THERMODYNAMICS
TUTORIAL 14

GAS MIXTURES

This tutorial is set at QCF Levels 5 to 6

On completion of this tutorial you should be able to apply

- Analyse mixtures of gases and vapours and the relationship between specific and molar properties.
- Determine the effects of mixtures of gases and vapours on the performance of cooling towers and condensers.
- Analyse air-conditioning plant.
- Use psychrometric charts.

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1 *Universal Gas Law*

$$pV = \frac{mR_oT}{\tilde{N}}$$

R_o is the universal constant 8 314.4 J/kmol K

Ñ is the relative molecular mass which is 18 for water vapour treated as a gas and 28.96 for dry air treated as a single gas.

2. *Partial Pressures*

The pressure exerted by a gas on the surface of containment is due to the bombardment of the surface by the molecules. The relative distance between molecules is very large so if two or more gases exist in the same space, their behaviour is unaffected by the others and so each gas produces a pressure on the surface according to the gas law above. Each gas occupies the total volume V and has the same temperature T. If two gases A and B are considered, the pressure due to each is:

$$p_a = \frac{m_aR_oT}{\tilde{N}_aV_a} \qquad p_b = \frac{m_bR_oT}{\tilde{N}_bV_b}$$

The total pressure on the surface of containment is $\mathbf{p = p_a + p_b}$

This is Daltons Law of partial pressures.

Now let's see how these laws are applied to mixtures of vapour and air.

3. *Air - Vapour Mixtures*

In the following work, water vapour is treated as a gas.

Consider a mixture of dry air and vapour. If the temperature of the mixture is cooled until the vapour starts to condense, the temperature must be the saturation temperature (dew point) and the partial pressure of the vapour p_s must be the value of p_s in the fluids tables at the mixture temperature.

If the mixture is warmed up at constant pressure so that the temperature rises, the vapour must become superheated. It can be shown that the partial pressure of the vapour and the dry air remains the same as at the saturation temperature.

Let condition (1) be at the saturation condition and condition (2) be at the higher temperature. p is constant so it follows that :

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

The initial partial pressure of the vapour is:

$$p_{s1} = \frac{m_sR_oT_1}{\tilde{N}_sV_1}$$

The final pressure of the vapour is:

$$p_{s2} = \frac{m_sR_oT_2}{\tilde{N}_sV_2}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ then } p_{s1} = p_{s2}$$

By the same process it can be shown that

$$p_{a1} = p_{a2}$$

If p is constant then the partial pressures are constant and the partial pressure of the vapour may easily be found by looking up the saturation pressure at the dew point if it is known.

When the air is contact with water, it will evaporate the water and the water will cool down until it is at the saturation temperature or dew point. When stable conditions are reached, the air becomes saturated and equal to the temperature of the water and so its temperature is the dew point (t_s) in fluids tables.

WORKED EXAMPLE No. 1

Moist air at 1 bar and 25°C passes over water and emerges at 1 bar and 18°C. Calculate the partial pressure of the air and vapour before cooling.

SOLUTION

When cooled 18°C must be the saturation temperature so the partial pressure of the vapour is p_s in the fluids tables and is 0.02063 bar.

The partial pressure of the vapour was the same before cooling so the partial pressure of the air must be

1 - 0.02063 = 0.97937 bar.

Now let's look at the definitions and use of humidity.

4. *Humidity*

There are two ways to express humidity ***Specific and Relative.***

4.1. Specific Humidity ω

ω = mass of water vapour/mass of dry air

Starting with the gas law

$$m = \frac{pV\tilde{N}}{R_oT}$$

$$\omega = \frac{p_sVR_o\tilde{N}_s}{p_aVR_oT\tilde{N}_a} = \frac{p_s\tilde{N}_s}{p_a\tilde{N}_a} = \frac{p_s \times 18}{p_a \times 28.96} = 0.622\frac{p_s}{p_a}$$

$$\omega = 0.622\frac{p_s}{p - p_s}$$

4.2. Relative Humidity ϕ

ϕ = mass of vapour/maximum possible mass of vapour

The maximum possible mass of water vapour which can be held by air is when the vapour is saturated and the temperature of the mixture is the saturation temperature.

