

# APPLIED THERMODYNAMICS

## TUTORIAL 1

### REVISION OF ISENTROPIC EFFICIENCY

#### ADVANCED STEAM CYCLES

#### INTRODUCTION

This tutorial is designed for students wishing to extend their knowledge of thermodynamics to a more advanced level with practical applications.

- Before you start this tutorial you should be familiar with the following.
- The basic principles of thermodynamics equivalent to level 2.
- Basic steam cycles, mainly the Rankine and Carnot cycles.
- Fluid property tables and charts mainly a set of standard thermodynamic tables and a h - s chart for steam which you must have in your possession.
- The use of entropy.

On completion of the tutorial you should be able to

- understand isentropic efficiency for turbines and compressors.
- describe the use of process steam.
- describe the use of back pressure turbines.
- describe the use of pass out turbines.
- solve steam cycles involving pass out and back pressure turbines.
- describe the use of feed heating and superheating in steam cycles.
- solve problems involving feed heating and re-heating.

You may be very familiar with all these studies in which case you should proceed directly to section 2. For those who wish to revise the basics, section 1 should be completed. This covers

- entropy.
- isentropic processes.
- property diagrams.
- isentropic efficiency.

## **1. REVISION OF ENTROPY**

### **1.1 DEFINITION**

Entropy is a property which measures the usefulness of energy. It is defined most simply as

$$dS = dQ/dT \quad \text{where}$$

S is entropy

T is temperature

Q is heat transfer

The units of entropy is hence J/k. The units of specific entropy are J/kg K.

### **1.2. ISENTROPIC PROCESS**

ISENTROPIC means constant entropy. Usually (but not always) this means a process with no heat transfer. This follows since if dQ is zero so must be dS.

A process with no heat transfer is called ADIABATIC. An adiabatic process with no friction is hence also ISENTROPIC.

### **1.3. PROPERTY DIAGRAMS**

The two most commonly used property diagrams are

- i. Enthalpy - Entropy (h - s) diagrams and
- ii. Temperature - Entropy (T - s) diagrams.

h-s diagrams are commonly used for steam work. The diagram will hence show the saturation curve. You should familiarise yourself with the h-s diagram for steam and ensure that you can use it to find values of h and s for any pressure, temperature or dryness fraction.

T-s diagrams are commonly used for gas.

### **1.4. ISENTROPIC EFFICIENCY**

Real expansion and compression processes have a degree of friction and this will

- generate heat which is in effect a heat transfer.
- increase the entropy.
- make the final enthalpy bigger than it would otherwise be.
- make the final temperature bigger than it would otherwise be if it is a gas or superheated vapour.

An adiabatic process with friction has no external heat transfer ( $\Phi$  Watts or Q Joules) but the internal heat generated causes an increase in entropy. Consider the expansion and compression processes on fig.1 and 2.

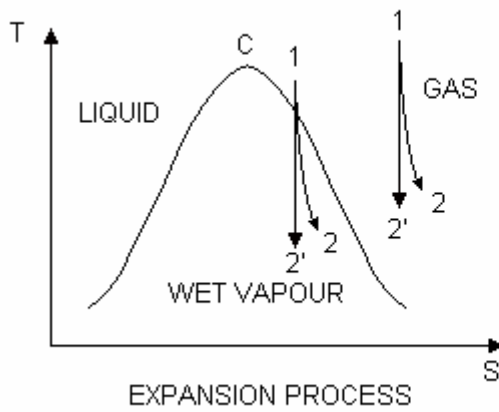


fig.1

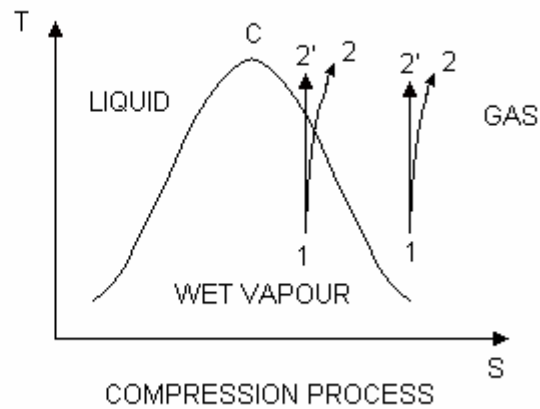


fig.2

The ideal change in enthalpy is

$$h_2' - h_1$$

The actual change is

$$h_2 - h_1$$

The isentropic efficiency is defined as

$$\eta_{is} = \frac{\Delta h \text{ (actual)}}{\Delta h \text{ (ideal)}} = \frac{h_2 - h_1}{h_2' - h_1}$$

for an expansion.

$$\eta_{is} = \frac{\Delta h \text{ (ideal)}}{\Delta h \text{ (actual)}} = \frac{h_2' - h_1}{h_2 - h_1}$$

for a compression.

In the case of a perfect gas  $h = c_p T$

hence

$$\eta_{is} = \frac{T_2 - T_1}{T_2' - T_1}$$

for an expansion

$$\eta_{is} = \frac{T_2' - T_1}{T_2 - T_1}$$

for a compression

Note that for an expansion this produces a negative number on the top and bottom lines that cancels out.

### **WORKED EXAMPLE No.1**

A steam turbine takes steam at 70 bar and 500°C and expands it to 0.1 bar with an isentropic efficiency 0.9. The process is adiabatic.

The power output of the turbine is 35 MW. Determine the enthalpy at exit and calculate the flow rate of steam in kg/s.

Note you need the tables and h-s chart for steam.

### **SOLUTION**

$$h_1 = 3410 \text{ kJ/kg (tables)} \quad s_1 = 6.796 \text{ kJ/kg K} \quad \text{for an ideal expansion } s_1 = s_2'$$

Assuming that the steam becomes wet during the expansion, then

$$s_2' = s_f + x' s_{fg} \text{ at 0.1 bar}$$

$$6.796 = 0.649 + x' 7.500 \text{ (tables)} \quad x' = 0.8196$$

Note if  $x'$  is larger than 1 then the steam is still superheated and the solution does not involve  $x$ .

Now find  $h_2'$ .

$$h_2' = h_f + x' h_{fg} \text{ at 0.1 bar}$$

$$h_2' = 192 + (0.8196)(2392) = 2152.2 \text{ kJ/kg.}$$

$$\text{Ideal change in enthalpy} \quad \Delta h' = 2152.2 - 3410 = -1257.5 \text{ kJ/kg}$$

$$\text{actual change in enthalpy} \quad \Delta h = 0.9(-1257.5) = -1131.7 \text{ kJ/kg}$$

$$\begin{aligned} \text{actual change in enthalpy} \quad \Delta h &= (h_2 - h_1) = -1131.7 \\ h_2 - 3410 &= -1131.7 \\ h_2 &= 2278.3 \text{ kJ/kg} \end{aligned}$$

From the steady flow energy equation (with which you should already be familiar) we have

$$\Phi + P = \Delta H/s$$

Since there is no heat transfer then this becomes

$$P = \Delta H/s = m (h_2 - h_1)$$

$$P = m(-1131.7) = -35\,000 \text{ kW}$$

hence

$$m = 30.926 \text{ kg/s}$$

(Note the sign convention used here is negative for energy leaving the system)

### WORKED EXAMPLE No.2

A gas turbine expands gas from 1 MPa pressure and 600°C to 100 kPa pressure. The isentropic efficiency 0.92. The mass flow rate is 12 kg/s. Calculate the exit temperature and the power output.

