THERMODYNAMICS
TUTORIAL 12
GAS TURBINE POWER PLANT AND CYCLES

This tutorial is set at QCF levels 4 to 6

On completion of this tutorial you should be able to analyse the thermodynamic performance of gas turbine power cycles and explain the practical aspect of the plant.

Contents

1. Introduction

Gas turbines are used for many purposes from aeroplane engines to industrial electric power generating sets. They are at the heart of most jet engines but also for propeller driven engines.

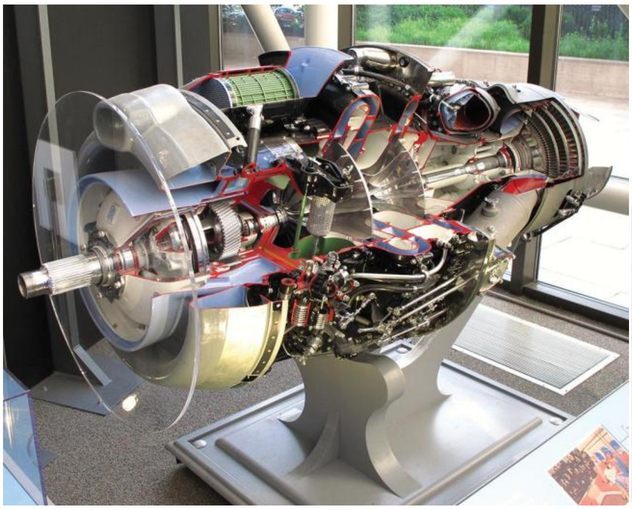


Figure 1 Gas Turbine Prop Engine

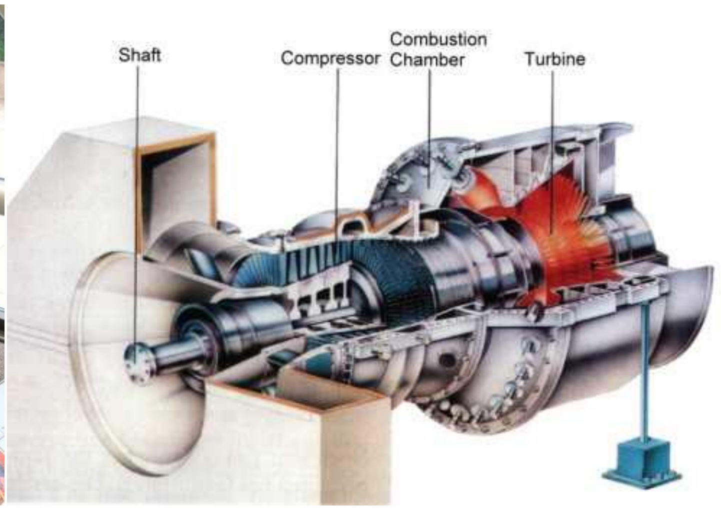


Figure 2 Industrial Gas Turbine Set

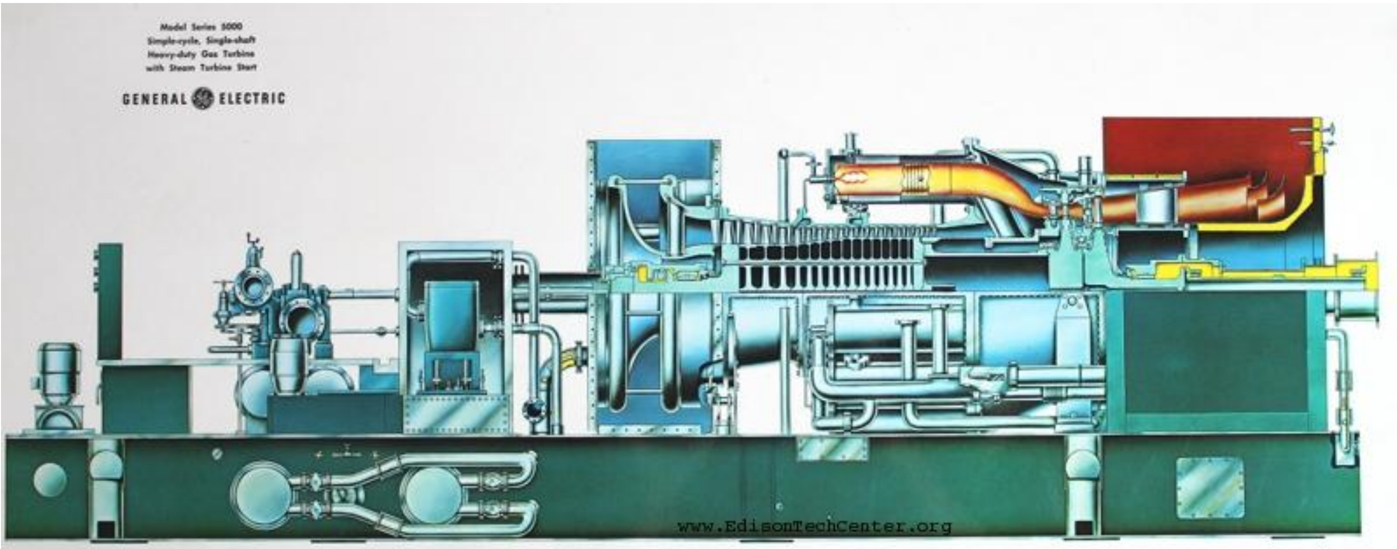


Figure 3 Large Industrial Electric Generator Set

2. *Revision of Expansion and Compression Processes With Friction.*

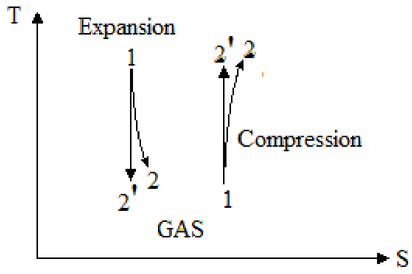
When a gas is expanded from pressure p_1 to pressure p_2 adiabatically, the temperature ratio is given by the formula

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$$

The same formula may be applied to a compression process. Always remember that when a gas is expanded it gets colder and when it is compressed it gets hotter. The temperature change is $\Delta T = T_2 - T_1$

If there is friction the isentropic efficiency (η_{is}) is expressed as

$$\eta_{is} = \frac{\Delta T(\text{ideal})}{\Delta T(\text{actual})} \text{ (compression)} \qquad \eta_{is} = \frac{\Delta T(\text{actual})}{\Delta T(\text{ideal})} \text{ (expansion)}$$



On a T – S diagram they are as shown on figure 4.