mass = Volume/specific volume = V/*v*

When saturated

$v = v_g$ at the mixture temperature.

$$\phi = \frac{m_s}{m_g} = \frac{V}{v_s} \div \frac{V}{v_a} = \frac{v_a}{v_s}$$

Alternatively

$$v = V/m$$

$$v_s = \frac{p_s\tilde{N}_s}{R_oT} \text{ and } v_g = \frac{p_g\tilde{N}_g}{R_oT}$$

$$\phi = \frac{p_s}{p_g}$$

p_s = partial pressure of the actual vapour

p_g = partial pressure when saturated.

$$\omega = 0.622\frac{p_s}{p - p_s} \text{ and } \phi = \frac{p_s}{p_g}$$

$$\phi = \frac{\omega(p - p_s)}{0.622p_g}$$

WORKED EXAMPLE No. 2

Moist air at 1 bar and 25°C is cooled to 18°C by passing it over water at 18°C. It emerges at 18°C and 1 bar with a relative humidity of 1.0. Assuming that net water is neither absorbed nor lost calculate the relative and specific humidity before cooling.

SOLUTION

This is the same as the previous problem so the dew point must be 18°C and the partial pressure of the vapour is p_s at 18°C and is 0.02063 bar.

$$p_a = 1 - 0.02063 = 0.97937 \text{ bar}$$

$$p_g \text{ at } 25^\circ\text{C} = 0.03166 \text{ bar}$$

It follows that if no net water is gained nor lost then the specific humidity must be the same before and after and is

$$\omega = 0.622\frac{p_s}{p_a} = 0.0131$$

$$\phi = \frac{p_s}{p_g} = 0.651$$

These are the humidity values which will result in no evaporation and no condensation.

If $\phi < 0.651$ then there will have been evaporation

If $\phi > 0.651$ then condensation will have taken place on contact with the water and cooling also.

4.3 Mass Balance

Consider the worked example 2 again only this time supposes the relative humidity at inlet is 0.5. This means that water is evaporated. Consider 1 kg of dry air passing through from inlet to outlet.

At outlet $\phi = 1$ and

$$p_{s2} = p_g \text{ at } 18^oC = 0.02063 \text{ bar.}$$

$$p_{a2} = 1 - 0.02063 = 0.97937 \text{ bar}$$

$$\omega_2 = 0.622\frac{p_s}{p_g} = 0.0131 = \frac{m_s}{m_a}$$

hence for 1 kg of dry air there must be 0.0131 kg of saturated vapour.

At inlet $\phi_1 = 0.5$

This time the mass of the vapour at inlet and outlet are not the same so the specific humidity is different at inlet.

$$\phi_1 = 0.5 = \frac{\omega_1(p - p_s)}{0.622p_g} \text{ at } 25^oC$$

Remember p_s is the saturation pressure at the dew point (18^oC) and p_g is the saturation pressure at the actual temperature (25^oC).

$$0.5 = \frac{\omega_1(1 - 0.02063)}{0.622 \times 0.03166} = 49.733\omega_1$$

Hence

$$\omega_1 = 0.01 = \frac{m_{s1}}{m_a}$$

Since the air mass is 1 kg throughout, then the mass of vapour at inlet is 0.01 kg.

It follows that the mass of water evaporated is 0.0131 - 0.01 = 0.0031 kg

4.4 Psychrometric Charts

In order to use these charts you need to know the ***Dry Bulb Temperatures (DBT)*** and the ***Wet Bulb Temperature (WBT)*** of the moist air. The dry bulb temperature refers to the actual temperature of the air originally found with a glass thermometer in a shaded position. The wet bulb temperature refers to the temperature you would get if the bulb was covered in a wet porous tube (e.g. muslin cloth) with the bottom immersed in water that is drawn up the tube and evaporated. This temperature can be obtained more rapidly by blowing air over the bulb to create evaporation. The wet bulb temperature is the lowest temperature that can be achieved by evaporating water. The difference in the DBT and WBT depends on the moisture content of the air and they would be the same if the relative humidity was 100%. Actual calculation of the WBT is quite complicated and involves considering the heat transfer involved.

