THERMODYNAMICS

TUTORIAL 11
STEAM TURBINE POWER CYCLES

This is a comprehensive tutorial covering most aspects of steam cycles and it is set at QCF Levels 5 and 6

On completion of this tutorial you should be able to:

- Explain the Ideal and basic steam power cycle.
- Define the parameters needed to solve problems involving steam power plant
- Explain the practical modifications made to steam power plant and the steam cycle
- Explain advanced steam power plant and their cycles
- Solve problems involving steam power plant taking into account the affect of friction on the processes.

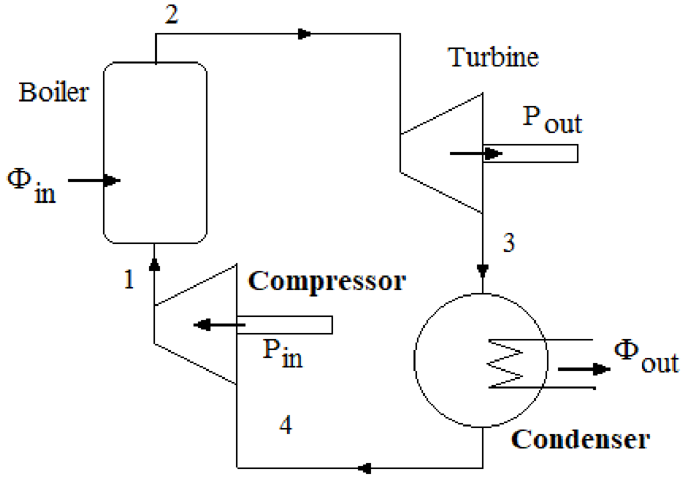
Contents

1. *Steam Cycles*

1.1 *The Carnot Steam Cycle*

In previous tutorials you learned that a Carnot cycle gave the highest thermal efficiency possible for an engine working between two temperatures. The cycle consisted of isothermal heating and cooling and reversible adiabatic expansion and compression.

Consider a cycle that uses vapour throughout. Evaporation and condensation at constant pressure is also constant temperature. Isothermal heating and cooling is theoretically possible. The cycle would consist of the same 4 processes as before only this time each process would be carried out in a separate steady flow plant item with the vapour flowing from one to the other in a closed loop as shown below.



The four processes are:

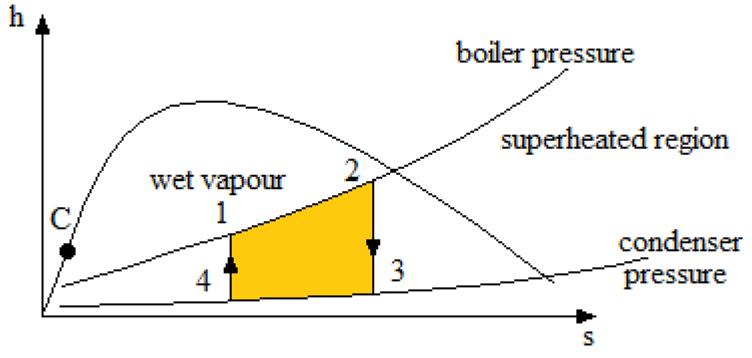
1 - 2 Evaporation at constant pressure and temperature requiring heat input

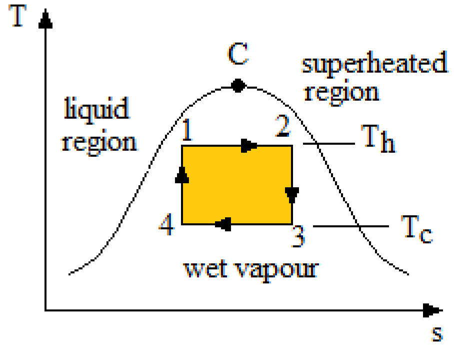
2 – 3 Reversible adiabatic expansion in the turbine giving power output

3 - 4 Cooling and condensing at constant pressure and temperature in the condenser requiring heat output.

4 - 1 Reversible adiabatic compression requiring power input.

In order that no temperature changes occur in the evaporator and condenser, the vapour must be wet at inlet and outlet. Over-cooling will produce liquid at temperatures below the saturation temperature and over-heating will superheat it beyond the saturation temperature. The cycle will be a rectangle on the T - s diagram and as shown on the h - s diagram.





The limits are that at point (2) it may be dry saturated vapour but not superheated. At point 1 it may be saturated water but not under-cooled. If these limits are not used, then the vapour has a dryness fraction at each point. Since heat transfer only occurs at the evaporator and condenser the heat transfer rates are given by the following expressions.

$$\Phi_{in} = \dot{m}(h_2 - h_1) = T_h \ \Delta S \quad \text{(Boiler)}$$

$$\Phi_{out} = \dot{m}\,(h_3 - h_4) = T_c \,\Delta S \quad \text{(Condenser)}$$

T_h is the boiler temperature and T_c is the condenser temperature.

The thermal efficiency may be found from the 1st. Law.

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{T_c \Delta S}{T_h \Delta S} = 1 - \frac{T_c}{T_h}$$

This expression is the same as for the gas version.

WORKED EXAMPLE No. 1

A Carnot cycle is conducted on steam as follows. The evaporator produces dry saturated steam at 10 bar. The steam is expanded reversibly and adiabatically in a turbine to 1 bar. The exhaust steam is partially condensed and then compressed back to 10 bar. As a result of the compression, the wet steam is changed completely into saturated water.

Assuming a flow rate of 1 kg/s throughout determine the condition and specific enthalpy at each point in the cycle. Calculate the energy transfers for each stage. Show that the efficiency is correctly predicted by the expression

$$\eta_{th} = 1 - \frac{T_c}{T_h}$$

SOLUTION

We will refer to the previous diagrams throughout. Determine the properties using tables, charts or online software as convenient.

Evaporator

$h_2 = h_g$ at 10 bar (since it is dry saturated) = 2 778 kJ/kg.

$s_2 = s_g$ at 10 bar (since it is dry saturated) = 6.586 kJ/kg K.

$h_1 = h_f$ at 10 bar (since it is saturated water) = 763 kJ/kg.

$$\Phi_{in} = 1\,(2\,778 - 763) = 2\,015 \text{ kW}$$

Turbine

Since the expansion is isentropic then $s_2 = s_3$ = 6.586 kJ/kg K

$s_3 = 6.586 = s_f + x_3 s_{fg}$ at 1 bar

$6.586 = 1.303 + x_3(6.056)$ hence $x_3 = 0.872$

$h_3 = h_f + x_3 h_{fg}$ at 1 bar = 417 + (0.872)(2 258) = 2 387 kJ/kg

$$P(\text{output}) = 1(2\,778 - 2\,387) = 391.2 \text{ kW}$$

Compressor

Since the compression is isentropic then $s_4 = s_1$

$s_1 = s_f$ at 10 bar (since it is saturated water) = 2.138 kJ/kg K.

$s_4 = s_1 = 2.138 = s_f + x_4 s_{fg}$ at 1 bar

$2.138 = 1.303 + x_4(6.056)$ hence $x_4 = 0.138$

$h_4 = h_f + x_4 h_{fg}$ at 1 bar = 417 + (0.139)(2 258) = 728.3 kJ/kg

Power Input = 1(763 - 728.3) = 34.7 kW

Condenser

Heat output = 1(2 387 - 728.3) = 1 658.7 kW

Energy Balances rounded off to nearest kW.

Total energy input = 34.7 + 2 015 = 2 050 kW

Total energy output = 391.2 + 1 658.7 = 2 050 kW

Net Power output = 391.2 - 34.7 = 356 kW

Net Heat input = 2 015 – 1 658.7 = 356 kW

$$\eta_{th} = \frac{P_{net}}{\Phi_{net}} = \frac{356}{2\,015} = 17.7\%$$

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{1\,658}{2\,015} = 17.7\%$$

The hottest temperature in the cycle is t_s at 10 bar = 179.9 ^oC or 452.9 K

The coldest temperature in the cycle is t_s at 1 bar = 99.6 ^oC or 372.6 K

$$\eta_{th} = 1 - \frac{T_c}{T_h} = 1 - \frac{372.6}{452.9} = 17.7\%$$

SELF ASSESSMENT EXERCISE No. 1

1. A steam power plant uses the Carnot cycle. The boiler puts 25 kW of heat into the cycle and produces wet steam at 300°C. The condenser produces wet steam at 50°C.

   Calculate the following.

   i. The efficiency of the plant. (43.6%)
   ii. The net power output. (10.9 kW)
   iii. The heat removed by the condenser. (14 kW)

2. A steam power plant is based on the Carnot cycle. The boiler is supplied with saturated water at 20 bar and produces dry saturated steam at 20 bar. The condenser operates at 0.1 bar. Assuming a mass flow rate of 1 kg/s calculate the following.

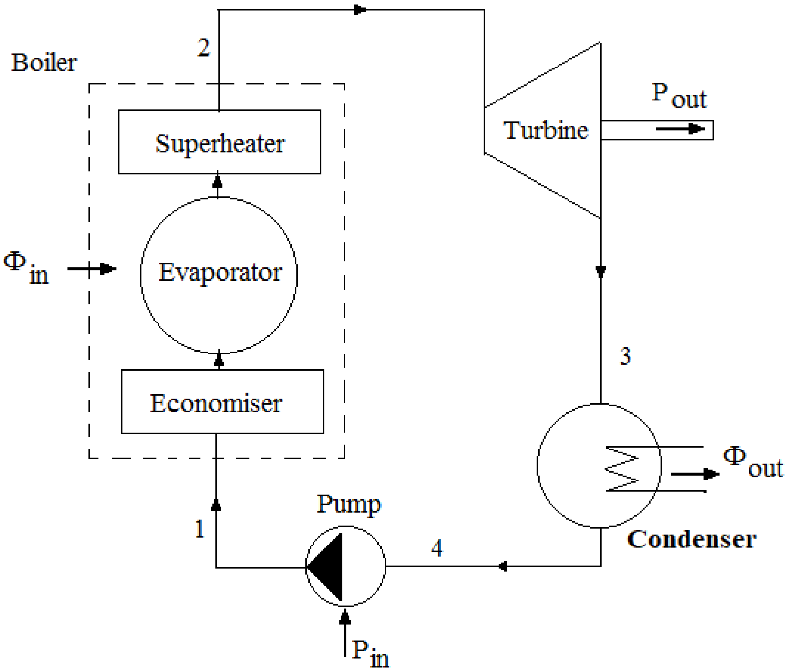
   i. The thermal efficiency. (34.3%)
   ii. The power output of the turbine. (792 kW)
   iii. The heat transfer rate into the boiler. (1.89 MW)

1.2 The Rankine Cycle

The Rankine Cycle is a practical cycle and most steam power plants are based on it. The problems with the Carnot Cycle are as follows.