Figure 4

Further studies of friction will show an alternative way of expressing this is with ***Polytropic Efficiency*** $\boldsymbol{\eta_{is}}$

For a process from (1) to (2) the temperature ratio is expressed as follows.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma\eta_\infty}} \text{ (compression)} \qquad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_\infty}{\gamma}} \text{ (expansion)}$$

η_∞ is called the polytropic efficiency. We will not use this here.

WORKED EXAMPLE No. 1

A gas turbine expands 4 kg/s of air from 12 bar and 900°C to 1 bar adiabatically with an isentropic efficiency of 87%. Calculate the exhaust temperature and the power output. $\gamma = 1.4$ $c_p = 1005$ J/kg K

SOLUTION

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = 1\,173\left(\frac{1}{12}\right)^{1-\frac{1}{1.4}} = 1\,173\left(\frac{1}{12}\right)^{0.2958} = 562.48 \text{ K}$$

Ideal temperature change = 1 173 - 562.48 = 610.52 K

Actual temperature change = 87% × 610.52 = 531.15 K

Exhaust temperature = 1173 - 531.15 = 641.85 K

The steady flow energy equation states

$$\Phi + P = \text{change in enthalpy/s}$$

Since it is an adiabatic process $\Phi = 0$ so

$$P = \Delta H/s = m\, c_p\, \Delta T = 4 \times 1\,005 \times (-531.15) = -2.135 \times 10^6 \text{ W or } 2.135 \text{ MW}$$

WORKED EXAMPLE No. 2

A gas turbine expands gas from 1 MPa pressure and 600^{o}C to 100 kPa pressure. The isentropic efficiency is 0.92. The mass flow rate is 12 kg/s. Calculate the exit temperature and the power output.

Take c_v = 718 J/kg K and c_p = 1 005 J/kg K

SOLUTION

$$\gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.4$$

The process is adiabatic so the ideal temperature $T_{2'}$ is given by:

$$T_2{}' = T_1(r_p)^{1-\frac{1}{\gamma}} \quad r_p = \frac{p_2}{p_1} = 0.1 \text{ (the pressure ratio)} \quad T_2{}' = 873(0.1)^{1-\frac{1}{1.4}} = 451.9 \text{ K}$$

Now we use the isentropic efficiency to find the actual final temperature.

$$\eta_{is} = 0.92 = \frac{T_2 - T_1}{T_2{}' - T_1} = \frac{T_2 - 873}{451.9 - 873}$$

$$T_2 = 485.6 \text{ K}$$

Now we use the S. F. E. E. to find the power output.

$$\Phi + P = \dot{m}\, c_p(T_2 - T_1) \quad \text{The process is adiabatic } \Phi = 0.$$

$$P = 12(1.005)(485.6 - 873) = -4\,672 \text{ kW (out of system)}$$

SELF ASSESSMENT EXERCISE No. 1

1. A gas turbine expands 6 kg/s of air from 8 bar and 700^{o}C to 1 bar isentropically.
Calculate the exhaust temperature and the power output. γ = 1.4 c_p = 1005 J/kg K
(Answers 537.1 K and 2.628 MW)

2. A gas turbine expands 3 kg/s of air from 10 bar and 920^{o}C to 1 bar adiabatically with an isentropic efficiency of 92%.
Calculate the exhaust temperature and the power output. γ = 1.41 c_p = 1010 J/kg K
(Answers 657.3 K and 1.62 MW)

3. A gas turbine expands 7 kg/s of air from 9 bar and 850^{o}C to 1 bar adiabatically with an isentropic efficiency of 87%.
Calculate the exhaust temperature and the power output.
γ = 1.4 c_p = 1 005 J/kg K
(Answers 667.5 K and 3.204 MW)

3. *The Basic Gas Turbine Cycle*

The ideal and basic cycle is called the ***Brayton*** Cycle and the ***Joule*** cycle. It is also known as the constant pressure cycle because the heating and cooling processes are conducted at constant pressure. A simple layout is shown on figure 5.

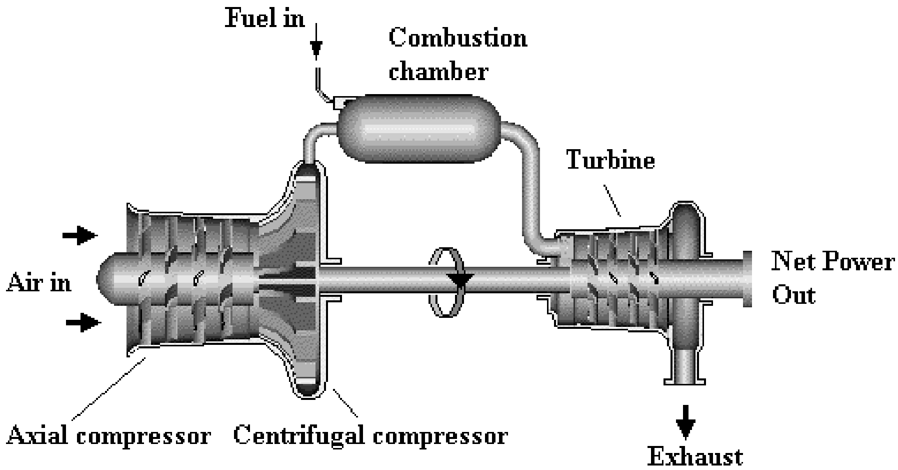


Figure 5 Illustrative diagram.

The cycle in block diagram form is shown on figure 6.

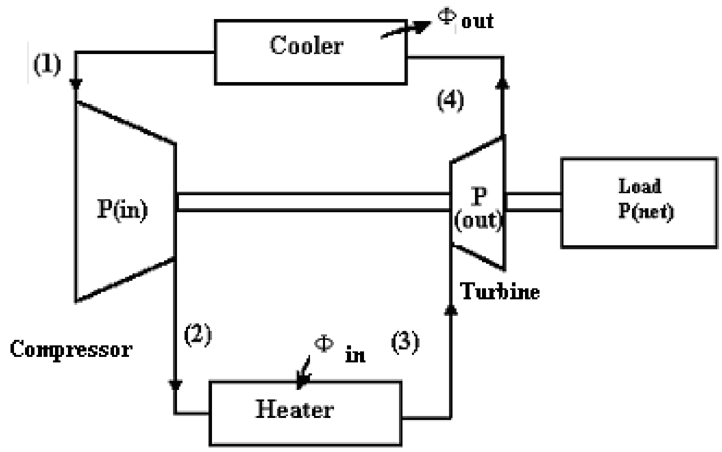


Figure **6 Block diagram**

There are 4 ideal processes in the cycle.