A psychrometer measures both temperatures. A **sling psychrometer** requires manual operation to create the airflow over the bulbs, but a **powered psychrometer** includes a fan for this function. Knowing both temperatures enables you to find the relative humidity from the psychrometric chart appropriate to the air pressure. A *Hygrometer* is an instrument for determining relative humidity and may incorporate the DBT and WBT. Modern electronic tools for Heating, Ventilating and Air Conditioning will do all the measurements and calculations.

A psychrometric chart enables you to determine the humidity and other properties of moist air. A simplified chart is shown next. Simply locate the point pin pointed by the two temperatures and read off specific humidity, the relative humidity, specific volume and specific enthalpy. In practice you could use a calculator programme by searching the internet for humidity calculator.

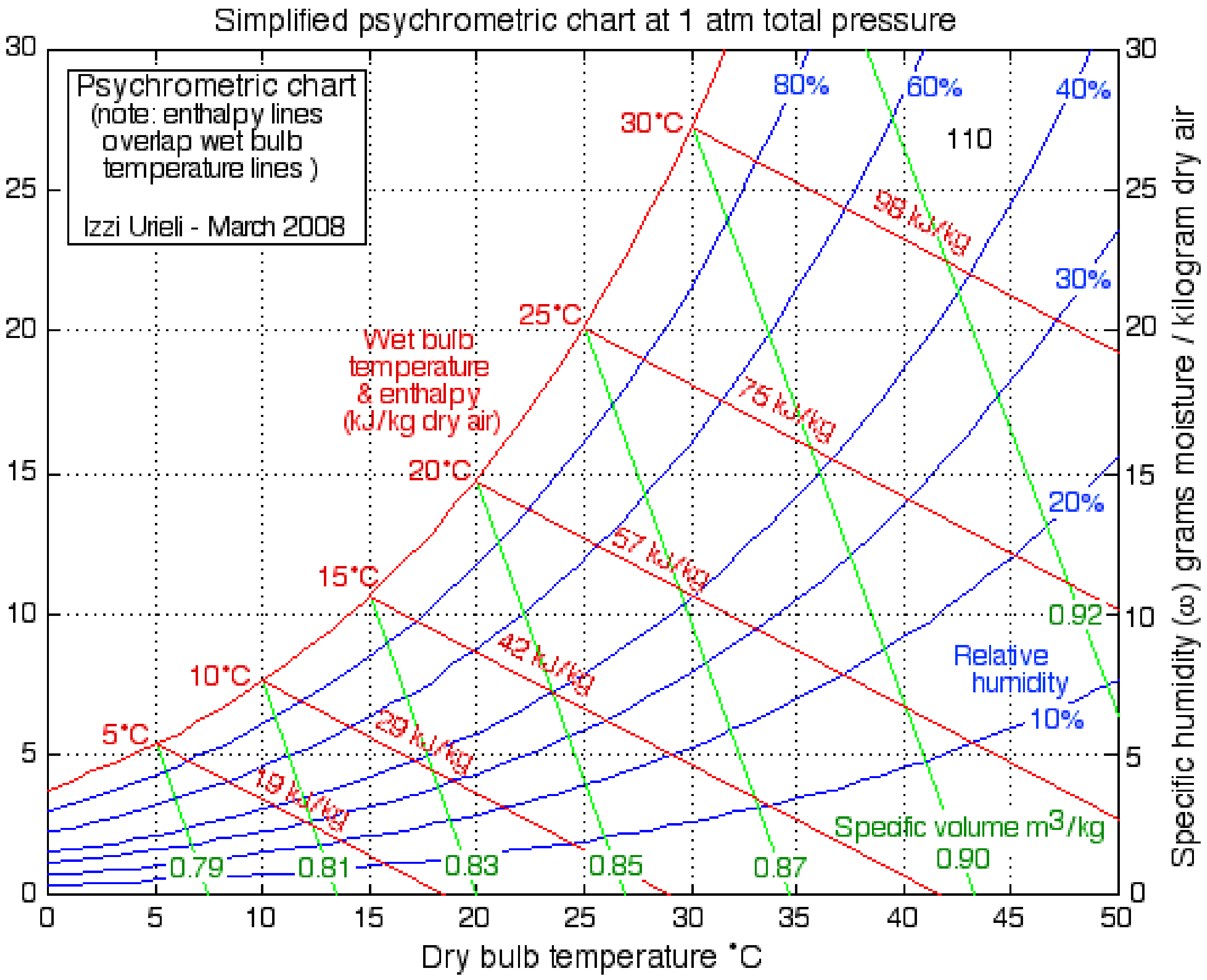

Figure No. 1

SELF ASSESSMENT EXERCISE No. 1

1. Repeat the worked example 2 but this time the relative humidity 0.8 at inlet. Is water condensed or evaporated?

   (m_{s1} = 0.01609 kg so water is condensed)

2. Define specific humidity ω and prove that

$$\omega = \frac{\tilde{N}_s p_s}{\tilde{N}_a (p - p_a)}$$

Humid air at 1 bar flows through an insulated vessel over a pool of water and emerges saturated. The temperatures are 25°C and 18°C at inlet and outlet respectively. The mass of water is maintained constant at 18°C all the time. Calculate the relative humidity at inlet assuming constant pressure throughout.
(Ans. 0.651)

5. *Energy Balance*

Consider a simple air conditioner. Moist air is drawn in and cooled so that water condenses out. The air at this point must be at the dew point.

