UNIT 61: ENGINEERING THERMODYNAMICS

Unit code: D/601/1410

QCF level: 5

Credit value: 15

OUTCOME 4
STEAM AND GAS TURBINE POWER PLANT

TUTORIAL No. 10 – ISENTROPIC EFFICIENCY

4 **Understand the operation of steam and gas turbine power plant**

Principles of operation: impulse and reaction turbines; condensing; pass-out and back pressure steam turbines; single and double shaft gas turbines; regeneration and re-heat in gas turbines; combined heat and power plants

Circuit and property diagrams: circuit diagrams to show boiler/heat exchanger; superheater; turbine; condenser; condenser cooling water circuit; hot well; economiser/feedwater heater; condensate extraction and boiler feed pumps; temperature - entropy diagram of Rankine cycle

Performance characteristics: Carnot, Rankine and actual cycle efficiencies; turbine isentropic efficiency; power output; use of property tables and enthalpy-entropy diagram for steam

On completion of this tutorial you should be able to do the following.

- Explain the effect of friction on steam and gas expansions in turbines.
- Solve steam cycle problems taking into account friction.
- Solve gas turbine cycle problems taking into account friction.

1. ISENTROPIC EFFICIENCY

1.1 THE EFFECT OF FRICTION

When a fluid is expanded or compressed with fluid friction occurring, a degree of irreversibility is present. The result is the generation of internal heat equivalent to a heat transfer. **This always results in an increase in entropy**.

Figure 1 shows expansion and compression processes on a T-s diagram. In the case of vapour, the line crosses the saturation curve. In the case of gas, the process takes place well away from the saturation curve and indeed the saturation curve would not normally be shown for gas processes. Note that in every case, the ideal process is from (1) to (2') but the real process is from (1) to (2).

Friction does the following.

- Increases the entropy.
- Increases the enthalpy.
- The true process path on property diagrams is always to the right of the ideal process.
- When the final point (2) is in the gas (superheat) region, the result is a hotter temperature.
- When the final point (2) is in the wet region, the result is a dryer vapour.

Gas and vapour processes should be described by sketching them on an appropriate property diagram and these effects of friction clearly shown.

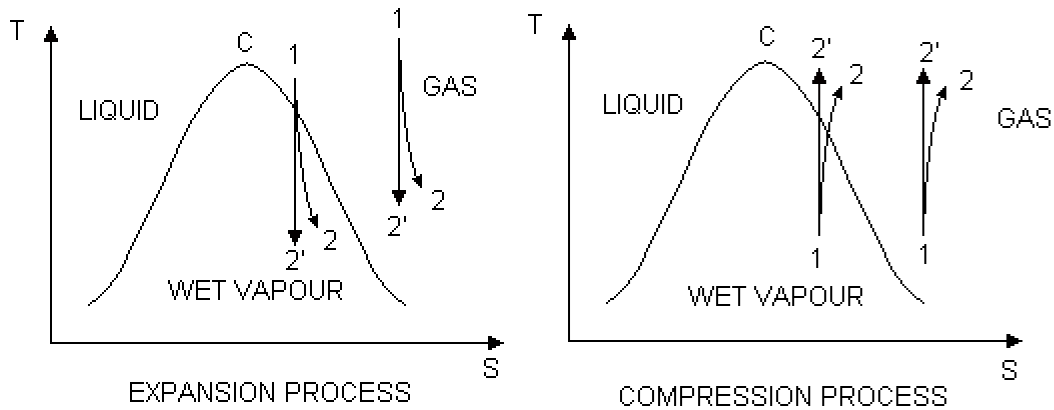

Figure 1

The same points are also apparent on the h-s diagram. Figure 2 shows a vapour expansion from (1) to (2) with the ideal being from (1) to (2'). Note how it ends up dryer at the same pressure with an increase in entropy. Vapour is not normally compressed so this is not shown.