1 - 2 Reversible adiabatic (isentropic) compression requiring power input.

$$P_{in} = \Delta H/s = m\ c_p\ (T_2 - T_1)$$

2 - 3 Constant pressure heating requiring heat input.

$$\Phi_{in} = \Delta H/s = m\ c_p\ (T_3 - T_2)$$

3 - 4 Reversible adiabatic (isentropic) expansion producing power output.

$$P_{out} = \Delta H/s = m\ c_p\ (T_3 - T_4)$$

4 - 1 Constant pressure cooling back to the original state requiring heat removal.

$$\Phi_{out} = \Delta H/s = m\ c_p\ (T_4 - T_1)$$

The pressure - volume, pressure - enthalpy and temperature-entropy diagrams are shown on figures 7a, 7b and 7c respectively.

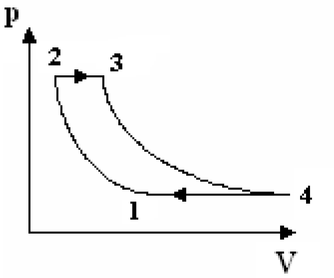


Figure 7a p-V diagram

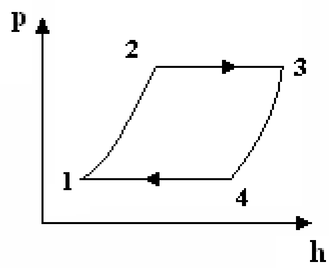


Figure 7b p-h diagram

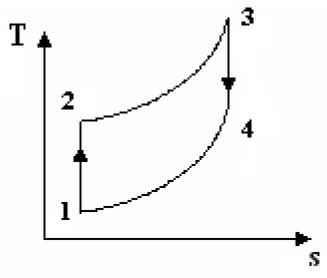


Figure 7c T-s diagram

Efficiency

The efficiency is found by applying the first law of thermodynamics.

$$\Phi_{net} = P_{net} \qquad \Phi_{in} - \Phi_{out} = P_{out} - P_{in}$$

$$\eta_{th} = \frac{P_{out}}{\Phi_{in}} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{\dot{m}c_p(T_4 - T_1)}{\dot{m}c_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

It assumed that the mass and the specific heats are the same for the heater and cooler.

It is easy to show that the temperature ratio for the turbine and compressor are the same.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = r_p^{\,1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{1-\frac{1}{\gamma}} = r_p^{\,1-\frac{1}{\gamma}} \qquad \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

r_p is the pressure compression ratio for the turbine and compressor.

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\left(\frac{T_3 T_1}{T_2} - T_1\right)}{\left(\frac{T_2 T_4}{T_1} - T_2\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \quad \text{hence} \quad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_p^{\,1-\frac{1}{\gamma}}} = 1 - r_p^{\,-\left(1-\frac{1}{\gamma}\right)}$$

A cycle conducted on pure air is known as an Air Standard Cycle and in this case putting γ =1.4 and

$$\eta_{th} = 1 - r_p^{\,-0.286}$$

This shows that the efficiency depends only on the pressure ratio which in turn affects the hottest temperature in the cycle.

WORKED EXAMPLE No. 3

A gas turbine uses the Joule cycle. The pressure ratio is 6/1. The inlet temperature to the compressor is 10^{o}C. The flow rate of air is 0.2 kg/s. The temperature at inlet to the turbine is 950^{o}C. Calculate the following.

i. The air standard cycle efficiency.

ii. The heat transfer into the heater.

iii. The net power output.

$\gamma = 1.4$ $\quad\quad$ $c_p = 1.005$ kJ/kg K

SOLUTION

$$\eta_{th} = 1 - r_p{}^{-0.286} = 1 - 6^{-0.286} = 0.4 \text{ or } 40\%$$

$$T_2 = T_1\, r_p{}^{0.286} = 283 \times 6^{0.286} = 472.4 \text{ K}$$

$$\Phi_{in} = \dot{m}c_p(T_3 - T_2) = 0.2 \times 1.005 \times (1\,223 - 472.4) = 150.8 \text{ kW}$$

$$\eta_{in} = \frac{P_{net}}{\Phi_{in}}$$

$$P_{net} = \eta_{in}\Phi_{in} = 0.4 \times 150.8 = 60.3 \text{ kW}$$

SELF ASSESSMENT EXERCISE No. 2

A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10^{o}C. After constant pressure heating, the pressure and temperature are 7 bar and 700^{o}C respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

1. The air standard cycle efficiency.

2. The heat transfer into the heater.

3. The net power output.

$\gamma = 1.4$ $\quad\quad$ $c_p = 1.005$ kJ/kg K

(Answers 42.7 % , 206.7 kW and 88.26 kW)

4. *The Effect of Friction on the Joule Cycle*

Turbine

The isentropic efficiency for a gas turbine is given by:

$$\eta_{is} = \frac{\Delta H(\text{actual})}{\Delta H(\text{ideal})} = \frac{\Delta T(\text{actual})}{\Delta T(\text{ideal})} = \frac{T_3 - T_4}{T_3 - T_4'}$$

Compressor

For a compressor the isentropic efficiency is inverted and becomes as follows.

$$\eta_{is} = \frac{\Delta H(\text{ideal})}{\Delta H(\text{actual})} = \frac{\Delta T(\text{ideal})}{\Delta T(\text{actual})} = \frac{T_2' - T_1}{T_2 - T_1}$$

Remember that friction always produces a smaller change in temperature than for the ideal case. This is shown on the T-s diagrams (figure 8a and 8b).

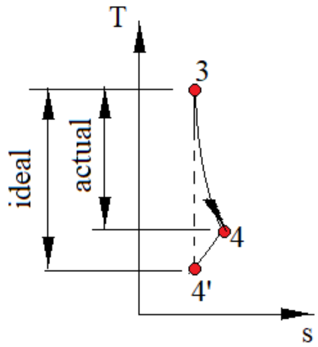


Figure 8a Turbine expansion.

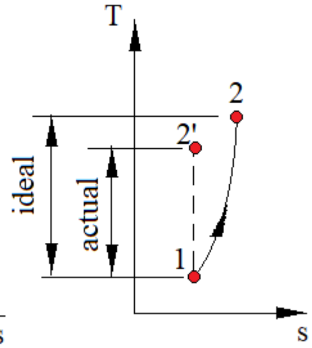


Figure 8b Compression process.

For the turbine $P_{out} = \dot{m}c_p(T_3 - T_4')\eta_{is}$ For the compressor $P_{out} = \dfrac{\dot{m}c_p(T_2' - T_1)}{\eta_{is}}$

The Cycle with Friction

It can be seen that the effect of friction on the gas turbine cycle is reduced power output and increased power input with an overall reduction in net power and thermal efficiency. Figures 9a and 9b show the effect of friction on T-s and p-h diagrams for the Joule cycle.