Take  $c_v = 718 \text{ J/kg K}$  and  $c_p = 1005 \text{ J/kg K}$

### SOLUTION

The process is adiabatic so the ideal temperature  $T_2'$  is given by

$$T_2 = T_1 (r_p)^{1-\gamma}$$

$r_p$  is the pressure ratio

$$r_p = p_2/p_1 = 0.1$$

$$\gamma = c_p/c_v = 1.005/0.718 = 1.4$$

$$T_2 = 873(0.1)^{1-1.4} = 451.9 \text{ K}$$

Now we use the isentropic efficiency to find the actual final temperature.

$$\eta_{is} = (T_2 - T_1)/(T_2' - T_1)$$

$$0.92 = (T_2 - 873)/(451.9 - 873)$$

$$T_2 = 485.6 \text{ K}$$

Now we use the SFEE to find the power output.

$$\Phi + P = m c_p(T_2 - T_1)$$

The process is adiabatic  $\Phi = 0$ .

$$P = 12(1.005)(485.6 - 873) = - 4672 \text{ kW (out of system)}$$

**SELF ASSESSMENT EXERCISE No.1**

1. Steam is expanded adiabatically in a turbine from 100 bar and 600°C to 0.09 bar with an isentropic efficiency of 0.88. The mass flow rate is 40 kg/s.

Calculate the enthalpy at exit and the power output.

(Ans. 51 MW)

2. A gas compressor compresses gas adiabatically from 1 bar and 15°C to 10 bar with an isentropic efficiency of 0.89. The gas flow rate is 5 kg/s.

Calculate the temperature after compression and the power input.

(Ans. -1.513 MW)

Take  $c_v = 718 \text{ J/kg K}$  and  $c_p = 1005 \text{ J/kg K}$

## 2. BACK-PRESSURE AND PASS-OUT TURBINES

It is assumed that the student is already familiar with steam cycles as this is necessary for this tutorial.

If an industry needs sufficient quantities of process steam (e.g. for sugar refining), then it becomes economical to use the steam generated to produce power as well. This is done with a steam turbine and generator and the process steam is obtained in two ways as follows.

- By exhausting the steam at the required pressure (typically 2 bar) to the process instead of to the condenser.

A turbine designed to do this is called a **BACK-PRESSURE TURBINE**.

- By bleeding steam from an intermediate stage in the expansion process.

A turbine designed to do this is called a **PASS-OUT TURBINE**.

The steam cycle is standard except for these modifications.

### 2.1. BACK-PRESSURE TURBINES

The diagram shows the basic circuit. The cycle could use reheat as well but this is not normal.

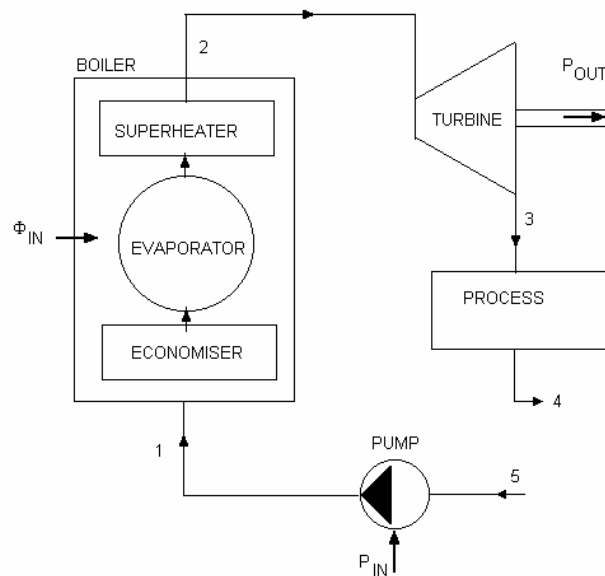


Figure 3

### **WORKED EXAMPLE No.3**

For a steam circuit as shown previously, the boiler produces superheated steam at 50 bar and 400°C. This is expanded to 3 bar with an isentropic efficiency of 0.9. The exhaust steam is used for a process.

The returning feed water is at 1 bar and 40°C. This is pumped to the boiler. The water leaving the pump is at 40°C and 50 bar. The net power output of the cycle is 60 MW. Calculate the mass flow rate of steam.

### **SOLUTION**

Referring to the cycle sketch previous for location points in the cycle we can find:

$$h_2 = 3196 \text{ kJ/kg} \quad s_2 = 6.646 \text{ kJ/kg K}$$

For an ideal expansion

$$s_1 = s_2 = 6.646 = s_f + x' s_{fg} \text{ at 3 bar}$$

$$6.646 = 1.672 + x'(5.321)$$

$$x' = 0.935$$

$$h_4 = h_f + x' h_{fg} \text{ at 3 bar}$$

$$h_4 = 561 + 0.935(2164)$$

$$h_4 = 2583.9 \text{ kJ/kg}$$

$$\text{ideal change in enthalpy} = 2583.9 - 3196 = -612 \text{ kJ/kg}$$

$$\text{actual change in enthalpy} = 0.9(-612) = -550.9 \text{ kJ/kg}$$

The power output of the turbine is found from the steady flow energy equation so :

$$P = m(-550.9) \text{ kW}$$

$$P = -550.9 m \text{ kW (output)}$$

Next we examine the enthalpy change at the pump.

$$h_1 = 168 \text{ kJ/kg at 1 bar and } 40^\circ\text{C}$$

$$h_2 = 172 \text{ kJ/kg at 50 bar and } 40^\circ\text{C.}$$

$$\text{Actual change in enthalpy} = 172 - 169 = 3 \text{ kJ/kg}$$

The power input to the pump is found from the steady flow energy equation so :

$$P = -m(3) \text{ kW}$$

$$P = -3 m \text{ kW(input)}$$

Net Power output of the cycle = 60 MW hence

$$60\,000 = 550.9 m - 3 m$$

$$m = 109.51 \text{ kg/s}$$



**SELF ASSESSMENT EXERCISE No.2**

A back pressure steam cycle works as follows. The boiler produces 8 kg/s of steam at 40 bar and 500°C. This is expanded to 2 bar with an isentropic efficiency of 0.88. The pump is supplied with feed water at 0.5 bar and 30°C and delivers it to the boiler at 31°C and 40 bar.