- It produces only small net power outputs for the plant size because dry saturated steam is used at inlet to the turbine.
- It is impractical to compress wet steam because the water content separates out and fills the compressor.
- It is impractical to control the condenser to produce wet steam of the correct dryness fraction.

In order to get around these problems, the Rankine Cycle uses superheated steam from the boiler to the turbine. The condenser completely condenses the exhaust steam into saturated water. The compressor is replaced with a water (feed) pump to return the water to the boiler. The result of this is reduced efficiency but greater quantities of power.



The plant layout is shown above. First let's briefly examine the boiler.

Boiler
For reasons of combustion efficiency, a practical boiler is made up of three sections.

a) Economiser
This is a water heater inside the boiler that raises the water temperature at the boiler pressure to just below the saturation temperature at that pressure.

b) Evaporator
This is a unit usually consisting of a drum and tubes in which the water is evaporated and the steam driven off.

c) Super-heater
This is a heater placed in the hottest part of the boiler that raises the temperature of the steam well beyond the saturation temperature.

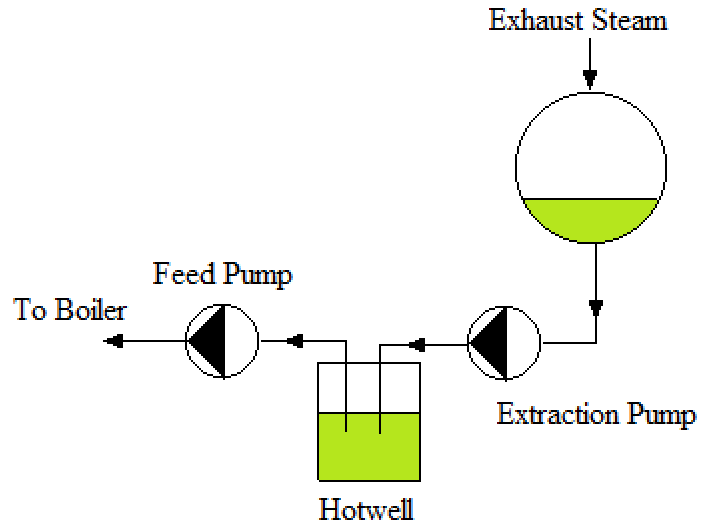
There are many boiler designs and not all of them have these features. The main point is that a heat transfer rate is needed into the boiler unit in order to heat up the water, evaporate it and superheat it. The overall heat transfer is

$$\Phi_{in} = \dot{m}\,(h_2 - h_1)$$

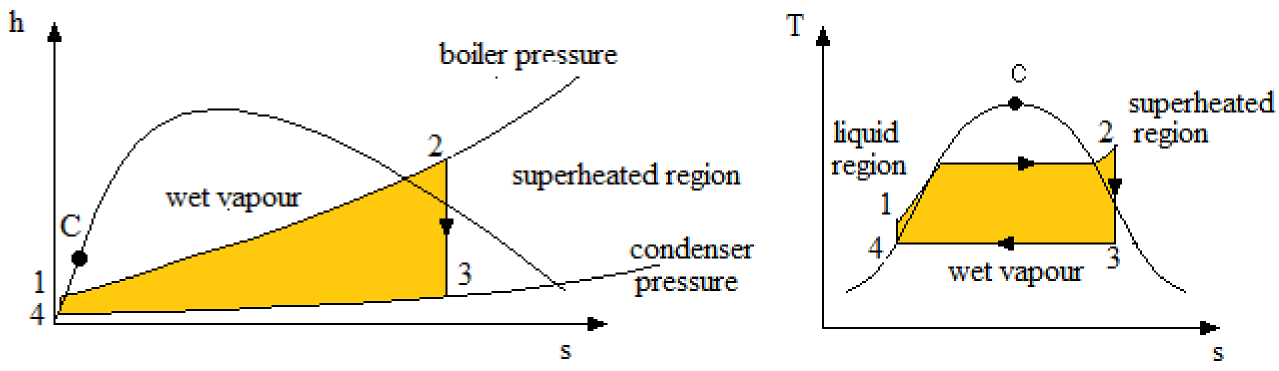
Next let's look at some other practical aspects of a steam power plant.

Extraction Pump and Hotwell

In a practical steam cycle the condensate in the condenser is extracted with an extraction pump and the water produced is the coldest point in the steam cycle. This is usually placed into a vessel where it can be treated and extra added to make up for leaks. This point is called the ***Hotwell*** because it contains hot water. The main feed pump returns this water to the boiler at high pressure. In the following work, extraction pumps and hotwells are not shown.



Now let's examine the cycle with the aid of property diagrams.



The process 4 to 1 is cramped into the corner of the h-s diagram and is not clear.

Boiler
Process (1) To (2) Heat Input

The water at point 1 is below the saturation temperature at the boiler pressure. The economiser first heats it up raising the temperature, enthalpy and entropy until it reached the saturation curve. The water is then evaporated and finally, the temperature is raised by superheating the steam to point 2.

$$\Phi_{in} = \dot{m}\ (h_2 - h_1)$$

Turbine
Process (2) To (3) Power Output

The second process is the expansion in the turbine and this is ideally reversible and adiabatic and is represented by a vertical line on the diagrams.

$$P_{out} = \dot{m}\ (h_2 - h_3)$$

Turbines in real plant are often in several stages and the last stage is specially designed to cope with water droplets in the steam that becomes wet as it gives up its energy. You must use the isentropic expansion theory in order to calculate the dryness fraction and enthalpy of the exhaust steam.

Condenser
Process (3) to (4) Heat Output

The third process is the condenser where the wet steam at point 3 is ideally turned into saturated water at the lower pressure (point 4). Condensers usually work at very low pressures (vacuums) in order to make the turbine give maximum power. The heat removed is given by

$$\Phi_{out} = \dot{m}\ (h_3 - h_4)$$

Since the condenser produces condensate (saturated water) then $h_4 = h_f$ at the condenser pressure.

Pump
Process (4) to (1) Power Input

The final process which completes the cycle is the pumping of the water (point 4) from the low condenser pressure to the boiler at high pressure (point 1). In reality there are many things which are done to the feed water before it goes back into the boiler and the pressure is often raised in several stages. For the Rankine Cycle we assume one stage of pumping which is adiabatic and the power input to the pump is

$$P_{in} = \dot{m}\ (h_1 - h_4)$$

The power required to pump the water is much less than that required to compress the vapour (if it was possible). The power input to the feed pump is very small compared to the power output of the turbine and you can often neglect it altogether. In this case we assume $h_1 = h_4$.

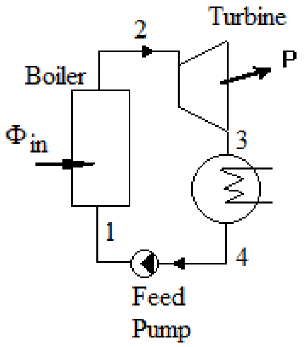
If you are not ignoring the power input, then you need to find h_1. If you know the exact temperature of the water at inlet to the boiler (outlet from the pump) then you may be able to look it up in tables. The nearest approximation is to look up h_f at the water temperature. Since the water is at high pressure, this figure will not be very accurate and you may correct it by adding the flow energy. We will look at this in greater detail later. Let's first do a simple example with no great complications.

WORKED EXAMPLE No. 2

A steam power plant is based on the Rankine cycle. The steam produced by the boiler is at 40 bar and 400°C. The condenser pressure is 0.035 bar. Assume isentropic expansion. Ignore the energy term at the feed pump.

Calculate the Rankine cycle efficiency and compare it to the Carnot efficiency for the same upper and lower temperature limits.

SOLUTION



Use tables, charts or online software to find the properties.

Turbine

$$h_2 = 3\,214 \text{ kJ/kg at 40 bar and } 400^oC$$

Since the expansion is isentropic then

$$s_2 = 6.769 \text{ kJ/kg K} = s_3 = 0.391 + 8.13\,x \qquad x = 0.785$$

$$h_3 = h_f + x\,h_{fg} = 112 + 0.785(2\,438) = 2\,024.6 \text{ kJ/kg}$$

Condenser

$$h_4 = h_f \text{ at 0.035 bar} = 112 \text{ kJ/kg}$$

Boiler

If the power input to the pump is neglected then

$$h_4 = h_1 = 112 \text{ kJ/kg}$$

$$\Phi_{in} = h_2 - h_1 = 3\,102 \text{ kJ/kg.}$$

P(output) = $h_2 - h_3$ = 1 189.4 kJ/kg

$$\eta_{th} = \frac{P_{out}}{\Phi_{in}} = \frac{1\,189.4}{3\,102} = 38.3\%$$

Carnot Efficiency

The hottest temperature in the cycle is 400°C (673 K) and the coldest temperature is t_s at 0.035 bar and this is 26.7 °C (299.7 K).

The Carnot efficiency is

$$\eta_{Carnot} = 1 - \frac{299.7}{673} = 55.5\% \quad \text{(Higher than expected)}$$

Now let's examine the feed pump in more detail.

Feed Pump

When water is compressed its volume hardly changes. This is the important factor that is different from the compression of a gas. Because the volume hardly changes, the temperature should not increase and the internal energy does not increase. The Steady flow Energy equation would then tell us that the power input to the pump is virtually equal to the increase in flow energy. We may write

$$P_{in} = \dot{m}\, v\Delta p$$

Since the volume of water in nearly all cases is 0.001 m^3/kg then this becomes

$$P_{in} = 0.001\, \dot{m}\, \Delta p = 0.001\, \dot{m}\, (p_1 - p_2)$$

If we use pressure units of bars then

$$P_{in} = 0.001\, \dot{m}\, (p_1 - p_2) \times 10^5 \text{ Watts}$$

Expressed in kilowatts this is

$$P_{in} = \dot{m}\, (p_1 - p_2) \times 10^{-1} \text{ kW}$$

From this we may also deduce the enthalpy of the water after the pump.

$$P_{in} = \dot{m}\, (h_1 - h_4)$$

Hence h_1 may be deduced.