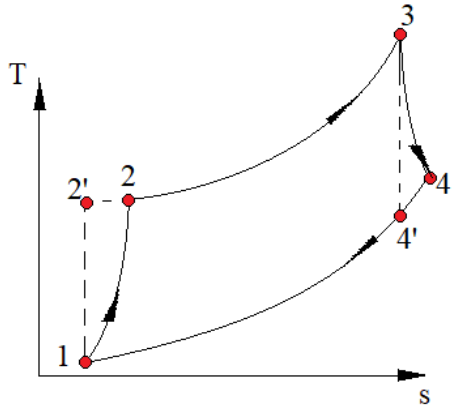


Figure 9a Temperature - Entropy

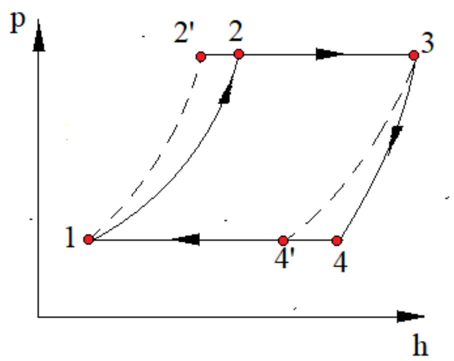


Figure 9b. Pressure - Enthalpy

Note the energy balance which exists is:

$$P(\text{in}) + \Phi(\text{in}) = P(\text{out}) + \Phi(\text{out}) \qquad P(\text{nett}) = P(\text{out}) - P(\text{in}) = \Phi(\text{nett}) = \Phi(\text{in}) - \Phi(\text{out})$$

WORKED EXAMPLE No. 4

A Joule Cycle uses a pressure ratio of 8. Calculate the air standard efficiency. The isentropic efficiency of the turbine and compressor are both 90%. The low pressure in the cycle is 120 kPa. The coldest and hottest temperatures in the cycle are 20°C and 1 200°C respectively. Calculate the cycle efficiency with friction and deduce the change. Calculate the net power output. γ = 1.4 and c_p = 1.005 kJ/kg K. Take the mass flow as 3 kg/s.

SOLUTION

T_1 = 293 K p_1 = 120 kPa T_3 = 1 473 K r_p = 8

No friction $\eta_{th} = 1 - r_p{}^{1/\gamma - 1} = 0.448$ or 48.8 %

With friction $T_2' = 293 \times 8^{0.286} = 531$ K

Compressor

$$\eta_{is} = 0.9 = \frac{T_2' - T_1}{T_2 - T_1} = \frac{531 - 293}{T_2 - 293} \qquad T_2 = 557.4 \text{ K}$$

$$T_4' = \frac{1\,473}{8^{0.286}} = 812.7 \text{ K}$$

Turbine

$$\eta_{is} = 0.9 = \frac{T_3 - T_4}{T_3 - T_4'} = \frac{1\,473 - T_4}{1\,473 - 812.7} \qquad T_4 = 878.7 \text{ K}$$

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{878.7 - 293}{1\,473 - 557.4} = 0.36$$

$\boldsymbol{\eta_{th}}$ **= 0.36 or 36%**

The change in efficiency is a reduction of 8.8%

$$\Phi(\text{in}) = \dot{m}\, c_p\, (T_3 - T_2) = 3 \times 1.005 \times (1\,473 - 557.4) = 2\,760 \text{ kW}$$

Nett Power Output = P(nett) = $\eta_{th} \times \Phi$(in) = 0.36 × 2 760 = 994 kW

SELF ASSESSMENT EXERCISE No. 3

A gas turbine uses a standard Joule cycle but there is friction in the compressor and turbine. The air is drawn into the compressor at 1 bar 15^{o}C and is compressed with an isentropic efficiency of 94% to a pressure of 9 bar. After heating, the gas temperature is 1000^{o}C. The isentropic efficiency of the turbine is also 94%. The mass flow rate is 2.1 kg/s. Determine the following.

1. The net power output.

2. The thermal efficiency of the plant.

$\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.

(Answers 612 kW and 40.4%)

5. *Variants of the Basic Cycle*

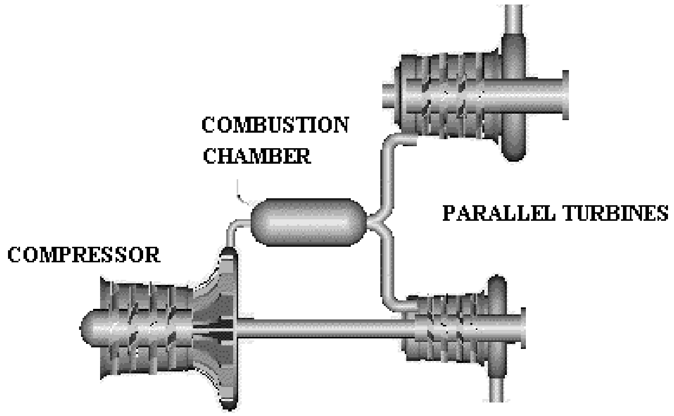
In this section we will examine how practical gas turbine engine sets vary from the basic Joule cycle.

Gas Constants

The first point is that in reality, although air is used in the compressor, the gas going through the turbine contains products of combustion so the adiabatic index and specific heat capacity is different in the turbine and compressor.

Free Turbines

Most designs used for gas turbine sets use two turbines, one to drive the compressor and a free turbine. The free turbine drives the load and it is not connected directly to the compressor. It may also run at a different speed to the compressor.
Figure 10a. shows such a layout with turbines in parallel configuration. Figure 10b shows the layout with series configuration.



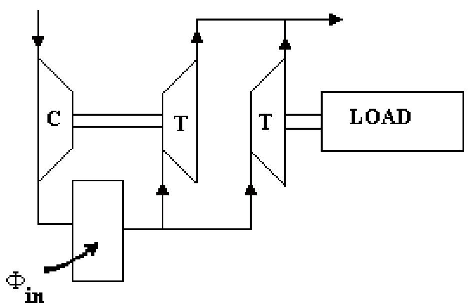
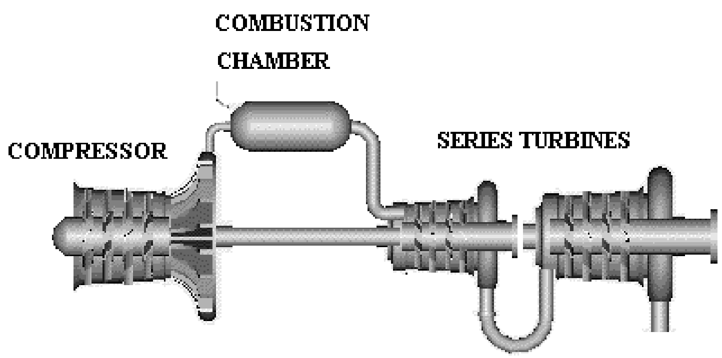


Figure 10a Parallel turbines



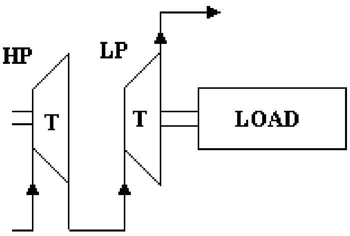


Figure 10b. Series turbines

Intercooling

Basically, if the air is compressed in stages and cooled between each stage, then the work of compression is reduced and the efficiency increased. The layout is shown on figure 11a.

