is $1.694\ m^3/kg$

hence $1.523 = 1.694x$ $\quad x = 0.899$

We may now find the enthalpies in the usual way.

h_1 at 20 bar and 300°C is 3025 kJ/kg
$h_2 = h_f + xh_{fg}$ at 1 bar (wet steam)
$h_2 = 417 + (0.899)(2258) = 2447$ kJ/kg

The change in enthalpy is $h_2 - h_1$ = -578 kJ/kg

SELF ASSESSMENT EXERCISE No.7

1. 3 kg/s of steam is expanded in a turbine from 10 bar and 200°C to 1.5 bar by the law $pV^{1.2}$=C. Determine the following.

   i. The initial and final volumes. (0.618 m^3 and 3 m^3)

   ii. The dryness fraction after expansion. (0.863)

   iii. The initial and final enthalpies. (2829 kJ/kg and 2388 kJ/kg)

   iv. The change in enthalpy. -1324 kW)

2. 1.5 kg/s of steam is expanded from 70 bar and 450°C to 0.05 bar by the law $pV^{1.3}$ = C. Determine the following.

   i. The initial and final volumes. (0.066 m^3/kg and 17.4 m^3/kg)

   ii. The dryness fraction after expansion. (0.411)

   iii. The initial and final enthalpies. (3287 kJ/kg and 1135 kJ/kg)

   iv. The change in enthalpy. (-3228 kW)

3. A horizontal cylindrical vessel is divided into two sections each 1m^3 volume, by a non-conducting piston. One section contains steam of dryness fraction 0.3 at a pressure of 1 bar, while the other contains air at the same pressure and temperature as the steam. Heat is transferred to the steam very slowly until its pressure reaches 2 bar.

   Assume that the compression of the air is adiabatic (γ=1.4) and neglect the effect of friction between the piston and cylinder. Calculate the following.

   i. The final volume of the steam. (1.39 m^3)

   ii. The mass of the steam. (1.97 kg)

   iii. The initial internal energy of the steam. (2053 kJ)

   iv The final dryness fraction of the steam. (0.798)

   v. The final internal energy of the steam. (4172 kJ)

vi. The heat added to the steam. (2119 kJ)

4.4. CLOSED SYSTEM WORK LAWS

4.4.1. EXPANSION OF PRESSURE WITH VOLUME

We will start by studying the expansion of a fluid inside a cylinder against a piston which may do work against the surroundings.

A fluid may expand in two ways.

a) It may expand rapidly and uncontrollably doing no useful work. In such a case the pressure could not be plotted against volume during the process. This is called an **UNRESISTED EXPANSION**

b) It may expand moving the piston. The movement is resisted by external forces so the gas pressure changes in order to overcome the external force and move the piston. In this case the movement is controlled and the variation of pressure with volume may be recorded and plotted on a p-V graph. Work is done against the surroundings. This process is called a **RESISTED EXPANSION.**

Consider the arrangement shown in fig. 15. Assume that there is no pressure outside. If the string holding the weight was cut, the gas pressure would slam the piston back and the energy would be dissipated first by acceleration of the moving parts and eventually as friction. The expansion would be unresisted.

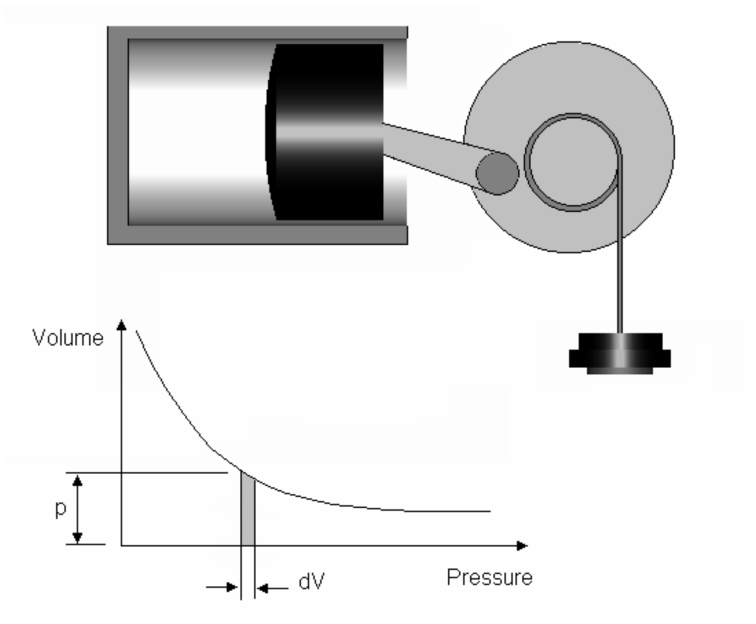

Fig. 15

If the weights were gradually reduced, the gas would push the piston and raise the remaining weights. In this way, work would be done against the surroundings (it ends up as potential energy in the weights). The process may be repeated in many small steps, with the change in volume each time being dV. The pressure although changing, is p at any time.