WORKED EXAMPLE No. 3

Repeat example 2, but this time do not ignore the feed pump and assume the boiler inlet condition is unknown.

SOLUTION

$$P_{in} = 1\text{kg/s}(40 - 0.035) \times 10^{-1} = 4 \text{ kW}$$

$$4 = 1 \text{ kg/s}(h_1 - h_4) = (h_1 - 112)$$

$$h_1 = 116 \text{ kJ/kg}$$

Reworking the energy transfers gives

$$\Phi_{in} = h_2 - h_1 = 3\,214 - 116 = 3\,098 \text{ kJ/kg.}$$

$$P_{nett} = P_{out} - P_{in} = 1\,189.4 - 4 = 1\,185.4 \text{ kJ/kg}$$

$$\eta = P_{nett}/\Phi_{in} = 1185.4/3098 = 38.3\ \%$$

$$\eta_{th} = \frac{P_{out}}{\Phi_{in}} = \frac{1\,185}{3\,098} = 38.3\%$$

Notice that the answers are not noticeably different from those obtained by ignoring the feed pump.

WORKED EXAMPLE No. 4

A steam power plant uses the Rankine Cycle. The details are as follows.

Boiler pressure	100 bar
Condenser pressure	0.07 bar
Temperature of steam leaving the boiler	400°C
Mass flow rate	55 kg/s

Calculate the cycle efficiency, the net power output and the specific steam consumption.

SOLUTION

Turbine

$$h_2 = 3\,097 \text{ kJ/kg at 100 bar and } 400^{\circ}\text{C}.$$

For an isentropic expansion we find the ideal condition at point 3 as follows.

$$s_2 = 6.213 \text{ kJ/kg K} = s_3 = 0.559 + 7.715\, x_3 \qquad x_3 = 0.733$$

$$h_3 = h_f + x_3 h_{fg} = 163 + 0.733(2\,409) = 1\,928 \text{ kJ/kg}$$

$$P_{out} = \dot{m}(h_2 - h_3) = 55(3\,097 - 1\,928) = 64.3 \text{ MW}$$

Condenser

$$h_4 = h_f \text{ at 0.07 bar} = 163 \text{ kJ/kg}$$

$$\Phi_{out} = \dot{m}(h_3 - h_4) = 55(1\,928 - 163) = 97.1 \text{ MW}$$

Pump

$$\text{Ideal power input} = \text{Flow Energy change} = \dot{m}v(\Delta p)$$

$$P_{in} = 55(0.001)(100 - 0.07) \times 10^5 = 550 \text{ kW}$$

$$P_{in} = \dot{m}(h_1 - h_4) = 55(h_1 - 163) \text{ hence } h_1 = 173 \text{ kJ/kg}$$

Boiler

$$\Phi_{in} = \dot{m}(h_2 - h_1) = 55(3\,097 - 173) = 160.8 \text{ MW}$$

Efficiency

$$P_{nett} = P_{out} - P_{in} = 64.3 - 0.55 = 63.7 \text{ MW}$$

$$\eta = P_{nett} / \Phi_{in} = 63.7/160.8 = 39.6\ \%$$

Alternatively $\quad P_{nett} = \Phi_{in} - \Phi_{out} = 160.8 - 97.1 = 63.7 \text{ MW}$

This should be the same as P_{nett} since the net energy entering the cycle must equal the net energy leaving.

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{97.1}{160.8} = 39.6\%$$

Specific Steam Consumption

This is given by

$$SSC = \frac{P_{net}}{\dot{m}} = \frac{63.78}{55} = 1.159 \text{ MJ/kg}$$

SELF ASSESSMENT EXERCISE No. 2

1. A simple steam plant uses the Rankine Cycle and the data for it is as follows.

Flow rate	45 kg/s
Boiler pressure	50 bar
Steam temperature from boiler	300^oC
Condenser pressure	0.07 bar

Assuming isentropic expansion and pumping, determine the following.

i. The power output of the turbine. (44.9 MW)
ii. The power input to the pump. (225 kW)
iii. The heat input to the boiler. (124 MW)
iv. The heat rejected in the condenser. (79 MW)
v. The thermal efficiency of the cycle. (36%)

2. A simple steam power plant uses the Rankine Cycle. The data for it is as follows.

Flow rate	3 kg/s
Boiler pressure	100 bar
Steam temperature from boiler	600^oC
Condenser pressure	0.04 bar

Assuming isentropic expansion and pumping, determine the following.

i. The power output of the turbine. (4.6 MW)
ii. The power input to the pump. (30 kW)
iii. The heat input to the boiler. (10.5 MW)
iv. The heat rejected in the condenser. (5.9 MW)
v. The thermal efficiency of the cycle. (44%)

3.
a) Explain why practical steam power plants are based on the Rankine Cycle rather than the Carnot Cycle.

b) A simple steam power plant uses the Rankine Cycle. The data for it is

Boiler pressure	15 bar
Steam temperature from boiler	300^oC
Condenser pressure	0.1 bar
Net Power Output	1.1 MW

Calculate the following.

i. The cycle efficiency. (29.7 %)

ii. The steam flow rate. (1.3 kg/s)

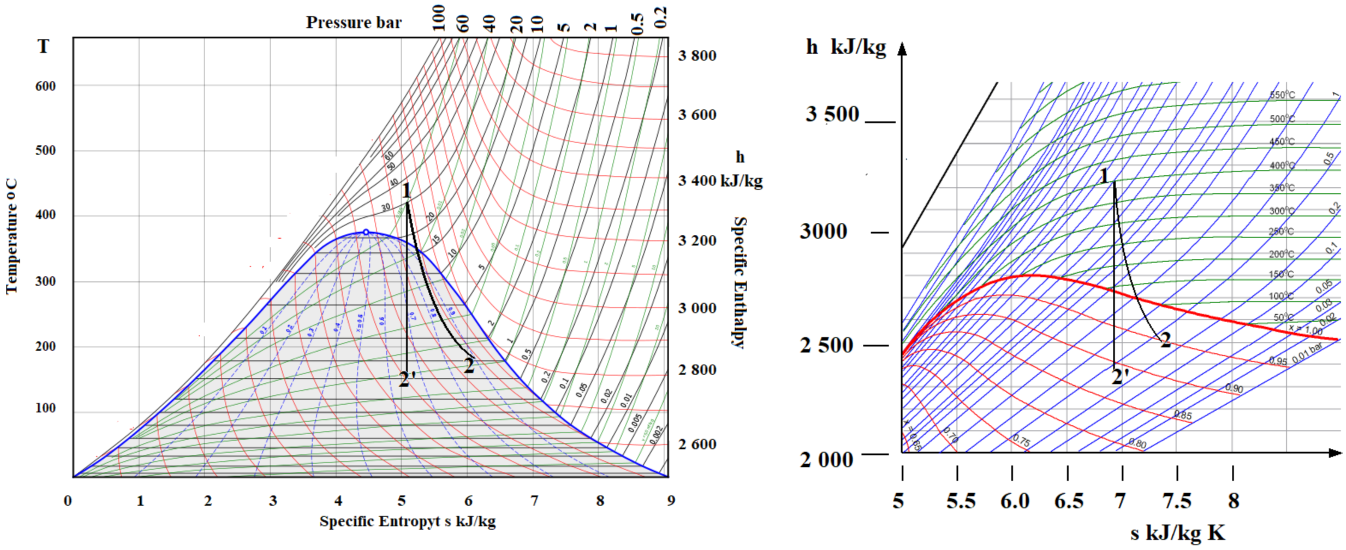
2. *Rankine Cycle with Friction*

2.1 *Revision of Isentropic Efficiency*

Isentropic efficiency has been covered in full in a previous tutorial. This revision is a reminder of how to to use it with steam expansions to determine the change in enthalpy. Friction during the expansion will:

- generate heat which is in effect a heat transfer
- increase the entropy
- make the final enthalpy bigger than it would otherwise be
- make the final temperature bigger than it would otherwise be if it is superheated vapour

The expansion is like this on a T – s and h – s diagram



The ideal change in enthalpy is $h_2' - h_1$ The actual change is $h_2 - h_1$

The ideal change in entropy is $s_2' - s_1 = 0$ The actual change is $s_2 - s_1$

The isentropic efficiency is defined as

$$\eta_{is} = \frac{\Delta h(\text{actual})}{\Delta h(\text{ideal})} = \frac{h_2 - h_1}{h_{2'} - h_1} \text{ for an expansion}$$

Note that for an expansion this produces a negative number on the top and bottom lines that cancels out.

You will need this to solve the advanced steam cycles following.

WORKED EXAMPLE No. 5

A turbine expands steam adiabatically from 70 bar and 500°C to 0.1 bar with an isentropic efficiency of 0.9. The power output is 35 MW. Determine the steam flow rate.

SOLUTION

The solution is easier with a h - s chart but we will do it with tables only.

h_1 = 3 410 kJ/kg at 70 bar and 500°C. s_1 = 6.796 kJ/kg K at 70 bar and 500°C.

For an ideal expansion from (1) to (2') we calculate the dryness fraction as follows.

$s_1 = s_2 = s_f + x's_{fg}$ at 0.1 bar. 6.796 = 0.649 + x'(7.5) x' = 0.8196

Note that you can never be certain if the steam will go wet. It may still be superheated after expansion. If x' came out to be larger than unity, then because this is impossible, it must be superheated and you need to deduce its temperature by referring to the superheat tables.

Now we find the ideal enthalpy h_2'

$h_2' = h_f + x'_{hfg}$ at 0.1 bar. h_2' = 192 + 0.8196(2 392) = 2 152.2 kJ/kg

Now we use the isentropic efficiency to find the actual enthalpy h_2.

$$\eta_{is} = \frac{\Delta h(\text{ideal})}{\Delta h(\text{actual})} = 0.9 = \frac{2\,152.2 - 3\,410}{h_2 - 3\,410}$$

$$h_2 = 2\,278.3 \text{ kJ/kg}$$

Now we may use the SFEE to find the mass flow rate.

$$\Phi + P = m(h_2 - h_1)$$

Φ = 0 since it is an adiabatic process.

P = -35 000 kW (out of system) = $\dot{m}$(2 278.3 – 3 410)

$\dot{m}$ = 30.926 kg/s

SELF ASSESSMENT EXERCISE No. 3

1. Steam is expanded adiabatically in a turbine from 100 bar and 600°C to 0.09 bar with an isentropic efficiency of 0.88. The mass flow rate is 40 kg/s.