Calculate the net power output of the cycle. (Answer 5.24 MW)

## 2.2. PASS-OUT TURBINES

The circuit of a simple pass-out turbine plant is shown below. Steam is extracted between stages of the turbine for process use. The steam removed must be replaced by make up water at point 6.

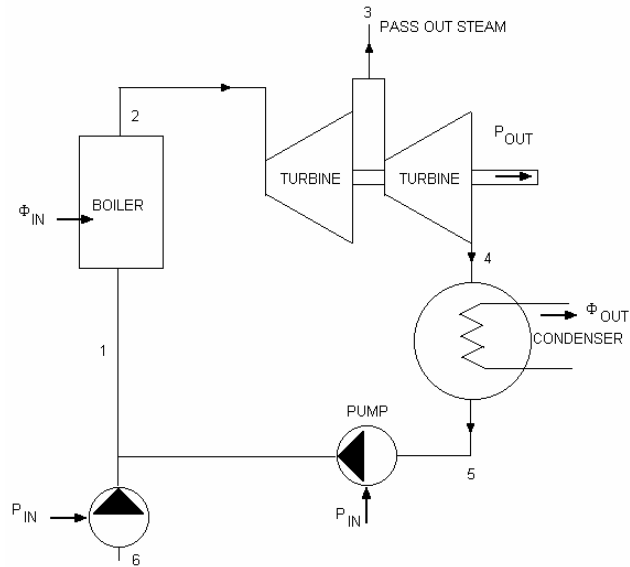


Figure 4

In order to solve problems you need to study the energy balance at the feed pumps more closely so that the enthalpy at inlet to the boiler can be determined. Consider the pumps on their own, as below.

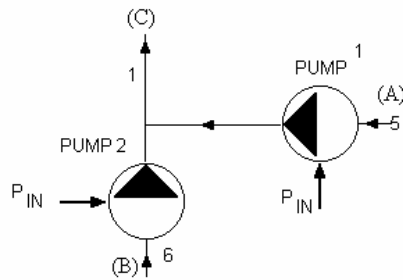


Figure 5

The balance of power is as follows.

$$P_1 + P_2 = \text{increase in enthalpy per second.} \\ = m_C h_C - m_A h_A - m_B h_B$$

From this the value of  $h_C$  or the mass  $m_C$  may be determined. This is best shown with a worked example.

### WORKED EXAMPLE No.4

The circuit below shows the information normally available for a feed pump circuit. Determine the enthalpy at entry to the boiler.

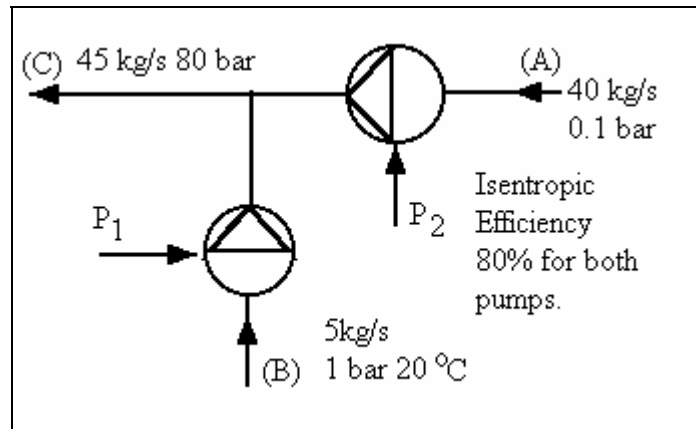


Figure 6

### SOLUTION

$$P_1(\text{ideal}) = (5)(0.001)(80-1)(100) = 39.5 \text{ kW}$$

$$P_1(\text{actual}) = 39.5/0.8 = 49.375 \text{ kW}$$

$$P_2(\text{ideal}) = (40)(0.001)(80 - 0.1)(100) = 319.6 \text{ kW}$$

$$P_2(\text{actual}) = 319.6/0.8 = 399.5 \text{ kW}$$

$$\text{Total power input} = 49.375 + 399.5 = 448.9 \text{ kW}$$

$$h_A = h_f = 192 \text{ kJ/kg at 0.1 bar}$$

$$h_B = 84 \text{ kJ/kg (from water tables or approximately } h_f \text{ at } 20^\circ\text{C) hence}$$

$$448.9 = 45 h_C - 40 h_A - 5h_B$$

$$448.9 = 45 h_C - 40(192) - 5(84)$$

$$h_C = 190 \text{ kJ/kg}$$

### WORKED EXAMPLE No.5

The following worked example will show you to solve these problems.

A passout turbine plant works as shown in fig. 4. The boiler produces steam at 60 bar and 500°C which is expanded through two stages of turbines. The first stage expands to 3 bar where 4 kg/s of steam is removed. The second stage expands to 0.09 bar. The isentropic efficiency is 0.9 for the overall expansion. Assume that the expansion is a straight line on the h - s chart.

The condenser produces saturated water. The make up water is supplied at 1 bar and 20°C. The isentropic efficiency of the pumps is 0.8. The net power output of the cycle is 40 MW. Calculate:

1. the flow rate of steam from the boiler.
2. the heat input to the boiler.
3. the thermal efficiency of the cycle.

### SOLUTION

#### TURBINE EXPANSION

$h_3 = 3421$  kJ/kg from tables.

$h_{5'} = 2165$  kJ/kg using isentropic expansion and entropy.

$0.9 = (3421 - h_5) / (3421 - 2165)$  hence  $h_5 = 2291$  kJ/kg

Sketching the process on the h - s chart as a straight line enables  $h_4$  to be picked off at 3 bar.  $h_4 = 2770$  kJ/kg.

#### POWER OUTPUT

$$P_{\text{out}} = m(h_3 - h_4) + (m - 4)(h_4 - h_5)$$

$$P_{\text{out}} = m(3421 - 2770) + (m - 4)(2770 - 2291)$$

$$P_{\text{out}} = 651 m + 479 m - 1916$$

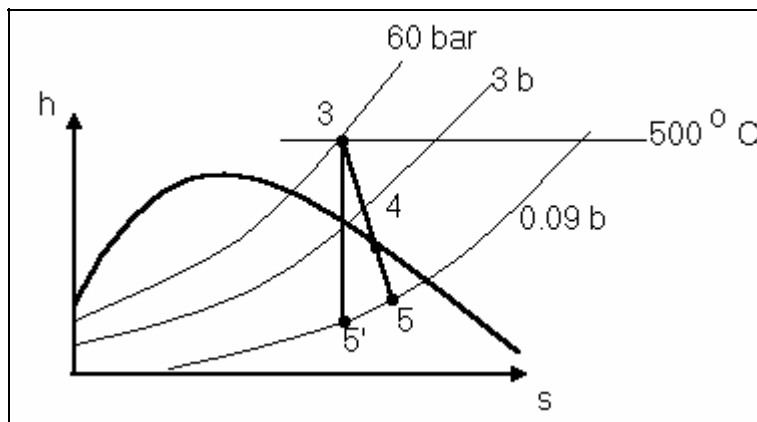


Figure 7

## POWER INPUT

The power input is to the two feed pumps.