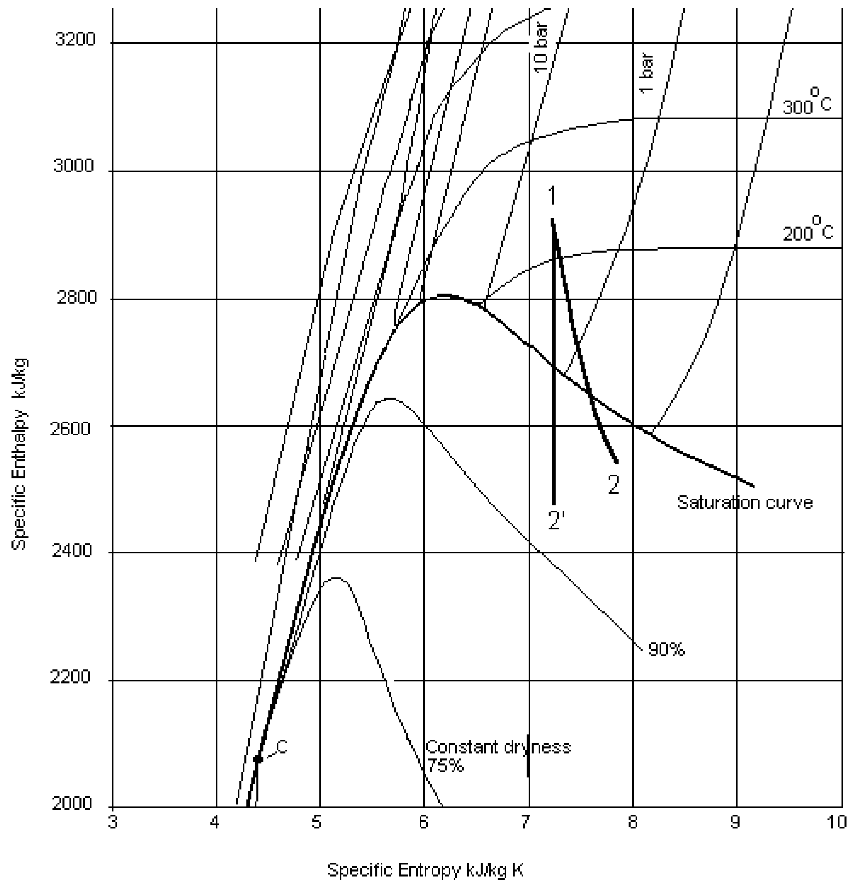

Figure 2

1.2 ISENTROPIC EFFICIENCY

An ideal reversible adiabatic process would be constant entropy as shown on the diagrams from (1) to (2').

When friction is present, the process is (1) to (2).

The ideal change in enthalpy is $\Delta h\text{ (ideal)} = h_2' - h_1$

The actual change is $\Delta h\text{(actual)} = h_2 - h_1$

The isentropic efficiency is defined as follows.

EXPANSION $$\eta_{is} = \frac{\Delta h\text{ (actual)}}{\Delta h\text{ (ideal)}} = \frac{h_2 - h_1}{h_{2'} - h_1}$$

COMPRESSION $$\eta_{is} = \frac{\Delta h\text{ (ideal)}}{\Delta h\text{ (actual)}} = \frac{h_{2'} - h_1}{h_2 - h_1}$$

For gas only $h = c_pT$

EXPANSION $$\eta_{is} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

COMPRESSION $$\eta_{is} = \frac{T_{2'} - T_1}{T_2 - T_1}$$

Note that for an expansion negative changes are obtained on the top and bottom lines that cancel.

If the work transfer rate is only due to the change in enthalpy we may also define isentropic efficiency as follows.

$$\eta_{is} = \frac{\text{Actual Power Output}}{\text{Ideal Power Output}} \quad \text{for a turbine}$$

$$\eta_{is} = \frac{\text{Ideal Power Input}}{\text{Actual Power Input}} \quad \text{for a compressor}$$

WORKED EXAMPLE No.1

A turbine expands steam adiabatically from 70 bar and 500°C to 0.1 bar with an isentropic efficiency of 0.9. The power output is 35 MW. Determine the steam flow rate.

SOLUTION

The solution is easier with a h-s chart but we will do it with tables only.

h_1 = 3410 kJ/kg at 70 bar and 500°C.

s_1 = 6.796 kJ/kg K at 70 bar and 500°C.

For an ideal expansion from (1) to (2') we calculate the dryness fraction as follows.

$s_1 = s_2 = s_f + x's_{fg}$ at 0.1 bar.

$6.796 = 0.649 + x'(7.5) \qquad x' = 0.8196$

Note that you can never be certain if the steam will go wet. It may still be superheated after expansion. If x' came out to be larger than unity, then because this is impossible, it must be superheated and you need to deduce its temperature by referring to the superheat tables.

Now we find the ideal enthalpy $h_{2'}$

$h_{2'} = h_f + x'_{hfg}$ at 0.1 bar.

$h_{2'} = 192 + 0.8196(2392) = 2152.2$ kJ/kg

Now we use the isentropic efficiency to find the actual enthalpy h_2.

$$\eta_{is} = \frac{\Delta h(ideal)}{\Delta h(actual)}$$

$$0.9 = \frac{2152.2 - 3410}{h_2 - 3410}$$

$$h_2 = 2278.3 \text{ kJ/kg}$$

Now we may use the SFEE to find the mass flow rate.

$\Phi + P = m(h_2 - h_1)$

$\Phi = 0$ since it is an adiabatic process.

P = -35 000 kW (out of system) = m(2278.3-3410)

m = 30.926 kg/s

WORKED EXAMPLE No.2

A turbine expands gas adiabatically from 1 MPa and 600°C to 100 kPa. The isentropic efficiency is 0.92. The mass flow rate is 12 kg/s. Calculate the power output.

c_p = 1.005 kJ/kg K c_v = 0.718 kJ/kg K.