   Calculate the enthalpy at exit and the power output.
   (Ans. 51 MW)

WORKED EXAMPLE No. 6

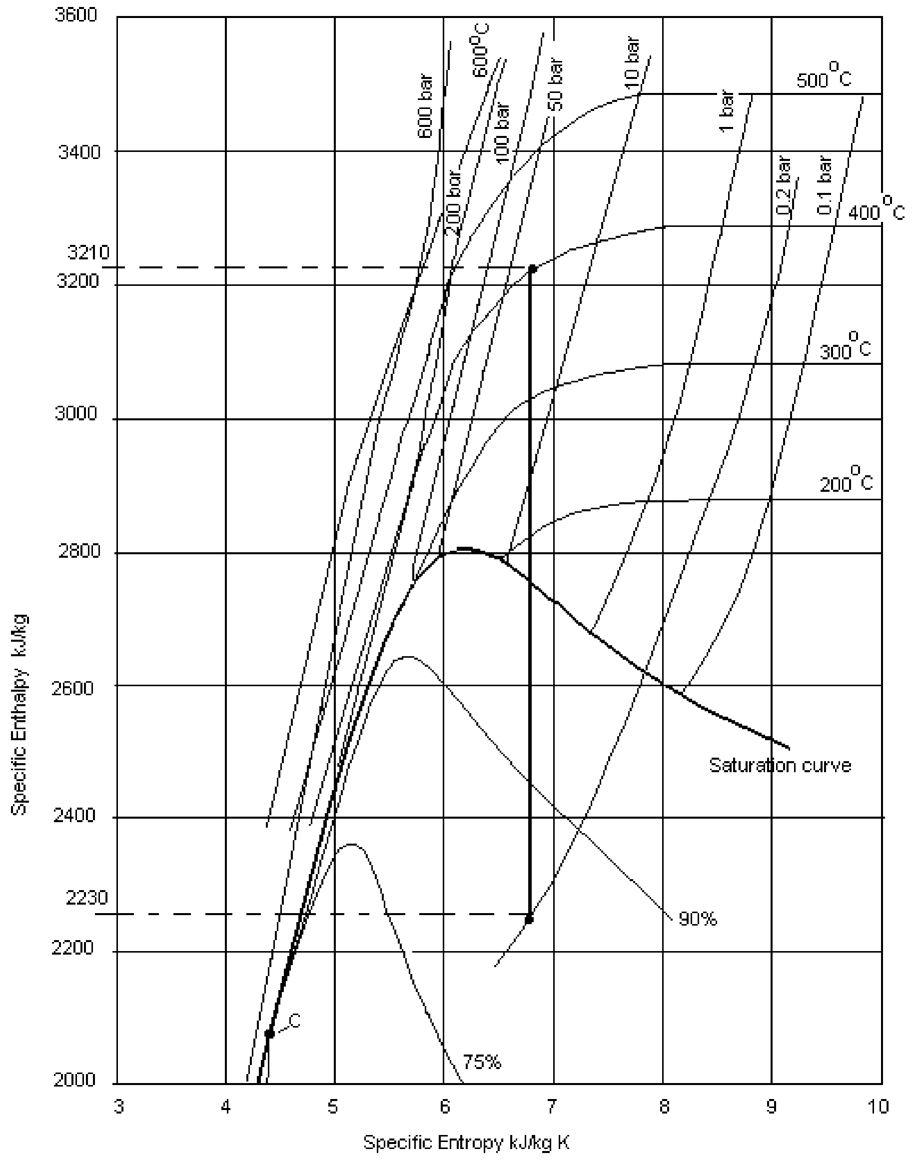
A simple steam power plant uses the Rankine cycle. The boiler supplies superheated steam to the turbine at 40 bar and 400^{o}C. The condenser operates at 0.2 bar and produces saturated water. The power input to the pump is negligible.

i. Calculate the thermal efficiency of the ideal cycle.

ii. Calculate the thermal efficiency when the turbine has an isentropic efficiency of 89%.

SOLUTION

The solution below uses the h - s chart to find the properties but you can verify them using on line methods.



Ideal Conditions

From the chart h_1 = 3 210 kJ/kg and h_2 = 2 230 kJ/kg/k

The ideal work output = 3 210 – 2 230 = 980 kJ/kg

When the power input to the pump is ignored, the power out is the net power and the enthalpy at inlet to the boiler is h_f at 0.2 bar

The heat input to the boiler = 3 210 – 251 = 2 959 kJ/kg

$$\eta_{th} = \frac{980}{2\,959} = 0.331 \text{ or } 33.1\%$$

Taking Account of Isentropic Efficiency

$$\eta_{is} = \frac{\text{Actual Work Output}}{\text{Ideal Work Output}} = \frac{\text{Actual Work Output}}{980}$$

$$0.89 = \frac{\text{Actual Work Output}}{980}$$

Actual Work Output = 980 × 0.89 = 872.2 kJ/kg

$$\eta_{th} = \frac{872.2}{2\,959} = 0.295 \text{ or } 29.5\%$$

SELF ASSESSMENT EXERCISE No. 4

1. A simple steam power plant uses the Rankine cycle. The boiler supplies superheated steam to the turbine at 100 bar and 550°C. The condenser operates at 0.05 bar and produces saturated water. The power input to the pump is negligible.

   i. Calculate the thermal efficiency of the ideal cycle. (42.5%)

   ii. Calculate the thermal efficiency when the turbine has an isentropic efficiency of 85%. (36.1%)

3. Back - Pressure and Pass - Out Turbines

It is assumed that the student is already familiar with steam cycles as this is necessary for this tutorial.

If an industry needs sufficient quantities of process steam (e.g. for sugar refining), then it becomes economical to use the steam generated to produce power as well. This is done with a steam turbine and generator and the process steam is obtained in two ways as follows.

- By exhausting the steam at the required pressure (typically 2 bar) to the process instead of to the condenser.

A turbine designed to do this is called a ***Back - Pressure Turbine*.**

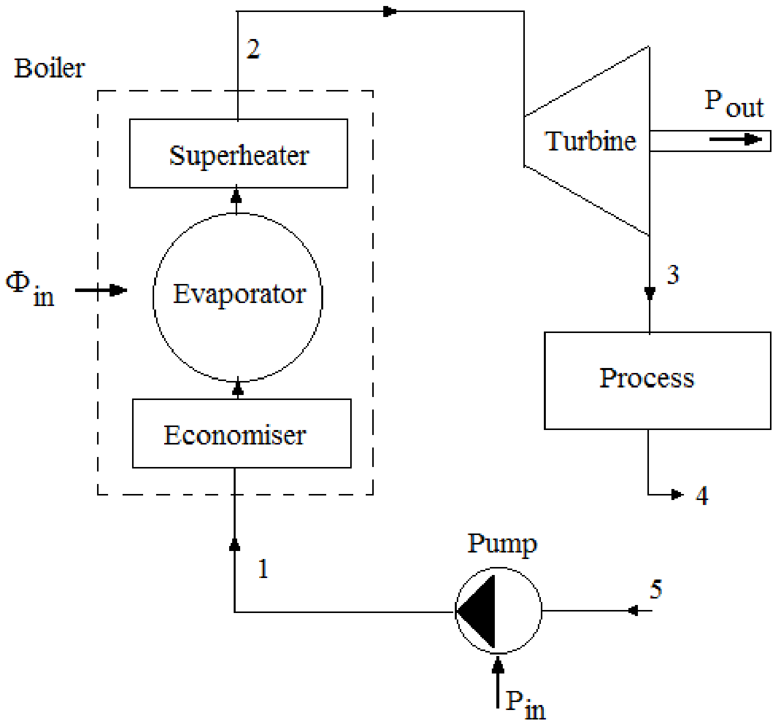
- By bleeding steam from an intermediate stage in the expansion process.

A turbine designed to do this is called a ***Pass - Out Turbine*.**

The steam cycle is standard except for these modifications.

2.1. Back-Pressure Turbines

The diagram shows the basic circuit. The cycle could use reheat as well but this is not normal.



WORKED EXAMPLE No. 7

For a steam circuit as shown previously, the boiler produces superheated steam at 50 bar and 400°C. This is expanded to 3 bar with an isentropic efficiency of 0.9. The exhaust steam is used for a process.

The returning feed water is at 1 bar and 40°C. This is pumped to the boiler. The water leaving the pump is at 40°C and 50 bar. The net power output of the cycle is 60 MW. Calculate the mass flow rate of steam.

SOLUTION

Referring to the cycle sketch previous for location points in the cycle we can find:

h_2= 3 196 kJ/kg $\qquad$ s_2 = 6.646 kJ/kg K

For an ideal expansion $\qquad$ $s_1 = s_2 = 6.646 = s_f + x's_{fg}$ at 3 bar

6.646 = 1.672 + x'(5.321) $\qquad$ x' = 0.935

$h_4 = h_f + x'h_{fg}$ at 3 bar $\qquad$ h_4 = 561 + 0.935(2 164) = 2 583.9 kJ/kg

Ideal change in enthalpy = 2 583.9 – 3 196 = -612 kJ/kg

Actual change in enthalpy = 0.9(-612) = -550.9 kJ/kg

The power output of the turbine is found from the steady flow energy equation so:

P = ṁ(-550.9) kW $\qquad$ P = -550.9 ṁ kW (output)

Next we examine the enthalpy change at the pump.

h_1 = 168 kJ/kg at 1 bar and 40°C $\qquad$ h_2 = 172 kJ/kg at 50 bar and 40°C.