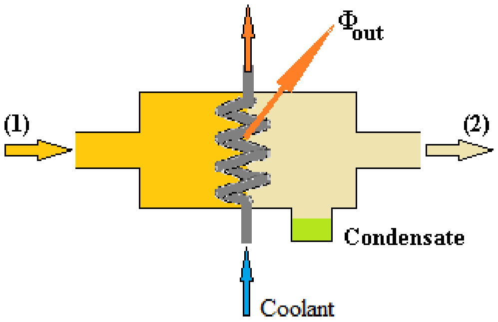

Figure 2

Applying the law of energy conservation we get

$$M_a c_a\theta_1 + m_{s2} h_{s1} = m_w c_w \theta_2 + m_a c_a \theta_2 + m_{s2} h_{s2} + \text{energy removed}$$

The suffixes a, s and w refer to air, vapour and water respectively. Treating vapour as a gas, the accepted value of the specific heat capacity is 1.864 kJ/kg K.

The enthalpy of steam relative to 0^o C is then $h_s = h_g + 1.864\,(\theta_s - t_s)$
t_s is the saturation temperature of the vapour.

It is probably best to use the thermodynamic tables or the h - s chart supplied in the exam whenever possible to find the enthalpy of vapour at low pressures and temperatures.

WORKED EXAMPLE No. 3

Moist air enters a conditioning unit at 25^oC and 1 bar with a relative humidity of 0.7. It is passed through a cooler causing the temperature to fall to 18^oC and condensate is formed.

Calculate the mass of condensate formed per kg of dry air and the energy removed per kg.

SOLUTION

Following the same method as in the previous examples, the mass of vapour at exit is

$$m_{s2} = 0.0131 \text{ kg per kg of dry air}$$

The vapour pressure at inlet is 1.0 bar.

At inlet the relative humidity is 0.7

$$\phi_1 = 0.7 = \frac{\omega_1(p - p_s)}{0.622 p_g} \text{ at } 25^o$$

$$\omega_1 = \frac{0.622 \times 0.03166 \times 0.7}{1 - 0.02063} = 0.014075$$

The mass of vapour at inlet is then $m_{s1} = 0.014075$ kg/kg

The condensate formed $m_W = 0.014075 - 0.0131 = 0.000975$ kg

Conducting an energy balance:

$$1 \times 1.005 \times 25 + 0.014075\ h_{s1} = 1 \times 1.005 \times 18 + 0.0131\ h_{s2} + m_W\ \theta_W + \Phi$$