This process is characterised by two important factors.

1. The process may be reversed at any time by adding weights and the potential energy is transferred back from the surroundings as work is done on the system. The fluid may be returned to its original pressure, volume, temperature and energy.

2. The fluid force on one side of the piston is always exactly balanced by the external force (in this case due to the weights) on the other side of the piston.

The expansion or compression done in this manner is said to be **REVERSIBLE and CARRIED OUT IN EQUILIBRIUM.**

4.4.2. WORK AS AREA UNDER THE p - V DIAGRAM

If the expansion is carried out in equilibrium, the force of the fluid must be equal to the external force F. It follows that F = pA.

When the piston moves a small distance dx, the work done is dW

$$dW = - F\,dx = - pA dx = - pdV.$$

The minus sign is because the work is leaving the system.

For an expansion from points 1 to 2 it follows that the total work done is given by

$$W = -\int_{V_1}^{V_2} pdV$$

We must remember at this stage that our sign convention was that work leaving the system is negative.

It should be noted that some of the work is used to overcome any external pressure such as atmospheric and the useful work is reduced. Consider the system shown in fig.15 again but this time suppose there is atmospheric pressure on the outside p_a.

In this case It follows that

$F + p_aA = pA.$

$F = pA. - p_aA$

When the piston moves a small distance dx, the the useful work done is –F dx

$- F\,dx = - (pAdx - p_aAdx) = - (p - p_a)dV.$

For an expansion from points 1 to 2 it follows that the useful work done is given by

$$W = -\int_{V_1}^{V_2} (p - p_a)dV$$

4.4.3. WORK LAWS FOR CLOSED SYSTEMS

If we solve the expression $W = -\int_{V_1}^{V_2} pdV$ we obtain the work laws for a closed system. The solution depends upon the relationship between p and V. The formulae now derived apply equally well to a compression process and an expansion process. Let us now solve these cases.

CONSTANT PRESSURE

$$W = -\int_{V_1}^{V_2} pdV$$

$$W = -p\int_{V_1}^{V_2} dV$$

W = - p (V_2-V_1)

CONSTANT VOLUME

If V is constant then dV = 0

W = 0.

HYPERBOLIC

This is an expansion which follows the law pV^1 = C and is **ISOTHERMAL** when it is a gas. Substituting p = CV^{-1} the expression becomes

$$W = -\int_{V_1}^{V_2} pdV = -C\int_{V_1}^{V_2} V^{-1}dV = -C\ln\left[\frac{V_2}{V_1}\right]$$

Since pV = C then

$$W = -pV\ln\left[\frac{V_2}{V_1}\right]$$

$$\text{since}\ \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

$$W = -pV\ln\left[\frac{p_1}{p_2}\right]$$

In the case of gas we can substitute pV = mRT and so

$$W = -mRT \ln\left[\frac{V_2}{V_1}\right] = -mRT \ln\left[\frac{p_1}{p_2}\right]$$

POLYTROPIC

In this case the expansion follows the law $pV^n = C$. The solution is as follows.

$$W = -\int_{V_1}^{V_2} pdV \quad \text{but } p = CV^{-n}$$

$$W = -C\int_{V_1}^{V_2} V^{-n}dV$$

$$W = -C\frac{\left[V_2^{-n+1} - V_1^{-n+1}\right]}{-n+1}$$

$$\text{Since } C = p_1V_1 \text{ or } p_2V_2$$

$$W = \frac{\left[p_2V_2 - p_1V_1\right]}{n-1}$$

For gas only we may substitute pV = mRT and so $W = mR\frac{\left[T_2 - T_1\right]}{n-1}$

ADIABATIC

Since an adiabatic case is the special case of a polytropic expansion with no heat transfer, the derivation is identical but the symbol γ is used instead of n.