Actual change in enthalpy = 172 - 169 = 3 kJ/kg

The power input to the pump is found from the steady flow energy equation so

P = -ṁ (3) kW = -3 ṁ kW(input)

Net Power output of the cycle = 60 MW Hence $\qquad$ 60 000 = 550.9 ṁ - 3 ṁ

ṁ = 109.51 kg/s

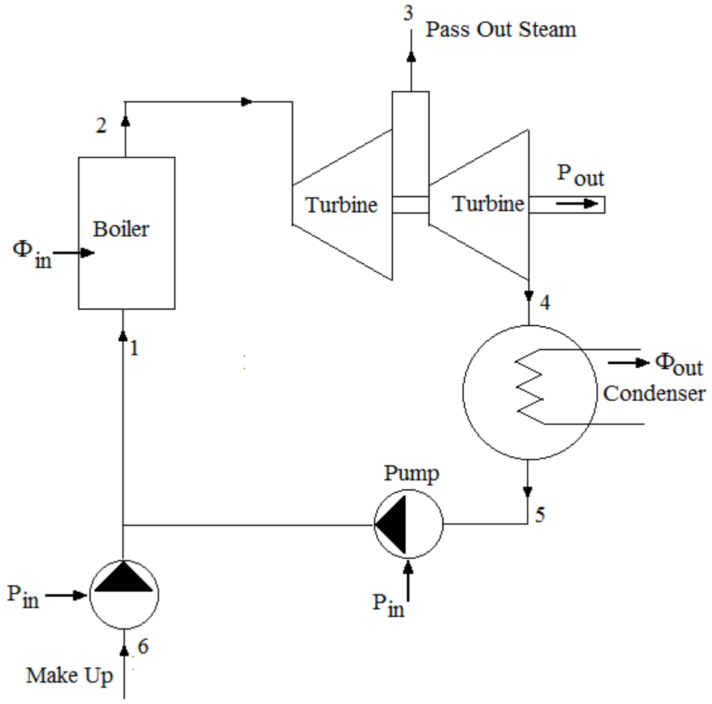
SELF ASSESSMENT EXERCISE No. 5

A back pressure steam cycle works as follows. The boiler produces 8 kg/s of steam at 40 bar and 500°C. This is expanded to 2 bar with an isentropic efficiency of 0.88. The pump is supplied with feed water at 0.5 bar and 30°C and delivers it to the boiler at 31°C and 40 bar.

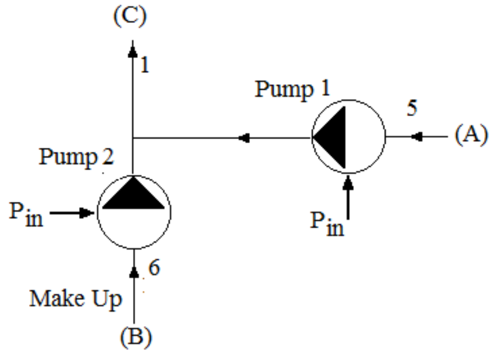
Calculate the net power output of the cycle. (Answer 5.24 MW)

2.2. Pass-Out Turbines

The circuit of a simple pass-out turbine plant is shown below. Steam is extracted between stages of the turbine for process use. The steam removed must be replaced by make up water at point 6.



In order to solve problems you need to study the energy balance at the feed pumps more closely so that the enthalpy at inlet to the boiler can be determined. Consider the pumps separately as shown below.



The balance of power is as follows.

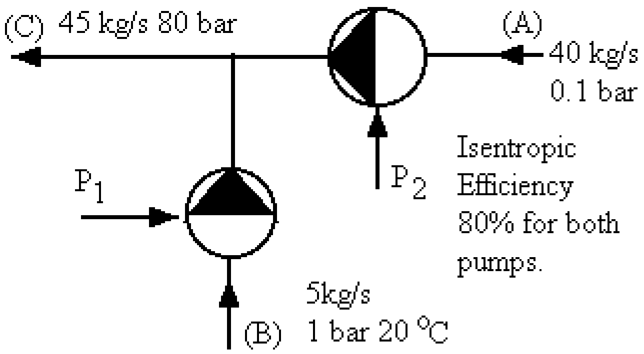
$$P_1 + P_2 = \text{increase in enthalpy per second.}$$

$$P_1 + P_2 = \dot{m}_C h_C - \dot{m}_A h_A - \dot{m}_B h_B$$

From this the value of h_C or the mass $\dot{m}_C$ may be determined. This is best shown with a worked example.

WORKED EXAMPLE No. 8

The circuit below shows the information normally available for a feed pump circuit. Determine the enthalpy at entry to the boiler.



SOLUTION

$$P_1(\text{ideal}) = (5)(0.001)(80 - 1)(100) = 39.5 \text{ kW}$$

$$P_1(\text{actual}) = 39.5/0.8 = 49.375 \text{ kW}$$

$$P_2(\text{ideal}) = (40)(0.001)(80 - 0.1)(100) = 319.6 \text{ kW}$$

$$P_2(\text{actual}) = 319.6/0.8 = 399.5 \text{ kW}$$

$$\text{Total power input} = 49.375 + 399.5 = 448.9 \text{ kW}$$

$$h_A = h_f = 192 \text{ kJ/kg at 0.1 bar}$$

h_B = 84 kJ/kg (from water tables or approximately h_f at 20°C) hence

$$448.9 = 45\,h_C - 40\,h_A - 5h_B$$

$$448.9 = 45\,h_C - 40(192) - 5(84)$$

$$h_C = 190 \text{ kJ/kg}$$

WORKED EXAMPLE No. 9

The following worked example will show you to solve these problems.

A pass out turbine plant works as shown previously. The boiler produces steam at 60 bar and 500^oC which is expanded through two stages of turbines. The first stage expands to 3 bar where 4 kg/s of steam is removed. The second stage expands to 0.09 bar. The isentropic efficiency is 0.9 for the overall expansion. Assume that the expansion is a straight line on the h - s chart.

The condenser produces saturated water. The make up water is supplied at 1bar and 20^oC. The isentropic efficiency of the pumps is 0.8. The net power output of the cycle is 40 MW. Calculate:

1. The flow rate of steam from the boiler.
2. The heat input to the boiler.
3. The thermal efficiency of the cycle.

SOLUTION

Turbine Expansion

h_3 = 3 421 kJ/kg from tables

h_5' = 2 165 kJ/kg using isentropic expansion and entropy.

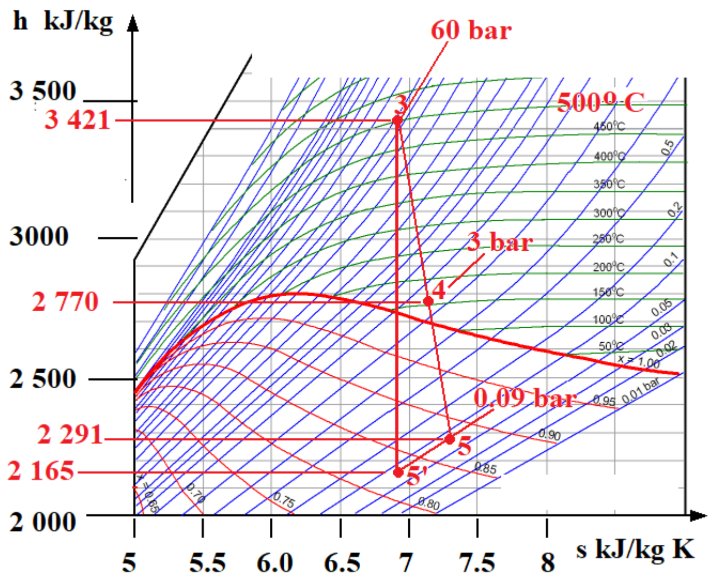
$$0.9 = \frac{3\ 421 - h_5}{3\ 41 - 2\ 165} \quad h_5 = 2\ 291\ kJ/kg$$

Sketching the process on the h - s chart as a straight line enables h_4 to be picked off at 3 bar.

h_4 = 2 770 kJ/kg.

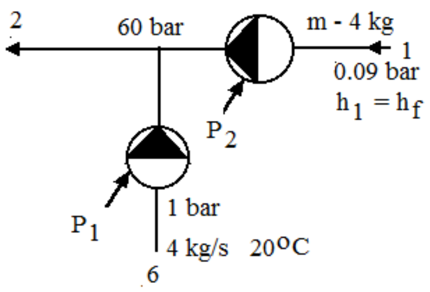
Power Output

$P_{out} = \dot{m}\,(h_3 - h_4) + (m - 4)(h_4 - h_5)$
$P_{out} = \dot{m}\,(3\ 421 – 2\ 770) + (\dot{m} - 4)(2\ 770 – 2\ 291)$
$P_{out} = 651\ \dot{m} + 479\ \dot{m} – 1\ 916$



Power Input

The power input is to the two feed pumps.



h_6 = 84 kJ/kg (water at 1 bar and 20°C)

$h_1 = h_f$ at 0.09 bar = 183 kJ/kg.

$$P_1 \text{ (ideal)} = \text{change in flow energy} = 4 \times 0.001 \times (60 - 1) \times 100 \text{ kW} = 23.6 \text{ kW}$$

$$P_1 \text{ (actual)} = 23.6 / 0.8 = 29.5 \text{ kW}$$

$$P_2\text{(actual)} = (\dot{m} - 4) \times 0.001 \times (60 - 0.09) \times 100/0.8 = 7.49\dot{m} - 29.96 \quad \text{kW}$$

Net Power

$$40\ 000 \text{ kW} = P_{out} - P_1 - P_2$$

$$40\ 000 = 651\dot{m} + 479\ \dot{m} - 1\ 916 - 29.5 - 7.49\ \dot{m} + 29.96$$

$$40\ 000 = 1\ 122.5\ \dot{m} - 1916 \quad \text{hence } \dot{m} = 37.34 \text{ kg/s}$$

Energy Balance on Pumps

$P_1 = 29.5$ kW $\qquad$ $P_2 = 249.4$ kW (using the value of m just found)

$$\dot{m}\ h_2 = (\dot{m} - 4)\ h_1 + P_1 + P_2$$

$$37.3\ h_2 = 33.34 \times 183 + 29.5 + 249.7$$

Hence

$$h_2 = 171 \text{ kJ/kg}$$

Heat Input

$$\text{Heat input} = \dot{m}\ (h_3 - h_2) = 121\ 355 \text{ kW}$$

Efficiency

$$\eta = \frac{40}{121.3} = 33\%$$

SELF ASSESSMENT EXERCISE No. 6

1. A steam turbine plant is used to supply process steam and power. The plant comprises an economiser, boiler, superheater, turbine, condenser and feed pump. The process steam is extracted between intermediate stages in the turbine at 2 bar pressure. The steam temperature and pressure at outlet from the superheater are 500^oC and 70 bar, and at outlet from the turbine the pressure is 0.1 bar. The overall isentropic efficiency of the turbine is 0.87 and that of the feed pump is 0.8.

   Assume that the expansion is represented by a straight line on the h-s chart. The make-up water is at 15^oC and 1 bar and it is pumped into the feed line with an isentropic efficiency 0.8 to replace the lost process steam.