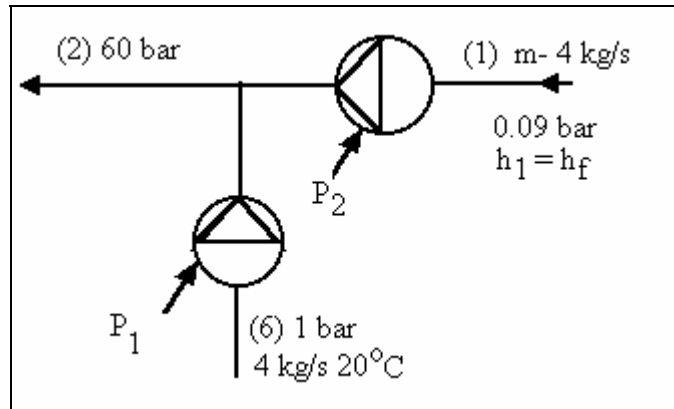


Figure 8

$$h_6 = 84 \text{ kJ/kg (water at 1 bar and } 20^\circ\text{C)}$$
$$h_1 = h_f \text{ at } 0.09 \text{ bar} = 183 \text{ kJ/kg.}$$

$$P_1 \text{ (ideal)} = \text{change in flow energy} = 4 \times 0.001 \times (60 - 1) \times 100 \text{ kW} = 23.6 \text{ kW}$$

$$P_1 \text{ (actual)} = 23.6 / 0.8 = 29.5 \text{ kW}$$

$$P_2 \text{ (actual)} = (m-4) \times 0.001 \times (60 - 0.09) \times 100 / 0.8 = 7.49m - 29.96 \text{ kW}$$

## NET POWER

$$40\,000 \text{ kW} = P_{\text{out}} - P_1 - P_2$$
$$40\,000 = 651m + 479m - 1916 - 29.5 - 7.49m + 29.96$$
$$40000 = 1122.5m - 1916 \text{ hence } m = 37.34 \text{ kg/s}$$

## ENERGY BALANCE ON PUMPS

$$P_1 = 29.5 \text{ kW}$$

$$P_2 = 249.4 \text{ kW (using the value of } m \text{ just found)}$$

$$m h_2 = (m-4) h_1 + P_1 + P_2$$

$$37.3 h_2 = 33.34 \times 183 + 29.5 + 249.7$$

hence

$$h_2 = 171 \text{ kJ/kg}$$

## HEAT INPUT

$$\text{Heat input} = m(h_3 - h_2) = 121355 \text{ kW}$$

## EFFICIENCY

$$\text{Efficiency} = \eta = 40 / 121.3 = 33 \%$$

### **SELF ASSESSMENT EXERCISE No.3**

1. A steam turbine plant is used to supply process steam and power. The plant comprises an economiser, boiler, superheater, turbine, condenser and feed pump. The process steam is extracted between intermediate stages in the turbine at 2 bar pressure. The steam temperature and pressure at outlet from the superheater are 500°C and 70 bar, and at outlet from the turbine the pressure is 0.1 bar. The overall isentropic efficiency of the turbine is 0.87 and that of the feed pump is 0.8.

Assume that the expansion is represented by a straight line on the h-s chart. The make-up water is at 15°C and 1 bar and it is pumped into the feed line with an isentropic efficiency 0.8 to replace the lost process steam.

If due allowance is made for the feed pump-work, the net mechanical power delivered by the plant is 30 MW when the process steam load is 5 kg/s. Calculate the rate of steam flow leaving the superheater and the rate of heat transfer to the boiler including the economiser and superheater. Sketch clear T- s and h-s and flow diagrams for the plant. (29.46 kg/s 95.1 MW)

2. The demand for energy from an industrial plant is a steady load of 60 MW of process heat at 117°C and a variable demand of up to 30 MW of power to drive electrical generators. The steam is raised in boilers at 70 bar pressure and superheated to 500°C. The steam is expanded in a turbine and then condensed at 0.05 bar. The process heat is provided by the steam bled from the turbine at an appropriate pressure, and the steam condensed in the process heat exchanger is returned to the feed water line.

Calculate the amount of steam that has to be raised in the boiler. Assume an overall isentropic efficiency of 0.88 in the turbine. The expansion is represented by a straight line on the h-s diagram. Neglect the feed pump work.

(Answer 36 kg/s).

### 3. ADVANCED STEAM CYCLES

In this section you will extend your knowledge of steam cycles in order to show that the overall efficiency of the cycle may be optimised by the use of regenerative feed heating and steam re-heating.

Regenerative feed heating is a way of raising the temperature of the feed water before it reaches the boiler. It does this by using internal heat transfer within the power cycle. Steam is bled from the turbines at several points and used to heat the feed water in special heaters.

In this way the temperature of the feed water is raised along with the pressure in stages so that the feed water is nearly always saturated. The heat transfers in the heaters and in the boiler are conducted approximately isothermally.

Studies of the Carnot cycle should have taught you that an isothermal heat transfer is reversible and achieves maximum efficiency.

The ultimate way of conducting feed heating is to pass the feed water through a heat exchanger inside the turbine casing. In this way the temperature of the steam on one side of heat exchanger tubes is equal to the temperature of the water on the other side of the tubes. Although the temperature is changing as water and steam flow through heat exchanger, at any one point, the heat transfer is isothermal. If no superheating nor undercooling is used then the heat transfers in the boiler and condenser are also isothermal and efficiencies equal to those of the Carnot cycle are theoretically possible.

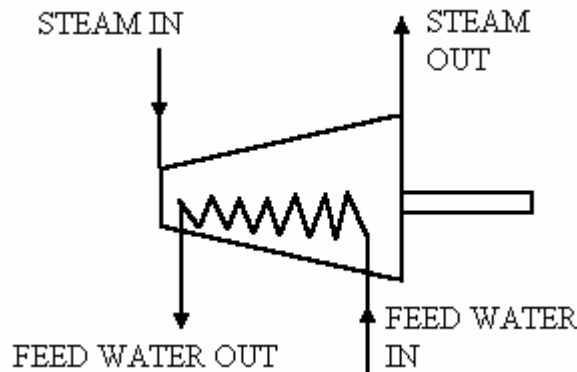


Figure 9

There are several reasons why this arrangement is impractical. Most of them are the same reasons why a Carnot cycle is impractical.

- i. The steam would be excessively wet in the turbine.
- ii. Placing a heat exchanger inside the turbine casing is mechanically impossible.
- iii. The power output would be small even though the cycle efficiency would be high.