$$W = \frac{\left[p_2V_2 - p_1V_1\right]}{\gamma - 1}$$

For gas only we may substitute pV = mRT and so $W = mR\frac{\left[T_2 - T_1\right]}{\gamma - 1}$

This is the special case of the polytropic process in which Q=0. $Q = 0 \quad W = \frac{mR\Delta T}{\gamma - 1}$

Substituting for Q and ΔU in the NFEE we find

$$Q + W = \Delta U \qquad 0 + \frac{mR\Delta T}{\gamma - 1} = mCv\Delta T \qquad \frac{R}{\gamma - 1} = Cv$$

$$\text{Since } R = Cp - Cv \qquad Cp - Cv = Cv(\gamma - 1) \qquad \frac{Cp}{Cv} = \gamma$$

This shows that the ratio of the principal specific heat capacities is the adiabatic index. It was shown earlier that the difference is the gas constant R. These important relationships should be remembered.

Cp – Cv = R

γ = Cp/Cv

WORKED EXAMPLE No.15

Air at a pressure of 500 kPa and volume 50 cm^3 is expanded reversibly in a closed system to 800 cm^3 by the law $pV^{1.3} = C$. Calculate the following.

a. The final pressure.

b. The work done.

SOLUTION

$p_1 = 500$ kPa $V_1 = 50 \times 10^{-6}\ m^3$ $V_2 = 800 \times 10^{-6}\ m^3$

$$p_1V_1^{1.3} = p_2V_2^{1.3} \qquad 500x10^3(50x10^{-6})^{1.3} = p_2(800x10^{-6})^{1.3}$$

$$p_2 = 13.6x10^3 \text{ or } 13.6kPa$$

$$W = \frac{(p_2V_2 - p_1V_1)}{n-1} = \left(\frac{13.6x10^3 x800x10^{-6} - 500x10^3 x50x10^{-6}}{1.3-1}\right)$$

$$W = -47 Joules$$

WORKED EXAMPLE No.16

Steam at 6 bar pressure and volume 100 cm^3 is expanded reversibly in a closed system to 2 dm^3 by the law $pV^{1.2} = C$. Calculate the work done.

SOLUTION

$p_1 = 6$ bar $V_1 = 100 \times 10^{-6}\ m^3$ $V_2 = 2 \times 10^{-3}\ m^3$

$$p_2 = \frac{p_1V_1^{1.2}}{V_2^{1.2}} = 6x\left(\frac{100x10^{-6}}{2x10^{-3}}\right)^{1.2} = 0.1648bar$$

$$W = \frac{(p_2V_2 - p_1V_1)}{n-1} = \frac{(0.1648x10^5 x2x10^{-3} - 6x10^5 x100x10^{-6})}{1.2-1}$$

$$W = -135.2 Joules$$

SELF ASSESSMENT EXERCISE No.8

1. 10 g of steam at 10 bar and 350°C expands reversibly in a closed system to 2 bar by the law $pV^{1.3}$=C . Calculate the following.

   i. The initial volume. (0.00282 m^3)

   ii. The final volume. (0.00974 m^3)

   iii. The work done. (-2.92 kJ)

2. 20 g of gas at 20°C and 1 bar pressure is compressed to 9 bar by the law $pV^{1.4}$ = C. Taking the gas constant R = 287 J/kg K calculate the work done. (Note that for a compression process the work will turn out to be positive if you correctly identify the initial and final conditions). (3.67 kJ)

3. Gas at 600 kPa and 0.05 dm^3 is expanded reversibly to 100 kPa by the law $pV^{1.35}$ = C. Calculate the work done. (-31.8 kJ)

4. 15 g of gas is compressed isothermally from 100 kPa and 20°C to 1 MPa pressure. The gas constant is 287 J/kg K. Calculate the work done. (2.9 kJ)

5. Steam at 10 bar with a volume of 80 cm^3 is expanded reversibly to 1 bar by the law pV=C. Calculate the work done. (-184.2 kJ)

6. Gas fills a cylinder fitted with a frictionless piston. The initial pressure and volume are 40 MPa and 0.05 dm^3 respectively. The gas expands reversibly and polytropically to 0.5 MPa and 1 dm^3 respectively. Calculate the index of expansion and the work done. (1.463 and -3.24 kJ)

7. An air compressor commences compression when the cylinder contains 12 g at a pressure is 1.01 bar and the temperature is 20°C. The compression is completed when the pressure is 7 bar and the temperature 90°C. (1.124 and 1944 J)

   The characteristic gas constant R is 287 J/kg K. Assuming the process is reversible and polytropic, calculate the index of compression and the work done.