   If due allowance is made for the feed pump-work, the net mechanical power delivered by the plant is 30 MW when the process steam load is 5 kg/s. Calculate the rate of steam flow leaving the superheater and the rate of heat transfer to the boiler including the economiser and superheater. Sketch clear T- s and h-s flow diagrams for the plant. (29.46 kg/s   95.1 MW)

2. The demand for energy from an industrial plant is a steady load of 60 MW of process heat at 117^oC and a variable demand of up to 30 MW of power to drive electrical generators. The steam is raised in boilers at 70 bar pressure and superheated to 500^oC. The steam is expanded in a turbine and then condensed at 0.05 bar. The process heat is provided by the steam bled from the turbine at an appropriate pressure, and the steam condensed in the process heat exchanger is returned to the feed water line.

   Calculate the amount of steam that has to be raised in the boiler. Assume an overall isentropic efficiency of 0.88 in the turbine. The expansion is represented by a straight line on the h-s diagram. Neglect the feed pump work.

   (Answer 36 kg/s).

3. Advanced Steam Cycles

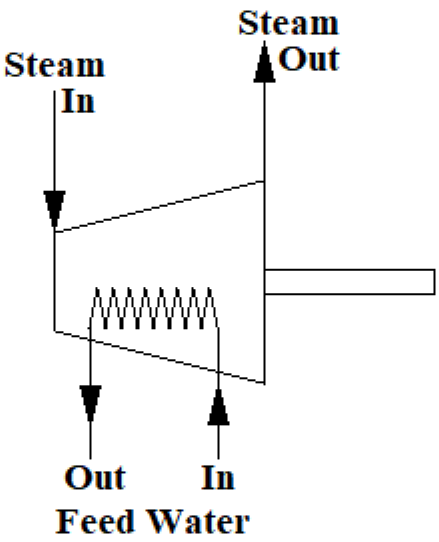
In this section you will extend your knowledge of steam cycles in order to show that the overall efficiency of the cycle may be optimised by the use of regenerative feed heating and steam re-heating.

Regenerative feed heating is a way of raising the temperature of the feed water before it reaches the boiler. It does this by using internal heat transfer within the power cycle. Steam is bled from the turbines at several points and used to heat the feed water in special heaters.

In this way the temperature of the feed water is raised along with the pressure in stages so that the feed water is nearly always saturated. The heat transfers in the heaters and in the boiler are conducted approximately isothermally.

Studies of the Carnot cycle should have taught you that an isothermal heat transfer is reversible and achieves maximum efficiency.

The ultimate way of conducting feed heating is to pass the feed water through a heat exchanger inside the turbine casing. In this way the temperature of the steam on one side of heat exchanger tubes is equal to the temperature of the water on the other side of the tubes. Although the temperature is changing as water and steam flow through heat exchanger, at any one point, the heat transfer is isothermal. If neither superheating nor undercooling is used then the heat transfers in the boiler and condenser are also isothermal and efficiencies equal to those of the Carnot cycle are theoretically possible.



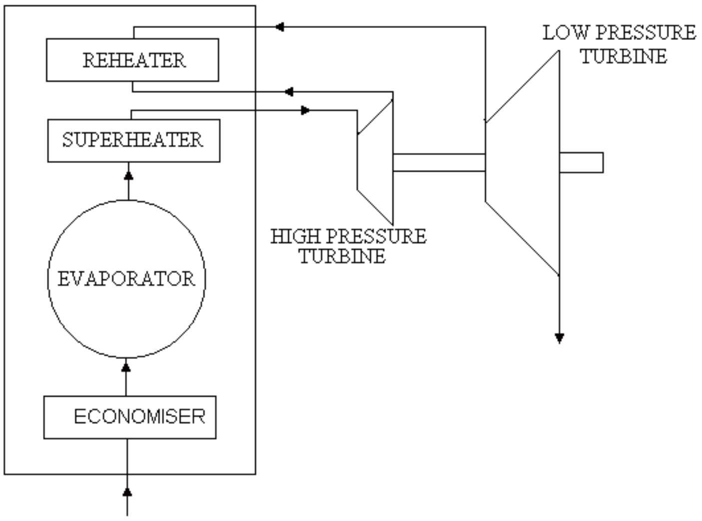
There are several reasons why this arrangement is impractical. Most of them are the same reasons why a Carnot cycle is impractical.

i. The steam would be excessively wet in the turbine.

ii. Placing a heat exchanger inside the turbine casing is mechanically impossible.

iii. The power output would be small even though the cycle efficiency would be high.

Steam reheating is another way of improving the thermodynamic efficiency by attempting to keep the steam temperature more constant during the heat transfer process inside the boiler.



Superheated steam is first passed through a high pressure turbine. The exhaust steam is then returned to the boiler to be reheated almost back to its original temperature. The steam is then expanded in a low pressure turbine. In theory, many stages of turbines and reheating could be done thus making the heat transfer in the boiler more isothermal and hence more reversible and efficient.

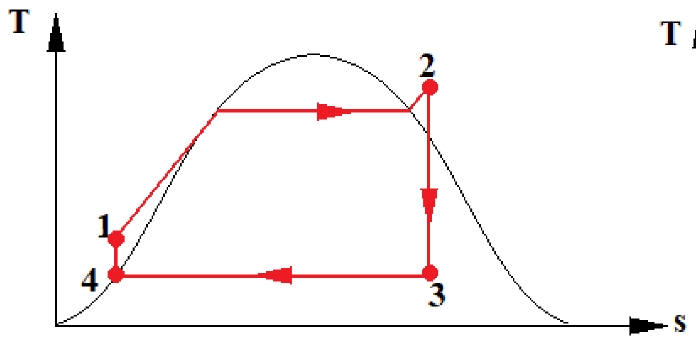
If a steam cycle used many stages of regenerative feed heating and many stages of reheating, the result would be efficiency similar to that of the Carnot cycle. Although practicalities prevent this happening, it is quite normal for an industrial steam power plant to use several stages of regenerative feed heating and one or two stages of reheating. This produces a significant improvement in the cycle efficiency.

There are other features in advanced steam cycles which further improve the efficiency and are necessary for practical operation. For example air extraction at the condenser, steam recovery from turbine glands, de-superheaters, de-aerators and so on. These can be found in details in textbooks devoted to practical steam power plant.

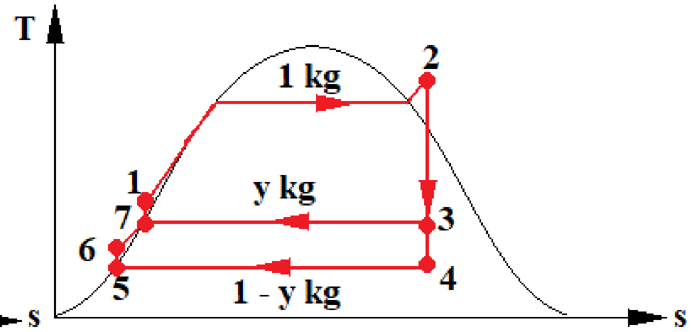
4. Feed Heating

4.1. Practical Designs

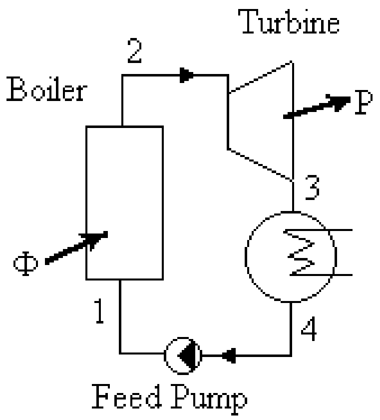
Practical feed heaters may be heat exchangers with indirect contact. The steam is condensed through giving up its energy and the hot water resulting may be inserted into the feed system at the appropriate pressure. The type which you should learn is the open or direct contact mixing type. The bled steam is mixed directly with the feed water at the appropriate pressure and condenses and mixes with the feed water. Compare a basic Rankine cycle with a similar cycle using one such feed heater.

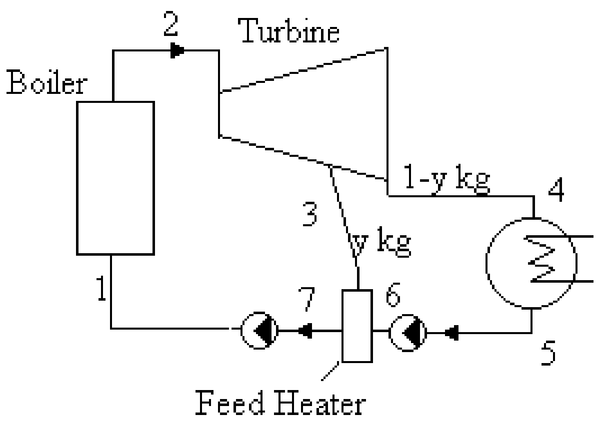


Rankine Cycle



Regenerative Cycle

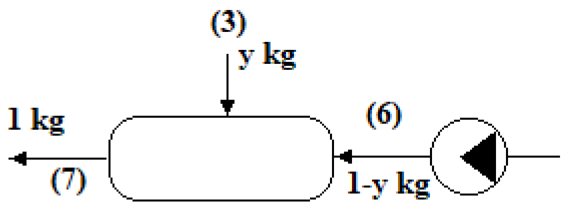




4. 2. Energy Balance for Mixing Feed Heater

Consider a simple mixing type feed heater. The bled steam at (3) is mixed directly with incoming feed water (6) resulting in hotter feed water (7).
Mass of bled steam = y kg
Mass of feed water entering= 1 - y kg
Doing an energy balance we find

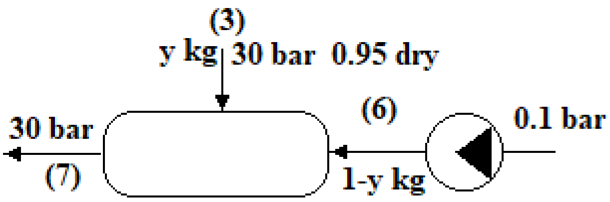
$$y\,h_3 + (1 - y)h_6 = h_7$$



WORKED EXAMPLE No. 10

A feed heater is supplied with condensate at 0.1 bar. The bled steam is taken from the turbine at 30 bar and 0.95 dry. Calculate the flow rate of bled steam needed to just produce saturated water at outlet.