Reheating

The reverse theory of Intercooling applies. If several stages of expansion are used and the gas reheated between stages, the power output and efficiency is increased. The layout is shown on figure 11b.

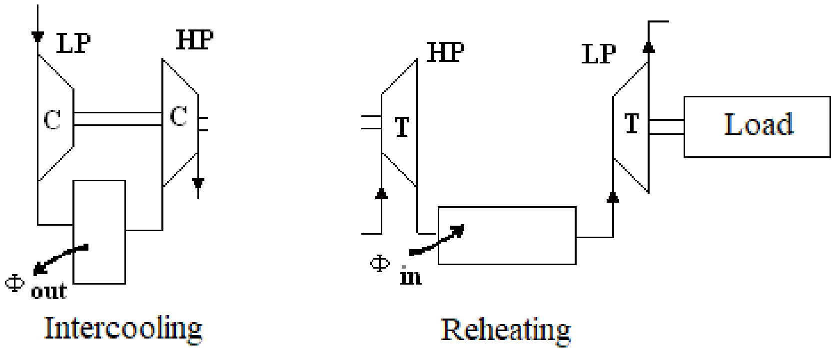


Figure 11a Intercooler   Figure 11b Reheater

WORKED EXAMPLE No. 5

A gas turbine draws in air from atmosphere at 1 bar and 10°C and compresses it to 5 bar with an isentropic efficiency of 80%. The air is heated to 1 200 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 80 kW of power. The isentropic efficiency is 85% for both stages.

Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

For the compressor γ = 1.4 and for the turbines γ = 1.333.

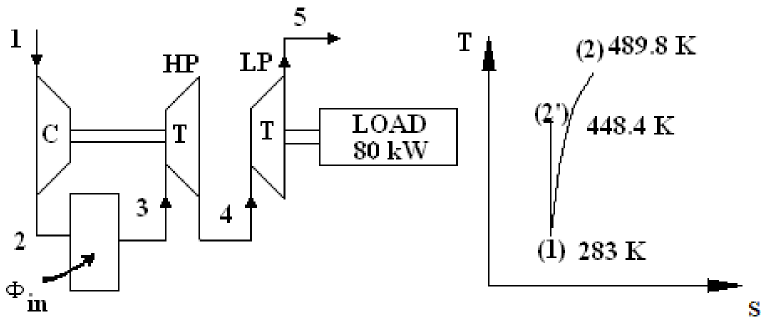
The gas constant R is 0.287 kJ/kg K for both.

Neglect the increase in mass due to the addition of fuel for burning.

SOLUTION

$$\frac{c_p}{c_v} = \gamma \text{ and } R = c_p - c_v \text{ hence } c_p = \frac{R}{1 - \frac{1}{\gamma}}$$

Hence c_p= 1.005 kJ/kg K for the compressor and 1.149 kJ/kg K for the turbines.



Compressor

$$T_2' = T_1 r_p^{1-\frac{1}{\gamma}} = 283 \text{ x } 5^{0.286} = 448.4 \text{ K}$$

$$\eta_{is} = 0.8 = \frac{T_2' - T_1}{T_2 - T_1} = \frac{448.4 - 283}{T_2 - 283} \text{ hence } T_2 = 489.8 \text{ K}$$

Power input to compressor = m c_p (T_2 - T_1) Power output of h.p. turbine = m c_p (T_3 - T_4)

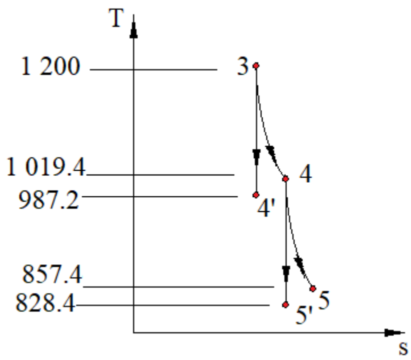
Since these are equal it follows that:

$$1.005(489.8 - 283) = 1.149(1\,200 - T_4) \qquad T_4 = 1\,019.1 \text{ K}$$

High Pressure Turbine

$$\eta_{is} = 0.85 = \frac{T_3 - T_4}{T_3 - T_4'} = \frac{1\,200 - 1\,019.1}{1\,200 - T_4'} \text{ hence } T_4' = 987.2 \text{ K}$$

$$\frac{T_4'}{T_3} = \left(\frac{p_4}{p_3}\right)^{1-\frac{1}{\gamma}} \qquad \frac{987.2}{1\,200} = \left(\frac{p_4}{5}\right)^{0.25} \text{ hence } p_4 = 2.29 \text{ bar}$$



Low Pressure Turbine

$$\frac{T_5'}{T_4} = \left(\frac{1}{2.29}\right)^{1-\frac{1}{\gamma}} \qquad \frac{T_5'}{1\,019.1} = 0.4367^{0.25} \text{ hence } T_5' = 828.4 \text{ K}$$

$$\eta_{is} = 0.85 = \frac{T_4 - T_5}{T_4 - T_5'} = \frac{1\,019.1 - T_5}{1\,019.1 - 828.4} \text{ hence } T_5 = 857\text{K}$$

Nett Power

The nett power is 80 kW hence

$80 = \dot{m}c_p(T_4 - T_5) = \dot{m} \times 1.149\ (1\,019.1 - 857)$ **$\dot{m}$ = 0.43 kg/s**

Heat Input Φ(in) = m c_p (T_3 - T_2) = 0.43 × 1.149 (1 200 - 489.8) = 350.9 kW

Thermal Efficiency

$$\eta_{th} = \frac{P_{net}}{\Phi_{in}} = \frac{80}{350.9} = 0.228 \text{ or } 22.8\%$$

SELF ASSESSMENT EXERCISE No. 4

A gas turbine draws in air from atmosphere at 1 bar and 15°C and compresses it to 4.5 bar with an isentropic efficiency of 82%. The air is heated to 1 100 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 100 kW of power. The isentropic efficiency is 85% for both stages.