$$h_{s1} = h_g + 1.864\ (\theta_s - t_s) \text{ at } 0.02063 \text{ bar.}$$

$$h_{s1} = 2\,533.9 + (25 - 18)(1.864) = 2\,546.9 \text{kJ/kg}$$

$h_{s2} = h_g$ at 18^oC since it is saturated. $h_{s2} = 2\,533.9$ kJ/kg

The balance becomes

$$1 \times 1.005 \times 25 + 0.014075\ (2\,546.9) =$$

$$1 \times 1.005 \times 18 + 0.0131\ (2\,533.9) + 0.000975 \times 4.168 \times 18 + \Phi$$

$\Phi = 9.87$ kJ per kg of dry air.

In air conditioning it is normal to heat the air to the required temperature before it leaves the unit.

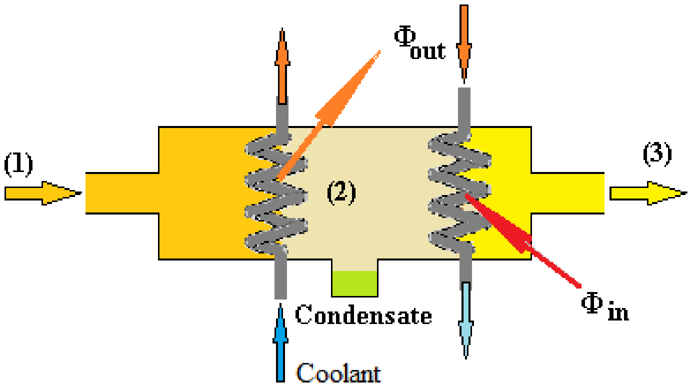

Figure 3

Suppose the air is heated to 22^oC before leaving. What is the heat transfer required in the heater? The masses are unchanged so we only need an energy balance between 2 and 3.

$$1 \times 1.005 \times 18 + 0.0131\ h_{s2} + \Phi = 1 \times 1.005 \times 22 + 0.0131\ h_{s3}$$

$$h_{s3} = h_g + 1.864\ (\theta_s - t_s) \text{ at } 0.02063 \text{ bar. } t_s = 18^oC$$

$$h_{s3} = 2\,533.9 + 1.864\ (22 - 18) = 2\,541.3 \text{ kJ/kg}$$

$h_{s2} = 2\,533.9$ kJ/kg

$$1 \times 1.005 \times 18 + 0.0131 \times 2\,533.9 + \Phi = 1 \times 1.005 \times 22 + 0.1031 \times 2\,541.3$$

$$51.28 + \Phi = 55.27$$

$$\Phi = 4.11 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No. 2

Air having a pressure, temperature and relative humidity of 1 bar, 26°C and 0.65 respectively, flows into an air conditioner at a steady rate and is dehumidified by cooling and removing water from it. The air is then heated to produce an outlet temperature and relative humidity of 24°C and 0.359 respectively. The pressure is constant throughout. Determine the heat transfers in the cooler and heater per kg of conditioned air at exit. Draw up a complete mass balance.

(36.22 kJ/kg and 16.24 kJ/kg)

6. *Cooling Towers*

Cooling towers fall into two types, dry and wet. Dry cooling towers are no more than very large air conditioners and the theory is the same as already outlined.

Wet cooling towers work on the principle of spraying warm water downwards so that heat and vapour is passed to the air which rises and carries away latent heat leaving the water at a lower temperature to collect in a pool at the bottom of the tower. The water is then recycled from the pool. The moist air leaves the top of the tower as a plume. The tower has a venturi shape to assist the process by causing a slight pressure reduction in the spray area followed by resurgence as the top widens. This causes condensation to form and make the plume visible. Some of the condensate rains down into the pool. We can say with certainty that the air leaves the tower with 100% humidity.

The best way to understand the problem is to do a worked example as follows.