SOLUTION



Assumptions

1. Energy input from pump is negligible.
2. No energy is lost.
3. The heater pressure is the same as the bled pressure.

In this case $h_6 = h_f$ at 0.1 bar = 192 kJ/kg

$h_7 = h_f$ at 30 bar = 1 008 kJ/kg

$h_3 = h_f + xh_{fg}$ at 30 bar

h_3 = 11 008 + 0.95(1 795) = 2 713.3 kJ/kg

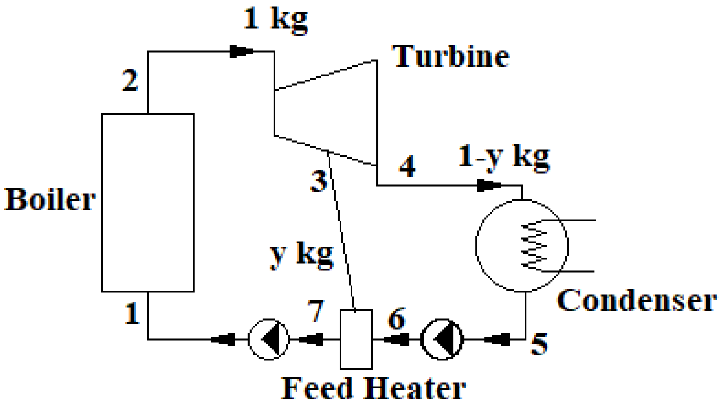
Energy Balance

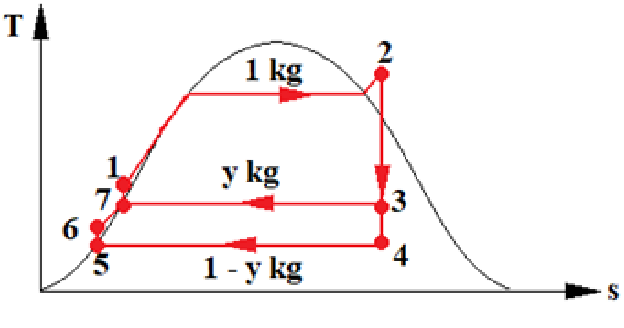
y(2 713.3) = (1 - y)(192) + 1 008

Hence y = 0.414 kg

Note that it is usual to calculate these problems initially on the basis of 1 kg coming from the boiler and returning to it.

4. 3. Cycle with One Feed Heater





Regenerative Cycle

If only one feed heater is used, the steam is bled from the turbine at the point in the expansion where it just becomes dry saturated and the saturation temperature is estimated as follows.

$$t_s(\text{bleed}) = \frac{t_s(\text{high pressure}) - t_s(\text{low pressure})}{2}$$

For example a cycle operating between 40 bar and 0.035 bar.

t_S (40 bar) = 250.3 ^oC

t_S (0.035 bar) = 26.7 ^oC

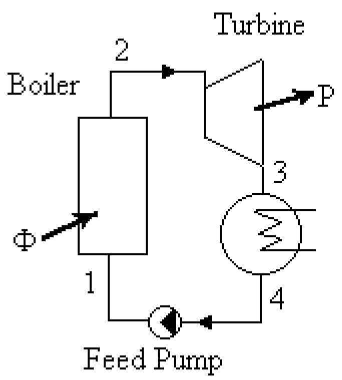
t_S (bleed) = (250.3 + 26.7)/2 = 138.5^oC

The pressure corresponding to this is 3.5 bar so this is the bleed pressure.

WORKED EXAMPLE No. 11

A Rankine cycle works between 40 bar, 400°C at the boiler exit and 0.035 bar at the condenser. Calculate the efficiency with no feed heating. Assume isentropic expansion. Ignore the energy term at the feed pump.

SOLUTION



$h_2 = 3\ 214$ kJ/kg $\qquad s_2 = 6.769$ kJ/kg K

$$s_2 = s_3 = 0.391 + 8.13\ x$$

$$x = 0.785$$

$$h_3 = h_f + x\ h_{fg} = 112 + 0.785(2438) = 2\ 024.6 \text{ kJ/kg}$$

$$h_4 = h_f \text{ at } 0.035 \text{ bar} = 112 \text{ kJ/kg}$$

$$\Phi = h_2 - h_1 = 3\ 102 \text{ kJ/kg into boiler.}$$

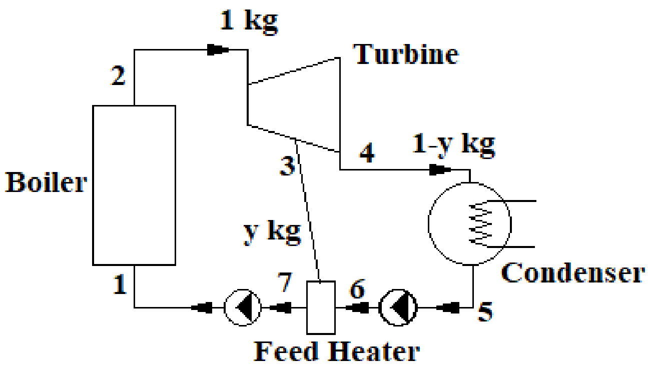
$$P = h_2 - h_3 = 1\ 189.4 \text{ kJ/kg (out of turbine)}$$

$$\eta_{th} = \frac{P_{out}}{\Phi_{in}} = \frac{1\ 189.4}{3\ 102} = 38.3\%$$

WORKED EXAMPLE No. 12

Repeat the last example but this time there is one feed heater.

SOLUTION



The bleed pressure was calculated in an earlier example and was 3.5 bar.

$s_2 = s_3 = 6.769$ kJ/kg K $= 1.727 + 5.214x_3$ $\quad$ $x_3 = 0.967$ (not quite dry).

$h_3 = h_f + x\,h_{fg} = 584 + 0.967(2\,148) = 2\,661$ kJ/kg $\quad$ $h_7 = h_f$ at 3.5 bar $= 584$ kJ/kg

Neglecting pump power

$h_6 = h_5 = h_f = 112$ kJ/kg $\qquad$ $h_1 = h_7 = 584$ kJ/kg

Conducting an energy balance we have

$$yh_3 + (1- y)\,h_6 = h_7 \qquad \text{hence } y = 0.185 \text{ kg}$$

$$\Phi = h_2 - h_1 = 2\,630 \text{ kJ/kg into boiler.}$$

Rather than work out the power from the turbine data, we may do it by calculating the heat transfer rate from the condenser as follows.

$$\Phi_{out} = (1- y)(h_4 - h_5) = 0.815(2\,024.6 - 112) = 1\,558.8 \text{ kJ/kg}$$

$$P = \Phi_{in} - \Phi_{out} = 1\,072 \text{ kJ/kg (out of turbine)}$$

$$\eta = \frac{P}{\Phi_{in}} = 40.8\%$$

Note that the use of the feed heater produced an improvement of 2.5 % in the thermodynamic efficiency.

SELF ASSESSMENT EXERCISE No. 7

A simple steam plant uses a Rankine cycle with one regenerative feed heater. The boiler produces steam at 70 bar and 500^{o}C. This is expanded to 0.1 bar isentropically. Making suitable assumptions, calculate the cycle efficiency. (Answer 41.8%)

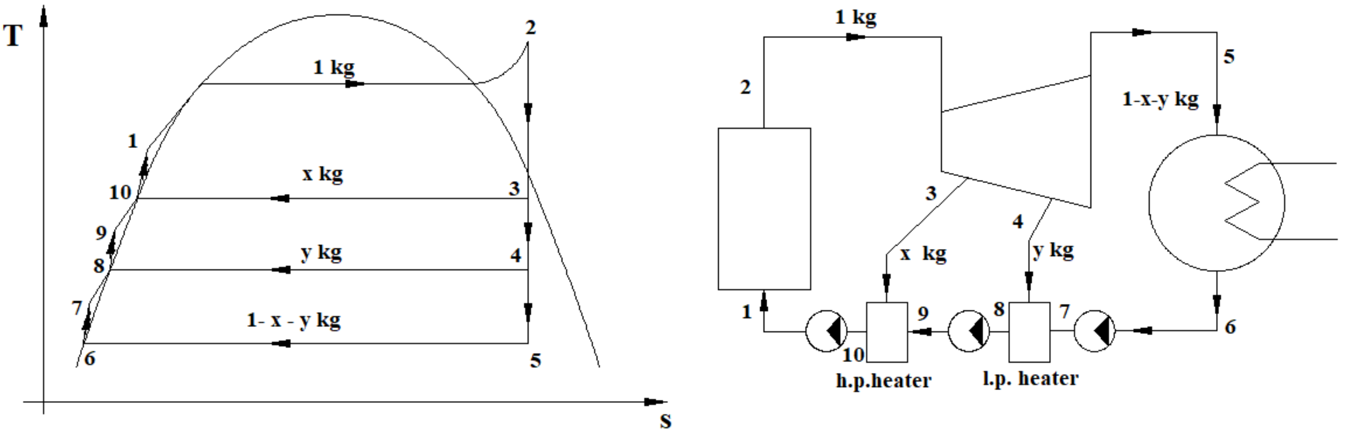
4. 4. Cycle with Two Feed Heaters

When two (or more) feed heaters are used, the efficiency is further increased. The principles are the same as those already explained. The mass of bled steam for each heater must be determined in turn starting with the high pressure heater. It is usual to assume isentropic expansion that enables you to pick off the enthalpy of the bled steam from the h-s chart at the pressures stated.

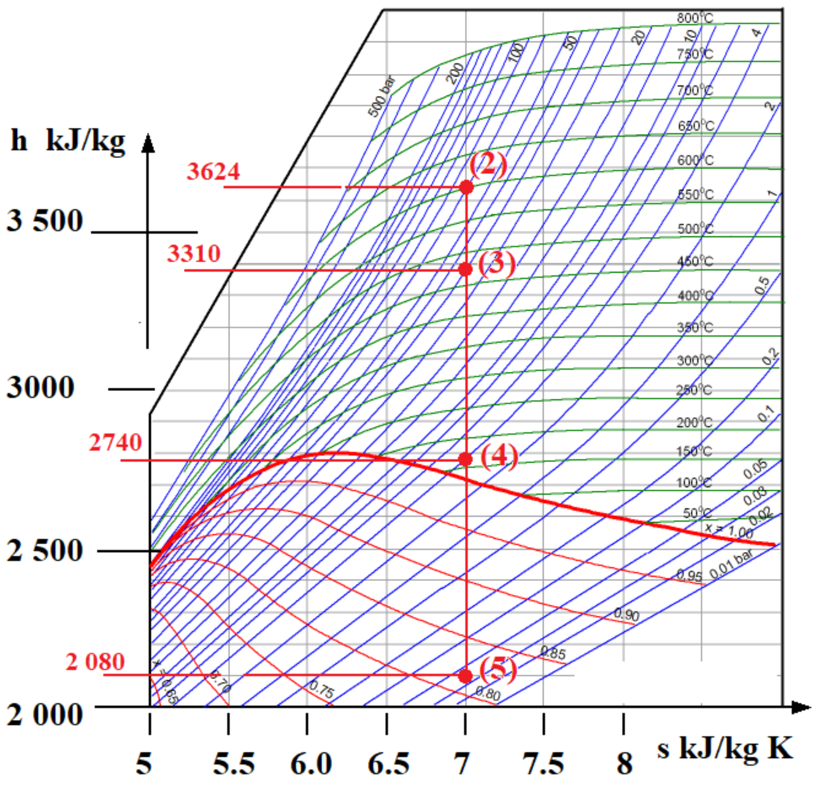
WORKED EXAMPLE No. 13

A steam power plant works as follows. The boiler produces steam at 100 bar and 600^{o}C. This is expanded isentropically to 0.04 bar and condensed. Steam is bled at 40 bar for the h. p. heater and 4 bar for the l. p. heater. Solve the thermodynamic efficiency.