For the compressor $\gamma = 1.4$ and for the turbines $\gamma = 1.3$. The gas constant R is 0.287 kJ/kg K for both.

Neglect the increase in mass due to the addition of fuel for burning.

Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

(Answers 0.642 kg/s and 20.1 %)

6. *Exhaust Heat Exchangers*

Because the gas leaving the turbine is hotter than the gas leaving the compressor, it is possible to heat up the air before it enters the combustion chamber by use of an exhaust gas heat exchanger. This results in less fuel being burned in order to produce the same temperature prior to the turbine and so makes the cycle more efficient. The layout of such a plant is shown on figure 12.

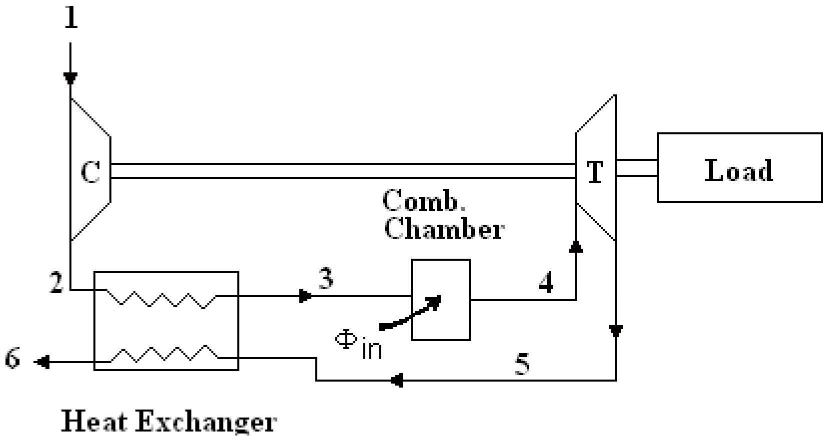


Figure 12 Plant layout

In order to solve problems associated with this cycle, it is necessary to determine the temperature prior to the combustion chamber (T_3).

A perfect heat exchanger would heat up the air so that T_3 is the same as T_5. It would also cool down the exhaust gas so that T_6 becomes T_2. In reality this is not possible so the concept of ***THERMAL RATIO*** is used. This is defined as the ratio of the enthalpy given to the air to the maximum possible enthalpy lost by the exhaust gas. The enthalpy lost by the exhaust gas is:

$$\Delta H = m_g c_{pg}(T_5 - T_6)$$

This would be a maximum if the gas is cooled down such that $T_6 = T_2$. Of course in reality this does not occur and the maximum is not achieved and the gas turbine does not perform as well as predicted by this idealisation.

$$\Delta H(\text{maximum}) = \Delta H = m_g c_{pg}(T_5 - T_6)$$

The enthalpy gained by the air is

$$\Delta H(\text{air}) = m_a c_{pa} (T_3 - T_2)$$

Hence the thermal ratio is

$$\text{T.R.} = \frac{m_a c_{pa}(T_3 - T_2)}{m_g c_{pg}(T_5 - T_2)}$$

The suffix 'a' refers to the air and g to the exhaust gas. Since the mass of fuel added in the combustion chamber is small compared to the air flow we often neglect the difference in mass and the equation becomes

$$\text{T.R.} = \frac{c_{pa}(T_3 - T_2)}{c_{pg}(T_5 - T_2)}$$

WORKED EXAMPLE No. 6

A gas turbine uses a pressure ratio of 7.5/1. The inlet temperature and pressure are respectively 10°C and 105 kPa. The temperature after heating in the combustion chamber is 1300 °C. The specific heat capacity c_p for the exhaust gas is 1.15 kJ/kg K. The adiabatic index is 1.4 for air and 1.33 for the gas. Assume isentropic compression and expansion. The mass flow rate is 1 kg/s. Use the chart below to determine c_p for air.

Calculate the air standard efficiency if no heat exchanger is used and compare it to the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.88 is used.

SOLUTION

Referring to the numbers used on figure 12 the solution is as follows.
Air standard efficiency

$$\eta_{AS} = 1 - r_p^{1-1/\gamma} = 1 - 7.5^{0.286} = 0.438 \text{ or } 43.8\ \%$$

Solution with heat exchanger

$$T_2 = T_1 r_p^{1-1/\gamma} = 283 \times 7.5^{0.286} = 503.6\ \text{K}$$

$$T_5 = \frac{T_4}{r_p^{1-1/\gamma}} = \frac{1\,573}{7.5^{0.286}} = 950.5\ \text{K}$$

Use the thermal ratio to estimate T_3 with a typical value of c_p = 1.005 kJ/kg K

$$0.88 = \frac{c_{pa}(T_3 - T_2)}{c_{pg}(T_5 - T_2)} = \frac{1.005(T_3 - 503.6)}{1.15(950.5 - 503.6)} \qquad T_3 = 953.6\ \text{K}$$

The chart below shows the effect of pressure and temperature on C_p. Post compressor pressure is about 7.5 bar and the mean temperature of air in the heat exchanger is about 728 K. From the chart c_p will be around 1.08 kJ/kg K

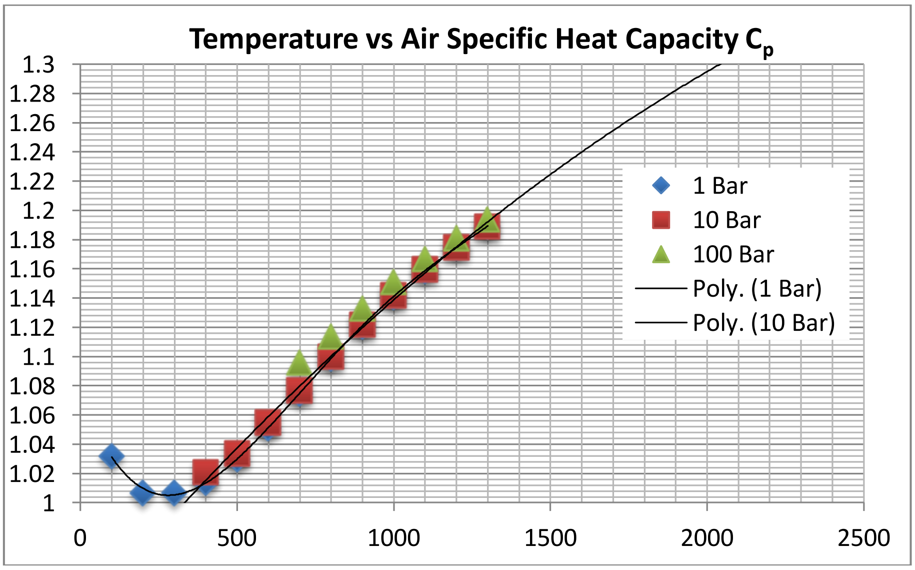


Figure 13

Recalculate T_3

$$0.88 = \frac{c_{pa}(T_3 - T_2)}{c_{pg}(T_5 - T_2)} = \frac{1.08(T_3 - 503.6)}{1.15(950.5 - 503.6)} \quad T_3 = 896.9 \text{ K}$$