WORKED EXAMPLE No. 4

A cooling tower must cool 340 kg of water per minute. The water is supplied at 42°C and it is sprayed down into the column of air which enters the bottom of the tower at a rate of 540m^3/min with a temperature of 18°C and relative humidity of 60%. The moist air leaves the top of the tower saturated at 27°C. The whole process occurs at a constant pressure of 1.013 bar. Determine the temperature of the cooled water in the pool and the rate at which make up water must be supplied to replace that evaporated.

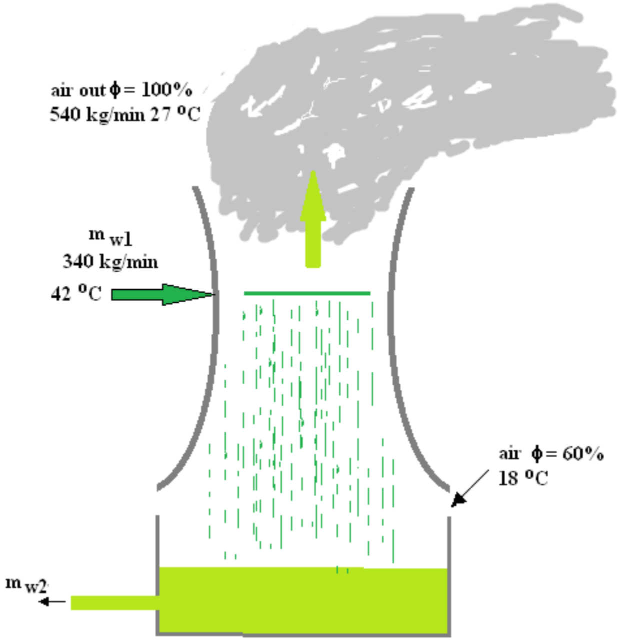

Figure 4

SOLUTION

R = 287 J/kg K for air and 462 J/kg K for vapour.

Inlet Air

p_{g1} = 0.02063 bar at 18°C $\phi_1 = 0.6 = p_{s1}/p_{g1}$ p_{s1} = 0.012378 bar

hence p_{a1} = 1.013 - 0.012378 = 1 bar

$$m_a = \frac{pV}{RT} = \frac{1 \times 10^5 \times 540}{287 \times 291} = 646.6 \text{ kg/min}$$

$$m_{s1} = \frac{pV}{RT} = \frac{0.012378 \times 10^5 \times 540}{462 \times 291} = 4.971 \text{ kg/min}$$

Outlet Air

$\phi 2 = 1$ $p_{s2} = p_{g2}$ = 0.03564 bar hence p_{a2} = 0.9774 bar

$$\phi_2 = 0.622 \times \frac{0.03564}{0.9774} = 0.02268$$

$m_{s2} = 0.02268 \times 646.6 = 14.66$ kg/min

Water evaporated = 14.66 - 4.971 = 9.693 kg/min

Make up water = 9.693 kg/min

$m_{w2} = 340 - 9.693 = 330.3$

Energy Balance

In this example enthalpy values from the steam tables and chart will be used.

$h_{w1} = h_f$ @ 42°C = 175.8 kJ/kg h_{w2} is unknown

$h_{a1} = 1.005 \times 27$ kJ/kg

h_{s1} = h @ 0.012378 bar & 18°C = 2 530 kJ/kg (from h-s chart)

$h_{s2} = h_g$ @ 27°C = 2 550.3 kJ/kg

Balancing energy we get

$(340 \times 175.8) + \{646.6 \times 1.005 \times (18 - 7)\} + \{4.971 \times 2530) - (14.66 \times 2550.3) = 330.3\, h_{w2}$

h_{w2} = 123.5 kJ/kg and from the tables the temperature must be 29.5 °C.

The temperature of the cooled water is 29.5 °C.

SELF ASSESSMENT EXERCISE No. 3

1. Derive the expression for specific humidity

$$\omega = 0.622\frac{p_s}{p_a}$$

Water flows at 5 000 kg/h and 40°C into a cooling tower and is cooled to 26°C. The unsaturated air enters the tower at 20°C with a relative humidity of 0.4. It leaves as saturated air at 30°C. The pressure is constant at 1 bar throughout.