SOLUTION



Regenerative Cycle



Draw the expansion (2) to (5) on the h – s chart and find

h_2 = 3 624 kJ/g h_3 = 3 310 kJ/kg h_4 = 2 740 kJ/kg h_5 = 2 080 kJ/g

Ignoring the energy input from the pump we find:

$$h_1 = h_{10} = h_f \ 40 \text{ bar} = 1\,087 \text{ kJ/kg}$$

$$h_9 = h_8 = h_f \ 4 \text{ bar} = 605 \text{ kJ/kg}$$

$$h_7 = h_6 = h_f \ 0.04 \text{ bar} = 121 \text{ kJ/kg}$$

H. P. Heater

$$xh_3 + (1-x)h_9 = h_{10} \quad 3\,310x + 605(1-x) = 1\,087 \qquad x = 0.178 \text{ kg}$$

L.P. Heater

$$(1-x)\,h_8 = yh_4 + (1-x-y)h_7 \qquad 0.822(605) = 2\,740\,y + (0.822 - y)(121) \qquad y = 0.152 \text{ kg}$$

Boiler

Heat input $\Phi_{in} = h_2 - h_1 = 3\,624 - 1\,087 = 2\,537$ kJ/kg

Condenser

Heat output $\Phi_{out} = (1-x-y))(h_5 - h_6) = 0.67(2\,080 - 121) = 1\,312.5$ kJ/kg

Power Output

$$P = \Phi_{in} - \Phi_{out} = 1\,224.5 \text{ kJ/kg}$$

$$\eta = \frac{P}{\Phi_{in}} = 48.3\%$$

SELF ASSESSMENT EXERCISE No. 8

1. Explain how it is theoretically possible to arrange a regenerative steam cycle which has cycle efficiency equal to that of a Carnot cycle.

   In a regenerative steam cycle steam is supplied from the boiler plant at a pressure of 60 bar and a temperature of 500^{o}C. Steam is extracted for feed heating purposes at pressures of 30 bar and 3.0 bar and the steam turbine exhausts into a condenser operating at 0.035 bar.

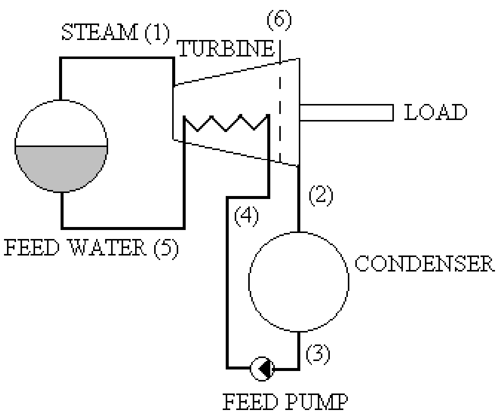
   Calculate the appropriate quantities of steam to be bled if the feed heaters are of the open type, and find the cycle efficiency; base all calculations on unit mass leaving the boiler.
   Assume isentropic expansion in the turbine and neglect the feed pump work.
   (Answers 0.169 kg/s, 0.145 kg/s and 45 %)

2. The sketch shows an idealised regenerative steam cycle in which heat transfer to the feed water in the turbine from the steam is reversible and the feed pump is adiabatic and reversible. The feed water enters the pump as a saturated liquid at 0.03 bar, and enters the boiler as a saturated liquid at 100 bar, and leaves as saturated steam.

   Draw a T-s diagram for the cycle and determine, not necessarily in this order, the dryness fraction in state 2, the cycle efficiency and the work per unit mass.

   (Answers 0.269 kg/s, 50% and 658.5 kJ/kg).

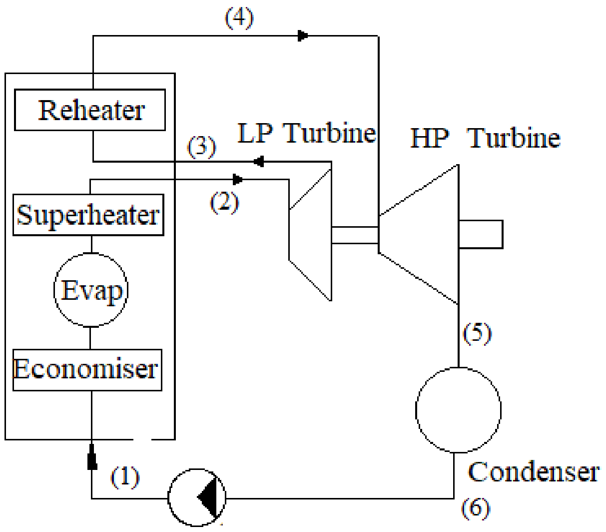
   Outline the practical difficulties that are involved in realising this cycle and explain how regenerative cycles are arranged in practice.



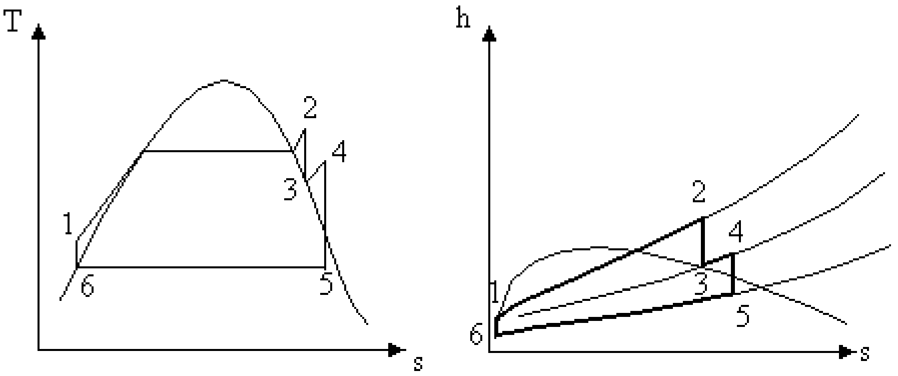
   Note point (6) is the point in the steam expansion where the feed water enters and presumably the temperatures are equal. There is further expansion from (6) to (2).

__5. Reheat Cycles__

We shall only examine cycles with one stage of reheating and two turbine stages, high pressure and low pressure. You should refer to text books on practical steam turbine layouts to see how low, medium and high pressure turbines are configured and laid out in order to produce axial force balance on the rotors. The diagram below shows a basic circuit with one stage of reheating.



You should be proficient at sketching the cycle on a T - s diagram and a h - s diagram. They are shown below for the cycle shown above.



The calculations for this cycle are not difficult. You need only take into account the extra heat transfer in the reheater.

WORKED EXAMPLE No. 14

A reheat cycle works as follows. The boiler produces 30 kg/s at 100 bar and 400^oC. This is expanded isentropically to 50 bar in the h. p. turbine and returned for reheating in the boiler. The steam is reheated to 400^oC. This is then expanded in the l.p. turbine to the condenser which operates at 0.2 bar. The condensate is returned to the boiler as feed.

Calculate the net power output and the cycle efficiency.

SOLUTION

$h_6 = h_f$ at 0.2 bar = 251 kJ/kg

h_2 = 3 097 kJ/kg at 100 bar and 400^oC.

From the h-s chart we find

h_3 = 2 930 kJ/kg $\quad$ h_4 = 3 196 kJ/kg $\quad$ h_5 = 2 189 kJ/kg

If we ignore the feed pump power then

Φin at boiler = $30(h_2 - h_1) + 30(h_4 - h_3)$ = 93 360 kW or 93.360 MW

Φ_{out} at condenser = $30(h_5 - h_6)$ = 58.14 kW

$P_{(net)} = \Phi_{in} - \Phi_{out}$ =35 220 kW or 35.22 MW

$$\eta = \frac{P_{net}}{\Phi_{in}} = 37.7\ \%$$

SELF ASSESSMENT EXERCISE No. 9

1. Repeat the worked example but this time do not ignore the feed pump term and assume an isentropic efficiency of 90% for each turbine and 80% for the pump.
   (Answers 32.3 MW 33.8%)

2. A water-cooled nuclear reactor supplies dry saturated steam at a pressure of 50 bar to a two-cylinder steam turbine. In the first cylinder the steam expands with an isentropic efficiency of 0.85 to a pressure of 10 bar, the power generated in this cylinder being 100 MW. The steam then passes at a constant pressure of 10 bar through a water separator from which all the water is returned to the reactor by mixing it with the feed water. The remaining dry saturated steam then flows at constant pressure through a reheater in which its temperature is raised to 250^oC before it expands in the second cylinder with an isentropic efficiency of 0.85 to a pressure of 0.1 bar, at which it is condensed before being returned to the reactor.

   Calculate the cycle efficiency and draw up an energy balance for the plant. Neglect the feed pump work.
   (Answer 30.3%)

3. Steam is raised in a power cycle at the supercritical pressure of 350 bar and at a temperature of 600^oC. It is then expanded in a turbine to 15 bar with an overall isentropic efficiency of 0.90. At that pressure some steam is bled to an open regenerative feed heater, and the remainder of the steam is, after reheating to 600^oC, expanded in a second turbine to the condenser pressure of 0.04 bar, again with an isentropic efficiency of 0.90. The feed pumps each have an overall isentropic efficiency of 0.90.

   Calculate the amount of steam to be bled into the feed heater, making the usual idealising assumptions. Also calculate the cycle efficiency. Use the h-s chart wherever possible and do not neglect feed pump work.

   (Answers 0.279 kg/s and 47%)