In order find the thermal efficiency, it is best to solve the energy transfers.

$$P(\text{in}) = mc_{pa}(T_2 - T_1) = 1 \times 1.08\ (503.6 - 283) = 238.2 \text{ kW}$$

$$P(\text{out}) = mc_{pg}(T_4 - T_5) = 1 \times 1.15\ (1\,573 - 950.5) = 715.9 \text{ kW}$$

$$P(\text{nett}) = P(\text{out}) - P(\text{in}) = 477.7 \text{ kW}$$

$$\Phi(\text{in})\text{combustion chamber}) = mc_{pg}(T_4 - T_3)$$

$$\Phi(\text{in}) = 1.15(1\,573 - 896.9) = 777.5 \text{ kW}$$

$$\eta_{th} = \frac{P_{net}}{\Phi_{in}} = \frac{8477.7}{777.5} = 0.614 \text{ or } 61.4\ \%$$

SELF ASSESSMENT EXERCISE No. 5

1. A gas turbine uses a pressure ratio of 7/1. The inlet temperature and pressure are respectively 10°C and 100 kPa. The temperature after heating in the combustion chamber is 1000 °C. The specific heat capacity c_p is 1.005 kJ/kg K and the adiabatic index is 1.4 for air and gas. Assume isentropic compression and expansion. The mass flow rate is 0.7 kg/s.

   Calculate the net power output and the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.
   (Answers 234 kW and 57%)

2. A gas turbine uses a pressure ratio of 6.5/1. The inlet temperature and pressure are respectively 15°C and 1 bar. The temperature after heating in the combustion chamber is 1200 °C. The specific heat capacity c_p for air is 1.005 kJ/kg K and for the exhaust gas is 1.15 kJ/kg K. The adiabatic index is 1.4 for air and 1.333 for the gas. The isentropic efficiency is 85% for both the compression and expansion process. The mass flow rate is 1kg/s.

   Calculate the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.75 is used.
   (Answer 48.3%)

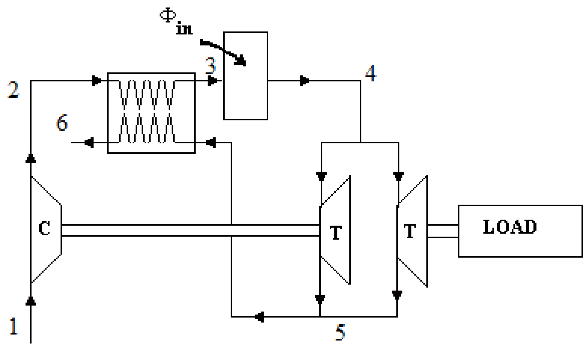
WORKED EXAMPLE No. 7

A gas turbine has a free turbine in parallel with the turbine which drives the compressor. An exhaust heat exchanger is used with a thermal ratio of 0.8. The isentropic efficiency of the compressor is 80% and for both turbines is 0.85.

The heat transfer rate to the combustion chamber is 1.48 MW. The gas leaves the combustion chamber at 1 100^oC. The air is drawn into the compressor at 1 bar and 25^oC. The pressure after compression is 7.2 bar.

The adiabatic index is 1.4 for air and 1.333 for the gas produced by combustion. The specific heat c_p is 1.005 kJ/kg K for air and 1.15 kJ/kg K for the gas. Determine the following.

i. The mass flow rate in each turbine.
ii. The net power output.
iii. The thermodynamic efficiency of the cycle.



SOLUTION

$T_1 = 298$ K
$T_2' = 298(7.2)^{(1-1/1.4)} = 524$ K
$T_4 = 1\,373$ K
$T_5' = 1\,373(1/7.2)^{(1-1/1.333)} = 838.5$ K

Compressor

$$\eta_{is} = 0.8 = \frac{T_2' - T_1}{T_2 - T_1} = \frac{524 - 298}{T_2 - 298} \quad \text{hence } T_2 = 580.5 \text{ K}$$

Turbines

Treat as one expansion with gas taking parallel paths.

$$\eta_{is} = 0.85 = \frac{T_4 - T_5}{T_4 - T_5'} = \frac{1\,373 - T_5}{1\,373 - 838.5} \quad \text{hence } T_5 = 918.7 \text{ K}$$

Heat Exchanger

$$\text{T.R.} = 0.8 = \frac{c_{pa}(T_3 - T_2)}{c_{pg}(T_5 - T_6)} = \frac{1.005(T_3 - 580.5)}{1.15(918.7 - 580.5)} \quad \text{hence } T_3 = 890.1 \text{ K}$$

Combustion Chamber

$$\Phi(\text{in}) = \dot{m}\, c_p\, (T_4 - T_3) = 1\,480 \text{ kW}$$

$$1\,480 = \dot{m}\,(1.15)(1\,373 - 890.1) \quad \text{hence } \dot{m} = 2.665 \text{ kg/s}$$

Compressor

$$P(\text{in}) = \dot{m}\, c_p\, (T_2 - T_1) = 2.665(1.005)(580.5 - 298) = 756.64 \text{ kW}$$

Turbine A

$$P(out) = 756.64 \text{ kW} = \dot{m}_A c_p (T_4 - T_5)$$

$$756.64 = = 2.665(1.15)(1\,373 - 918.7) \quad \text{hence } \dot{m}_A = 1.448 \text{ kg/s}$$

Hence mass flow through the free turbine is 1.2168 kg/s

$$P(net) = \text{Power from free turbine} = 1.2168(1.15)(1\,373\text{-}918.7) = 635.7 \text{ kW}$$

Thermodynamic Efficiency

$$\eta_{th} = \frac{P_{net}}{\Phi_{in}} = \frac{635.7}{1\,480} = 0.429 \text{ or } 42.9\ \%$$

SELF ASSESSMENT EXERCISE No. 6

1. List the relative advantages of open and closed cycle gas turbine engines.

Sketch the simple gas turbine cycle on a T-s diagram. Explain how the efficiency can be improved by the inclusion of a heat exchanger.

In an open cycle gas turbine plant, air is compressed from 1 bar and 15°C to 4 bar. The combustion gases enter the turbine at 800°C and after expansion pass through a heat exchanger in which the compressor delivery temperature is raised by 75% of the maximum possible rise. The exhaust gases leave the exchanger at 1 bar. Neglecting transmission losses in the combustion chamber and heat exchanger, and differences in compressor and turbine mass flow rates, find the following.