Steam reheating is another way of improving the thermodynamic efficiency by attempting to keep the steam temperature more constant during the heat transfer process inside the boiler.

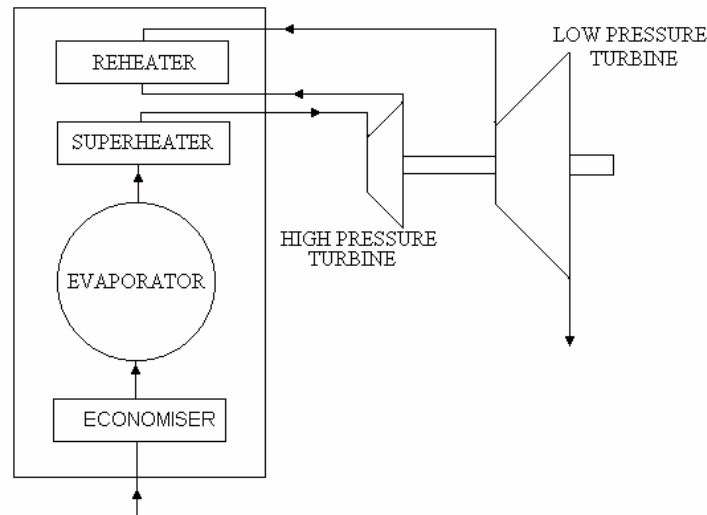


Figure 10

Superheated steam is first passed through a high pressure turbine. The exhaust steam is then returned to the boiler to be reheated almost back to its original temperature. The steam is then expanded in a low pressure turbine. In theory, many stages of turbines and reheating could be done thus making the heat transfer in the boiler more isothermal and hence more reversible and efficient.

If a steam cycle used many stages of regenerative feed heating and many stages of reheating, the result would be an efficiency similar to that of the Carnot cycle. Although practicalities prevent this happening, it is quite normal for an industrial steam power plant to use several stages of regenerative feed heating and one or two stages of reheating. This produces a significant improvement in the cycle efficiency.

There are other features in advanced steam cycles which further improve the efficiency and are necessary for practical operation. For example air extraction at the condenser, steam recovery from turbine glands, de-superheaters, de-aerators and so on. These can be found in details in textbooks devoted to practical steam power plant.



## 4. FEED HEATING

### 4.1. PRACTICAL DESIGNS

Practical feed heaters may be heat exchangers with indirect contact. The steam is condensed through giving up its energy and the hot water resulting may be inserted into the feed system at the appropriate pressure. The type which you should learn is the open or direct contact mixing type. The bled steam is mixed directly with the feed water at the appropriate pressure and condenses and mixes with the feed water. Compare a basic Rankine cycle with a similar cycle using one such feed heater.

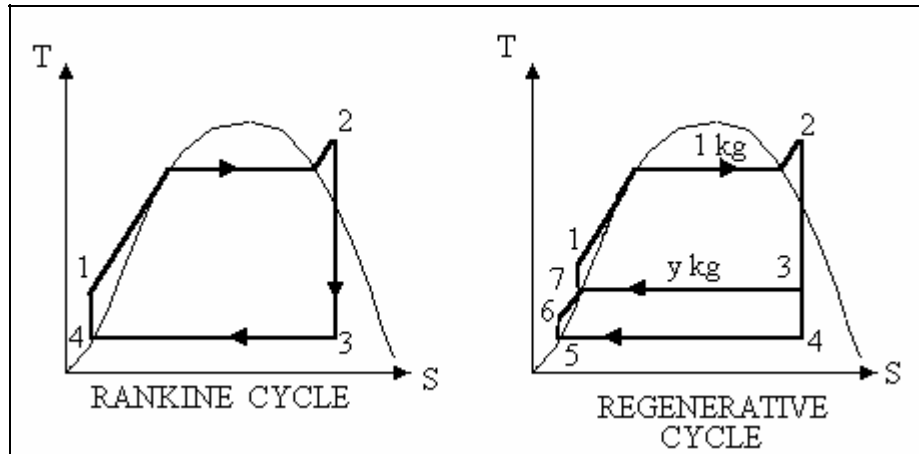


Figure 11

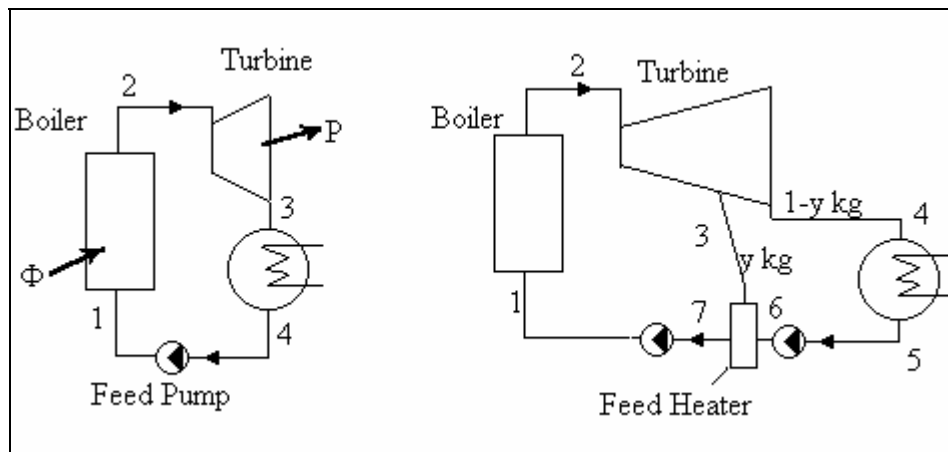
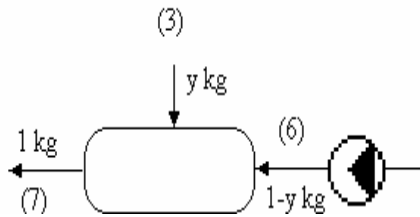


Figure 12

#### 4. 2. ENERGY BALANCE FOR MIXING FEED HEATER

Consider a simple mixing type feed heater. The bled steam at (3) is mixed directly with incoming feed water (6) resulting in hotter feed water (7).



Mass of bled steam =  $y$  kg  
 Mass of feed water entering =  $1 - y$  kg  
 Doing an energy balance we find

$$y h_3 + (1-y)h_6 = h_7$$

Figure 13

#### WORKED EXAMPLE No.6

A feed heater is supplied with condensate at 0.1 bar. The bled steam is taken from the turbine at 30 bar and 0.95 dry. Calculate the flow rate of bled steam needed to just produce saturated water at outlet.