Calculate

i. the mass flow of air per hour. (4 636 kg/h)

ii. the mass of water evaporated per hour. (100.5 kg/h)

2. The cooling water for a small condenser is sent to a small cooling tower. 7 m^3/s of air enters the tower with a pressure, temperature and relative humidity of 1.013 bar, 15°C and 0.55 respectively. It leaves saturated at 32°C. The water flows out of the tower at 7.5 kg/s at 13°C. Using a mass and energy balance, determine the temperature of the water entering the tower.
(Answer 33.9°C)

3. A fan supplies 600 dm^3/s of air with a relative humidity of 0.85, temperature 30°C and pressure 1.04 bar into an air conditioner. Moisture is removed from the air by cooling and both the air and condensate leave at the same temperature. The air is then heated to 20°C and has a relative humidity of 0.6. Determine the following.

i. The mass of dry air and water at entrance to the conditioner.
(0.6927 kg/s and 0.01546 kg/s)

ii. The mass of water vapour delivered at exit.
(0.00588 kg/s)

iii. The mass of water extracted from the cooler.
(0.00958 kg/s)

iv. The temperature at exit from the cooler.
(12°C)

v. The heat transfer in the cooler.

7. *Condensers*

It is inevitable that air will be drawn into steam condensers operating with a vacuum. The effect of this is to reduce the saturation pressure and temperature of the steam resulting in a colder condensate that would otherwise be obtained. This in turn means more heat required to turn it back into steam in the boiler and a reduced thermal efficiency for the power plant.

The air must be removed from the condenser in order to keep the partial pressure as small as possible. This is done with an extractor pump. Some vapour will be removed with the air but this loss is tolerable because of the energy saved.

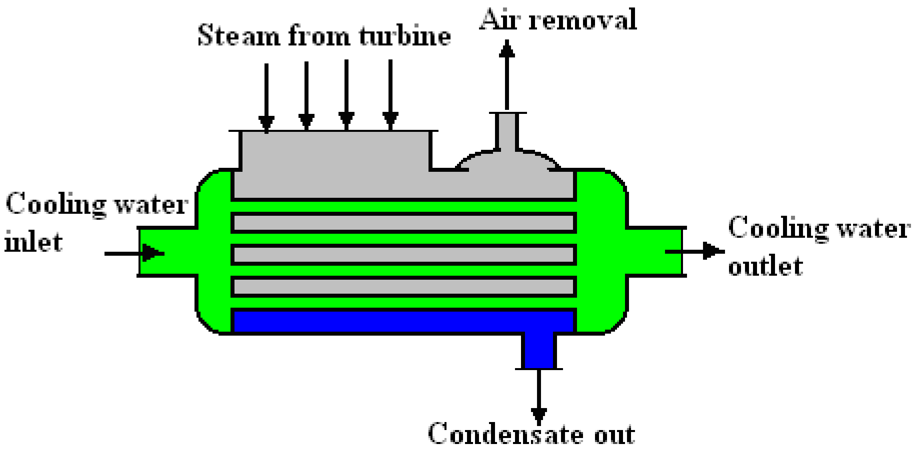

Figure 5

The solution to problems on condensers is similar to that for cooling towers and requires mass and energy balances. It is normal to neglect the partial pressure of the air at inlet as it makes little difference to the answers.

SELF ASSESSMENT EXERCISE No. 4

a. Discuss the reasons why air mixed with steam in a condenser is not desirable.

b. Wet steam with a dryness fraction of 0.9 enters a condenser at 0.035 bar pressure at a rate of 10 000 kg/h. The condensate leaves at 25°C. Air also enters with the steam at a rate of 40 kg/h. The air is extracted and cooled to 20°C. The partial pressure of the air at inlet is negligible and the process is at constant pressure. The cooling water is at 10°C at inlet and 21°C at outlet.

i. Determine the mass of vapour extracted with the air. (50 kg/h)

ii. Calculate the flow rate of the cooling water required. (475 484 kg/h)