WORKED EXAMPLE No.17

0.2 kg of gas at 100 ^oC is expanded isothermally and reversibly from 1 MPa pressure to 100 kPa. Take C_V = 718 J/kg K and R = 287 J/kg K.
Calculate

i. The work transfer.
ii. The change in internal energy.
iii. The heat transfer.

SOLUTION

$$W = -pV \ln\left(\frac{V_2}{V_1}\right) = -mRT \ln\left(\frac{V_2}{V_1}\right) = -mRT \ln\left(\frac{p_1}{p_2}\right)$$

$$W = -0.2x287x373\ln\left(\frac{1x10^6}{1x10^5}\right) = -49300 \text{ J or } -49.3 \text{ kJ}$$

The work is leaving the system so it is a negative work transfer.

Since T is constant $\Delta U = 0$ Q - 49.3 = 0 Q = 49.3 kJ

Note that 49.3 kJ of heat is transferred into the gas and 49.3 kJ of work is transferred out of the gas leaving the internal energy unchanged.

WORKED EXAMPLE No.18

Repeat worked example 17 but for an adiabatic process with γ = 1.4
Calculate

SOLUTION

$$T_2 = 373 \text{ x} \left(\frac{100 \text{ x } 10^3}{1 \text{ x } 10^6}\right)^{1-\frac{1}{\gamma}} = 193 \text{ K}$$

$$\text{W} = -\text{mRT}(\text{T}_2 - \text{T}_1) = -0.2 \text{ x } 287 \text{ x } \frac{(193 - 373)}{0.4}$$

$$\text{W} = -25\,830 \text{ J}$$

For an adiabatic process Q = 0

$\text{Q} + \text{W} = \Delta\text{U}$ hence ΔU = -25 830 J

Check $\Delta\text{U} = \text{mC}_\text{V}\Delta\text{T} = 0.2 \text{ x } 718 \text{ x } (193 - 373) = -25\,848$ J

WORKED EXAMPLE No.19

Repeat worked example 17 but for a polytropic process with n=1.25
Calculate

SOLUTION

$$T_2 = 373 \text{ x} \left(\frac{100 \text{ x } 10^3}{1 \text{ x } 10^6}\right)^{1-\frac{1}{n}} = 235.3 \text{ K}$$

$$W = -mRT(T_2 - T_1) = -0.2 \text{ x } 287 \text{ x } \frac{(235.3 - 373)}{0.4}$$

$$W = -31605 \text{ J}$$

$$\Delta U = mC_V\Delta T = 0.2 \text{ x } 718 \text{ x } (235.3 - 373) = -19773.7 \text{ J}$$

$$Q = \Delta U - W$$

$$Q = -19773.7 - (-31603) = 11831.3 \text{ J}$$

SELF ASSESSMENT EXERCISE No.9

Take C_V = 718 J/kg K and R = 287 J/kg K throughout.

1. 1 dm^3 of gas at 100 kPa and 20°C is compressed to 1.2 MPa reversibly by the law $pV^{1.2} = C$. Calculate the following.

   i. The final volume. (0.126 dm^3)
   ii. The work transfer. (257 J)
   iii. The final temperature. (170oC)
   iv. The mass. (1.189 g)
   v. The change in internal energy. (128 J)
   vi. The heat transfer. (-128 J)

2. 0.05 kg of gas at 20 bar and 1100°C is expanded reversibly to 2 bar by the law $pV^{1.3}$ =C in a closed system. Calculate the following.

   i. The initial volume. (9.85 dm^3)
   ii. The final volume. (58 dm^3)
   iii. The work transfer. (-27 kJ)
   iv. The change in internal energy. (-20.3 kJ)
   v. The heat transfer. (6.7 kJ)

3. 0.08 kg of air at 700 kPa and 800°C is expanded adiabatically to 100 kPa in a closed system. Taking γ = 1.4 calculate the following.

   i. The final temperature. (615.4 K)
   ii. The work transfer. (26.3 kJ)
   iii. The change in internal energy. (-26.3 J)

4. A horizontal cylinder is fitted with a frictionless piston and its movement is restrained by a spring as shown (Figure 16.)