#### SOLUTION

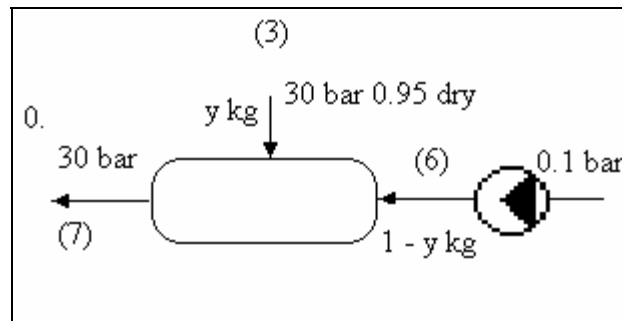


Figure 14

Assumptions

1. Energy input from pump is negligible.
2. No energy is lost.
3. The heater pressure is the same as the bled pressure.

In this case  $h_6 = h_f$  at 0.1 bar = 192 kJ/kg

$$h_7 = h_f \text{ at } 30 \text{ bar} = 1008 \text{ kJ/kg}$$

$$h_3 = h_f + x h_{fg} \text{ at } 30 \text{ bar}$$

$$h_3 = 11008 + 0.95(1795) = 2713.3 \text{ kJ/kg}$$

ENERGY BALANCE

$$y(2713.3) = (1-y)(192) + 1008$$

hence  $y = 0.414 \text{ kg}$

Note that it is usual to calculate these problems initially on the basis of 1 kg coming from the boiler and returning to it.

### 4. 3. CYCLE WITH ONE FEED HEATER

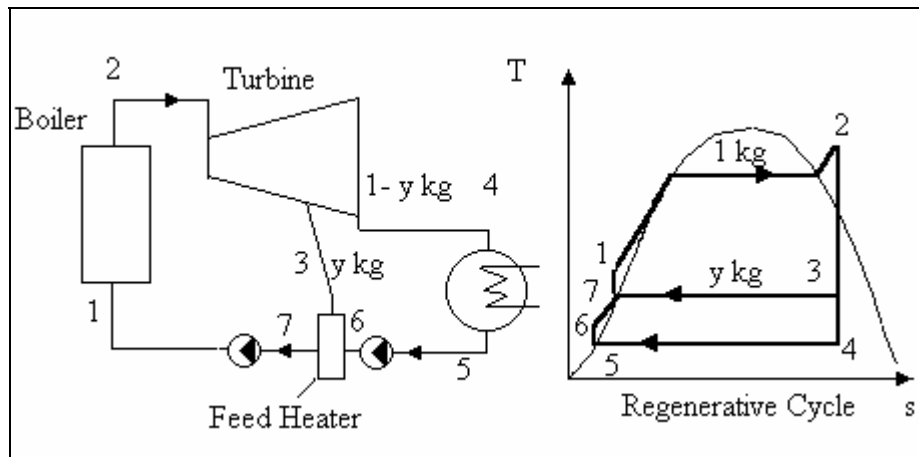


Figure 15

If only one feed heater is used, the steam is bled from the turbine at the point in the expansion where it just becomes dry saturated and the saturation temperature is estimated as follows.

$$t_s(\text{bleed}) = \{t_s(\text{high pressure}) + t_s(\text{low pressure})\}/2$$

For example a cycle operating between 40 bar and 0.035 bar.

$$t_s(40 \text{ bar}) = 250.3 \text{ }^\circ\text{C}$$

$$t_s(0.035 \text{ bar}) = 26.7 \text{ }^\circ\text{C}$$

$$t_s(\text{bleed}) = (250.3 + 26.7)/2 = 138.5^\circ\text{C}$$

The pressure corresponding to this is 3.5 bar so this is the bleed pressure.

### WORKED EXAMPLE No.7

A Rankine cycle works between 40 bar, 400°C at the boiler exit and 0.035 bar at the condenser. Calculate the efficiency with no feed heating. Assume isentropic expansion. Ignore the energy term at the feed pump.

### SOLUTION

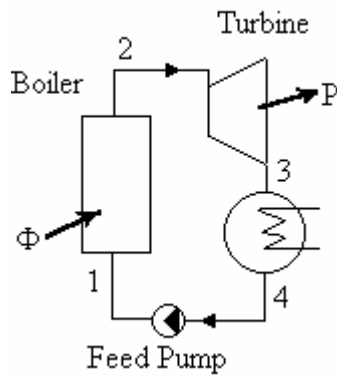


Figure 16

$$h_2 = 3214 \text{ kJ/kg} \quad s_2 = 6.769 \text{ kJ/kg K}$$

$$s_2 = s_3 = 0.391 + 8.13 x$$

$$x = 0.785$$

$$h_3 = h_f + x h_{fg} = 112 + 0.785(2438) = 2024.6 \text{ kJ/kg}$$

$$h_4 = h_f \text{ at } 0.035 \text{ bar} = 112 \text{ kJ/kg}$$

$$\Phi = h_2 - h_1 = 3102 \text{ kJ/kg into boiler.}$$

$$P = h_2 - h_3 = 1189.4 \text{ kJ/kg (out of turbine)}$$

$$\eta = P / \Phi = 38.3 \%$$

### WORKED EXAMPLE No.8

Repeat the last example but this time there is one feed heater.

### SOLUTION

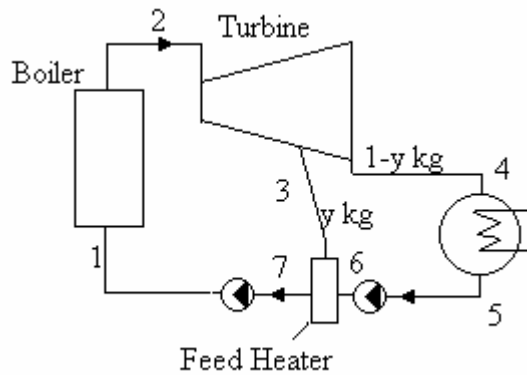


Figure 17

The bleed pressure was calculated in an earlier example and was 3.5 bar.

$$s_2 = s_3 = 6.769 \text{ kJ/kg K} = 1.727 + 5.214x_3$$

$$x_3 = 0.967 \text{ (not quite dry).}$$

$$h_3 = h_f + x h_{fg} = 584 + 0.967(2148)$$

$$h_3 = 2661 \text{ kJ/kg}$$

$$h_7 = h_f \text{ at 3.5 bar} = 584 \text{ kJ/kg}$$

Neglecting pump power

$$h_6 = h_5 = h_f = 112 \text{ kJ/kg}$$

$$h_1 = h_7 = 584 \text{ kJ/kg}$$

Conducting an energy balance we have

$$yh_3 + (1-y)h_6 = h_7 \quad \text{hence } y = 0.185 \text{ kg}$$

$$\Phi = h_2 - h_1 = 2630 \text{ kJ/kg into boiler.}$$

Rather than work out the power from the turbine data, we may do it by calculating the heat transfer rate from the condenser as follows.

$$\Phi_{\text{out}} = (1-y)(h_4 - h_5) = 0.815(2024.6 - 112) = 1558.8 \text{ kJ/kg}$$

$$P = \Phi_{\text{in}} - \Phi_{\text{out}} = 1072 \text{ kJ/kg (out of turbine)}$$

$$\eta = P / \Phi_{\text{in}} = 40.8 \%$$

Note that the use of the feed heater produced an improvement of 2.5 % in the thermodynamic efficiency.