(i) The specific work output. (135 kW per unit mass flow rate)
(ii) The work ratio (0.44)
(iii) The cycle efficiency (37.1%)

The compressor and turbine polytropic efficiencies are both 0.84.

Compressor $c_p = 1.005$ kJ/kg K $\gamma = 1.4$
Turbine $c_p = 1.148$ kJ/kg K $\gamma = 1.333$
Note for a compression

Note that for a compression $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$ and for an expansion $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_\infty}{\gamma}}$

2. ***This question involves knowledge of compressible flow through nozzles but is solvable with the information given. You will be achieving a high level if you complete this successfully***

A gas turbine for aircraft propulsion is mounted on a test bed. Air at 1 bar and 293 K enters the compressor at low velocity and is compressed through a pressure ratio of 4 with an isentropic efficiency of 85%. The air then passes to a combustion chamber where it is heated to 1175 K. The hot gas then expands through a turbine which drives the compressor and has an isentropic efficiency of 87%. The gas is then further expanded isentropically through a nozzle leaving at the speed of sound. The exit area of the nozzle is 0.1 m^2. Determine the following.

(i) The pressures at the turbine and nozzle outlets. (2.138 bar and 1.13 bar)
(ii) The mass flow rate. (27.23 kg/s)
(iii) The thrust on the engine mountings. (17 kN)

Assume the properties of air throughout.

The sonic velocity of air is given by $a = (\gamma RT)^{½}$. The temperature ratio before and after the nozzle is given by

$$\frac{T_{in}}{T_{out}} = \frac{2}{\gamma + 1}$$

3. (A). A gas turbine plant operates with a pressure ratio of 6 and a turbine inlet temperature of 927°C. The compressor inlet temperature is 27°C. The isentropic efficiency of the compressor is 84% and of the turbine 90%. Making sensible assumptions, calculate the following.

(i) The thermal efficiency of the plant. (29.3%)
(ii) The work ratio. (0.279)

Treat the gas as air throughout.

(B). If a heat exchanger is incorporated in the plant, calculate the maximum possible efficiency which could be achieved assuming no other conditions are changed. (44.8%)

Explain why the actual efficiency is less than that predicted.

8. *Ignition and Combustion*

The fuel is introduced into the combustion chamber and has to mix with the hot air flowing from the compressor. Depending on the fuel used there is a time delay before the fuel becomes hot enough to ignite. Ignition is started by generating a continuous electric arc. Once the combustion chamber reaches the auto ignition temperature the fuel self ignites and the arc can be stopped. Most fuels will ignite when they are premixed with warm air. The time taken is referred to as the auto-ignition delay time. Heavy fuels take longer than light gaseous fuels and strict safety procedures are used to establish steady combustion. This varies from 1 second for natural gas to 10 ms for LPG.

The ignition system consists of an exciter, the ignition lead, and the ignitor. The exciter produces high voltages across the electrodes through the ignition lead to the ignitor and produces an arc.

Starting Procedure

You will find a good description at this internet link
https://www.flight-mechanic.com/gas-turbine-engine-starters/

First the compressor must be rotated to get the pressure and air flow required for starting in the combustion chamber. Then ignition and introduction of fuel occurs. The starter must continue to assist the engine until the engine sustains itself. The starter must produce sufficient torque to accelerate the compressor up to speed as well compress the air.

If the starter was switched off before sufficient power was obtained to drive the compressor and accelerate it further the engine would not reach idling speed but might produce a false start. The cut off of the starter and ignition are automatic.

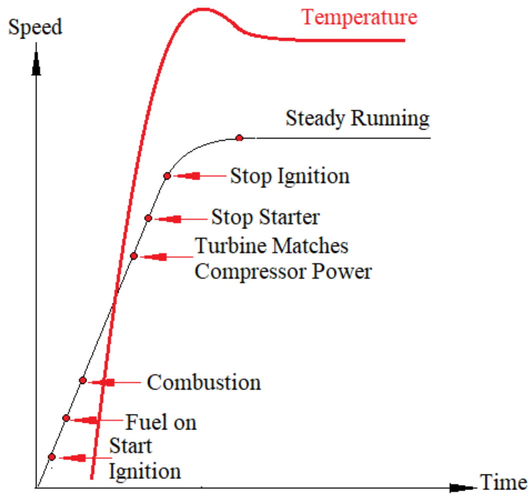


Figure 14

Starting Systems

Starters are normally a hydraulic motor, direct current electric motor or an air turbine.
Hydraulic motors are generally much smaller than electric motors of the same power level. Modern electrical alternatives using new technological advances have turned electrical solutions into a superior alternative to hydraulics.

Turbine starters are small and connected to the compressor spool through a gear box. Air or gas is supplied from an external source. They provide a high starting torque. They are lighter than electric starters but it is capable of developing considerable more torque than the electric starter. Figure 15 shows such starters.

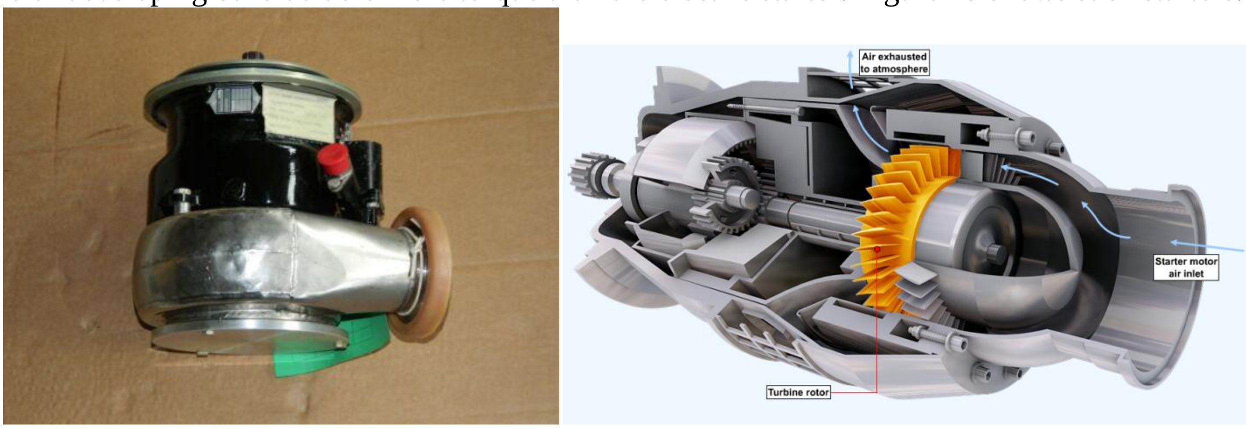


Figure 15