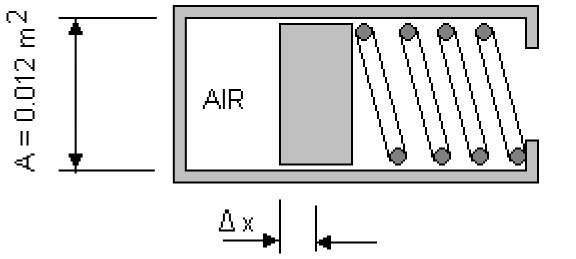

Figure 16

a. The spring force is directly proportional to movement such that $\Delta F/\Delta x = k$

Show that the change in pressure is directly proportional to the change in volume such that $\Delta p/\Delta V = k/A^2$

b. The air is initially at a pressure and temperature of 100 kPa and 300 K respectively. Calculate the initial volume such that when the air is heated, the pressure – volume graph is a straight line that extends to the origin. (0.5 ^{dm3})

c. The air is heated making the volume three times the original value. Calculate the following.

i. The mass. (0.58 g)
ii. The final pressure. (300 kPa)
iii. The final temperature. (2700 K)
iv. The work done. (-200 kJ)
v. The change in internal energy. (917 J)
vi. The heat transfer. (1.12 kJ)

4.5. CLOSED SYSTEM PROBLEMS INVOLVING VAPOUR

The solution of problems involving steam and other vapours is done in the same way as for gases with the important proviso that gas laws must not be used. Volumes and internal energy values should be obtained from tables and property charts. This is best illustrated with a worked example.

WORKED EXAMPLE No.20

1kg of steam occupies a volume of 0.2 m^3 at 9 bar in a closed system. The steam is heated at constant pressure until the volume is 0.3144 m^3. Calculate the following.

i. The initial dryness fraction.
ii. The final condition.
iii. The work transfer.
iv. The change in internal energy.
v. The heat transfer.

SOLUTION

First find the initial dryness fraction.

$V_1 = 0.2 = mx_1v_g$ at 9 bar $\quad x_1 = 0.2/(1 \times 0.2149)$

x_1= 0.931 (initial dryness fraction).

Now determine the specific volume after expansion.

$p_2 = 9$ bar (constant pressure) $\quad V_2 = 0.3144\ m^3$.

$V_2 = mv_2 \quad v_2 = 0.3144/1 = 0.3144 m^3/kg$

First, look in the superheat tables to see if this value exists for superheat steam. We find that at 9 bar and 350°C, the specific volume is indeed 0.3144 m^3/kg.

The final condition is superheated to 350°C .

Note that if v_2 was less than v_g at 9 bar the steam would be wet and x_2 would have to be found.

Next find the work.

$W = -p(V_2 - V_1) = -9 \times 10^5 (0.3144 - 0.2) = -102950$ J

W = -102.95 kJ (Energy leaving the system)

Next determine the internal energy from steam tables.

$U_1 = m\,u_1$ and $u_1 = u_f + x_1 u_{fg}$ at 9 bar

u_{fg} at 9 bar = $u_g - u_f = 2581 - 742 = 1839$ kJ/kg

$U_1 = 1\{742 + 0.931(1839)\} = 2454$ kJ

$U_2 = m\,u_2$ and $u_2 = u$ at 9 bar and 350 $^oC = 2877$ kJ/kg

$U_2 = m\,u_2 = 1(2877) = 2877$ kJ.

The change in internal energy = $U_2 - U_1$ = 423 kJ (increased)

Finally deduce the heat transfer from the NFEE

$Q + W = \Delta U$

hence $Q = \Delta U - W = 423 - (-102.95)$

Q = 526 kJ (energy entering the system)

SELF ASSESSMENT EXERCISE No.10

1. 0.2 kg of dry saturated steam at 10 bar pressure is expanded reversibly in a closed system to 1 bar by the law $pV^{1.2}$ =C. Calculate the following.

   i. The initial volume. (38.9 dm^3)

   ii. The final volume. (264 dm^3)

   iii. The work transfer. (-62 kJ)

   iv. The dryness fraction. (0.779)

   v. The change in internal energy. (-108 kJ)

   vi. The heat transfer. (-46 kJ)

2. Steam at 15 bar and 250°C is expanded reversibly in a closed system to 5 bar. At this pressure the steam is just dry saturated. For a mass of 1 kg calculate the following.

   i. The final volume. (0.375 m^3)
   ii. The change in internal energy. (-165 kJ)
   iii. The work done. (-187 kJ)
   iv. The heat transfer. (22.1 kJ)