#### **SELF ASSESSMENT EXERCISE No.4**

A simple steam plant uses a Rankine cycle with one regenerative feed heater. The boiler produces steam at 70 bar and 500°C. This is expanded to 0.1 bar isentropically. Making suitable assumptions, calculate the cycle efficiency. (41.8%)

#### **4. 4. CYCLE WITH TWO FEED HEATERS**

When two (or more) feed heaters are used, the efficiency is further increased. the principles are the same as those already explained. The mass of bled steam for each heater must be determined in turn starting with the high pressure heater. It is usual to assume isentropic expansion that enables you to pick off the enthalpy of the bled steam from the h-s chart at the pressures stated.

### WORKED EXAMPLE No.9

A steam power plant works as follows. The boiler produces steam at 100 bar and 600°C. This is expanded isentropically to 0.04 bar and condensed. Steam is bled at 40 bar for the h.p. heater and 4 bar for the l.p. heater. Solve the thermodynamic efficiency.

### SOLUTION

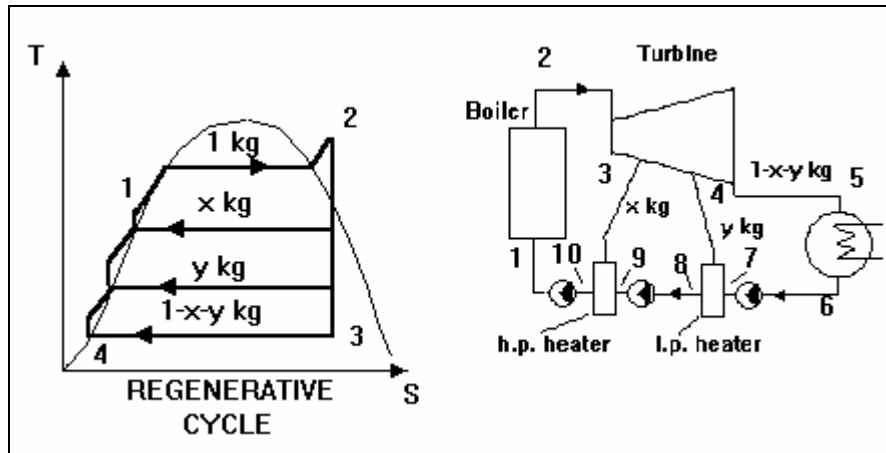


Figure 18

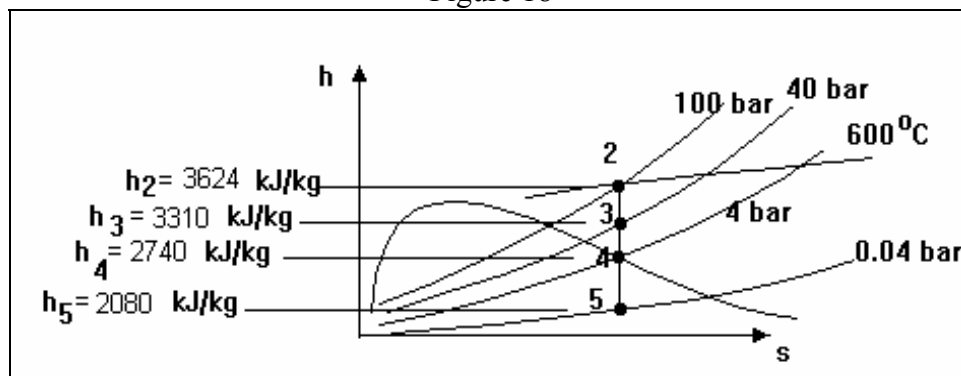


Figure 19

Ignoring the energy input from the pump we find:

$$h_1 = h_{10} = h_f \text{ 40 bar} = 1087 \text{ kJ/kg}$$

$$h_9 = h_8 = h_f \text{ 4 bar} = 605 \text{ kJ/kg}$$

$$h_7 = h_6 = h_f \text{ 0.04 bar} = 121 \text{ kJ/kg}$$

#### H.P. HEATER

$$xh_3 + (1-x)h_9 = h_{10}$$

$$3310x + 605(1-x) = 1087$$

$$\text{hence } x = 0.178 \text{ kg}$$

L.P. HEATER

$$(1-x)h_8 = yh_4 + (1-x-y)h_7$$
$$0.822(605) = 2740y + (0.822-y)(121)$$
$$y = 0.152 \text{ kg}$$

BOILER

heat input  $\Phi_{in} = h_2 - h_1 = 3624 - 1087 = 2537 \text{ kJ/kg}$

CONDENSER

heat output  $\Phi_{out} = (1-x-y)(h_5 - h_6)$

$$\Phi_{out} = 0.67(2080-121) = 1312.5 \text{ kJ/kg}$$

POWER OUTPUT

$$P = \Phi_{in} - \Phi_{out} = 1224.5 \text{ kJ/kg}$$

$$\eta = P / \Phi_{in} = 48.3 \%$$



### **SELF ASSESSMENT EXERCISE No. 5**

1. Explain how it is theoretically possible to arrange a regenerative steam cycle which has a cycle efficiency equal to that of a Carnot cycle.

In a regenerative steam cycle steam is supplied from the boiler plant at a pressure of 60 bar and a temperature of 500°C. Steam is extracted for feed heating purposes at pressures of 30 bar and 3.0 bar and the steam turbine exhausts into a condenser operating at 0.035 bar.

Calculate the appropriate quantities of steam to be bled if the feed heaters are of the open type, and find the cycle efficiency; base all calculations on unit mass leaving the boiler.

Assume isentropic expansion in the turbine and neglect the feed pump work.

(Answers 0.169 kg/s, 0.145 kg/s and 45 %)

2. The sketch shows an idealised regenerative steam cycle in which heat transfer to the feed water in the turbine from the steam is reversible and the feed pump is adiabatic and reversible. The feed water enters the pump as a saturated liquid at 0.03 bar, and enters the boiler as a saturated liquid at 100 bar, and leaves as saturated steam.

Draw a T-s diagram for the cycle and determine, not necessarily in this order, the dryness fraction in state 2, the cycle efficiency and the work per unit mass.

(Answers 0.269 kg/s, 50% and 658.5 kJ/kg).

Outline the practical difficulties that are involved in realising this cycle and explain how regenerative cycles are arranged in practice.

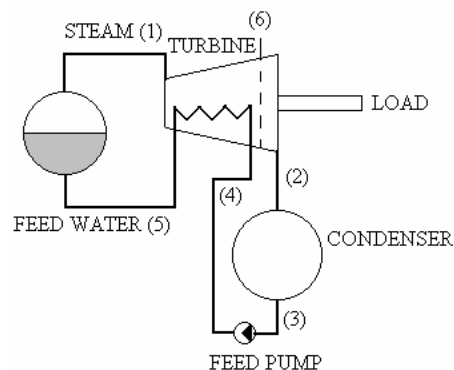


Figure 20

Note point (6) is the point in the steam expansion where the feed water enters and presumably the temperatures are equal. There is further expansion from (6) to (2).

## 5. REHEAT CYCLES

We shall only examine cycles with one stage of reheating and two turbine stages, high pressure and low pressure. You should refer to text books on practical steam turbine layouts to see how low, medium and high pressure turbines are configured and laid out in order to produce axial force balance on the rotors. The diagram below shows a basic circuit with one stage of reheating.

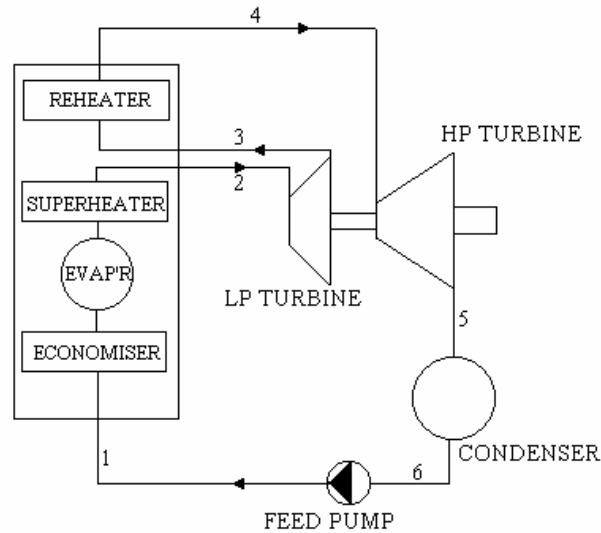


Figure 21

You should be proficient at sketching the cycle on a  $T - s$  diagram and a  $h - s$  diagram. They are shown below for the cycle shown above.

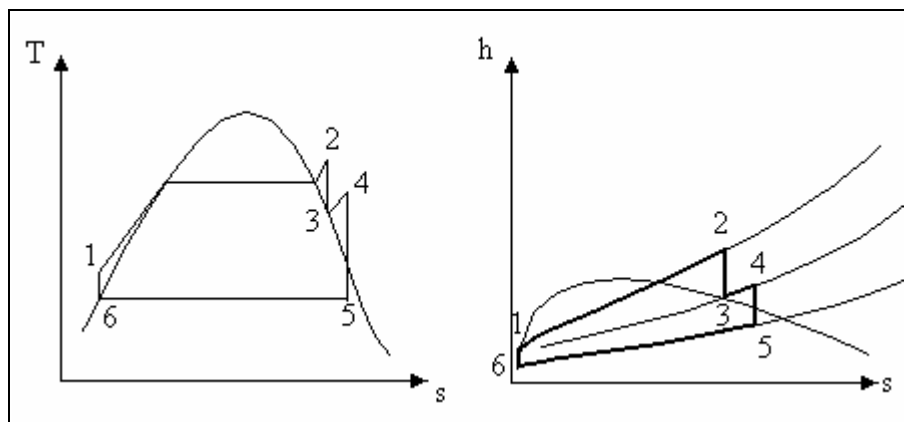


Figure 22

The calculations for this cycle are not difficult. You need only take into account the extra heat transfer in the reheater.

### **WORKED EXAMPLE No.10**

A reheat cycle works as follows. The boiler produces 30 kg/s at 100 bar and 400°C. This is expanded isentropically to 50 bar in the h.p. turbine and returned for reheating in the boiler. The steam is reheated to 400°C. This is then expanded in the l.p. turbine to the condenser which operates at 0.2 bar. The condensate is returned to the boiler as feed.

Calculate the net power output and the cycle efficiency.

### **SOLUTION**

$$h_6 = h_f \text{ at } 0.2 \text{ bar} = 251 \text{ kJ/kg}$$
$$h_2 = 3097 \text{ kJ/kg at } 100 \text{ bar and } 400^\circ\text{C.}$$

From the h-s chart we find

$$h_3 = 2930 \text{ kJ/kg} \quad h_4 = 3196 \text{ kJ/kg} \quad h_5 = 2189 \text{ kJ/kg}$$

If we ignore the feed pump power then

$$\Phi_{\text{in at boiler}} = 30(h_2 - h_1) + 30(h_4 - h_3) = 93\,360 \text{ kW or } 93.360 \text{ MW}$$

$$\Phi_{\text{out at condenser}} = 30(h_5 - h_6) = 58.14 \text{ kW}$$

$$P_{(\text{net})} = \Phi_{\text{in}} - \Phi_{\text{out}} = 35220 \text{ kW or } 35.22 \text{ MW}$$

$$\eta = P_{(\text{net})} / \Phi_{\text{in}} = 37.7 \%$$

### **SELF ASSESSMENT EXERCISE No. 6**

1. Repeat worked example No.10 but this time do not ignore the feed pump term and assume an isentropic efficiency of 90% for each turbine and 80% for the pump.

( Answers 32.1 MW ,35%)

2. A water-cooled nuclear reactor supplies dry saturated steam at a pressure of 50 bar to a two-cylinder steam turbine. In the first cylinder the steam expands with an isentropic efficiency of 0.85 to a pressure of 10 bar, the power generated in this cylinder being 100 MW. The steam then passes at a constant pressure of 10 bar through a water separator from which all the water is returned to the reactor by mixing it with the feed water. The remaining dry saturated steam then flows at constant pressure through a reheater in which its temperature is raised to 250°C before it expands in the second cylinder with an isentropic efficiency of 0.85 to a pressure of 0.1 bar, at which it is condensed before being returned to the reactor.

Calculate the cycle efficiency and draw up an energy balance for the plant. Neglect the feed pump work. (Answer 30.3%)

3. Steam is raised in a power cycle at the supercritical pressure of 350 bar and at a temperature of 600°C. It is then expanded in a turbine to 15 bar with an overall isentropic efficiency of 0.90. At that pressure some steam is bled to an open regenerative feed heater, and the remainder of the steam is, after reheating to 600°C, expanded in a second turbine to the condenser pressure of 0.04 bar, again with an isentropic efficiency of 0.90. The feed pumps each have an overall isentropic efficiency of 0.90.

Calculate the amount of steam to be bled into the feed heater, making the usual idealising assumptions. Also calculate the cycle efficiency. Use the h-s chart wherever possible and do not neglect feed pump work.

(Answers 0.279 kg/s and 47%)