SOLUTION

The process is adiabatic so the ideal temperature $T_{2'}$ is given by

$$T_{2'}=T_1(r_p)^{1-1/\gamma}$$

r_p is the pressure ratio
$r_p = p_2/p_1 = 0.1$

$$\gamma = c_p /c_v = 1.005/0.718 = 1.4$$

$$T_{2'} =873(0.1)^{1-1/1.4} = 451.9 \text{ K}$$

Now we use the isentropic efficiency to find the actual final temperature.

$$\eta_{is} =(T_2 - T_1)/(T_{2'} - T_1)$$

$$0.92 = (T_2 - 873)/(451.9 - 873)$$

$$T_2 = 485.6 \text{ K}$$

Now we use the SFEE to find the power output.

$$\Phi + P = m\, c_p(T_2 - T_1)$$

The process is adiabatic $\Phi = 0$.

P = 12(1.005)(485.6 - 873) = - 4672 kW (out of system)

WORKED EXAMPLE No.3

A simple steam power plant uses the Rankine cycle. The boiler supplies superheated steam to the turbine at 40 bar and 400^{o}C. The condenser operates at 0.2 bar and produces saturated water. The power input to the pump is negligible.

i. Calculate the thermal efficiency of the ideal cycle.

ii. Calculate the thermal efficiency when the turbine has an isentropic efficiency of 89%.

SOLUTION

The solution is easier with a h-s chart.

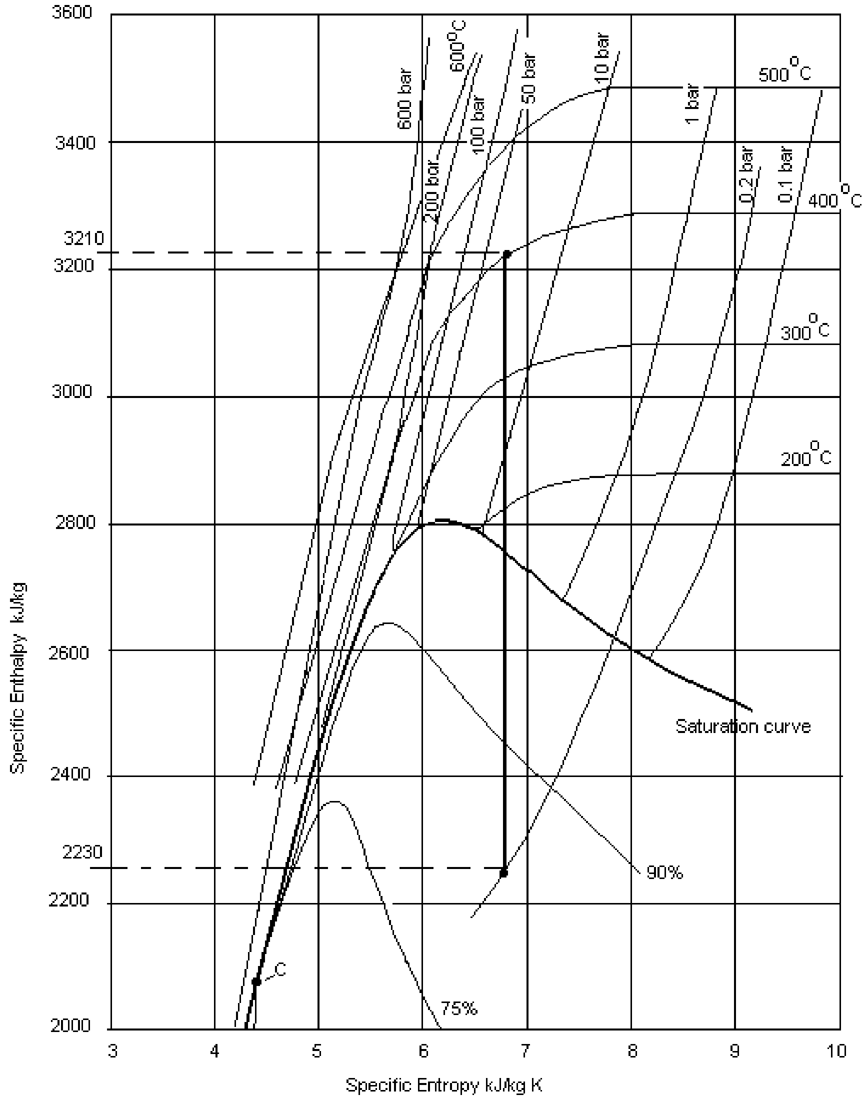

Figure 3

IDEAL CONDITIONS

From the chart h_1= 3210 kJ/kg and h_2 = 2230 kJ/kg/k

The ideal work output = 3210 – 2230 = 980 kJ/kg

When the power input to the pump is ignored, the power out is the net power and the enthalpy at inlet to the boiler is h_f at 0.2 bar

The heat input to the boiler = 3210 – 251 = 2959 kJ/kg

$$\eta_{th} = \frac{980}{2959} = 0.331 \text{ 0r } 33.1\%$$

TAKING ACCOUNT OF ISENTROPIC EFFICIENCY

$$\eta_{is} = \frac{\text{Actual work output}}{\textit{Ideal} \text{ work output}} = \frac{\text{Actual work output}}{980}$$

$$0.89 = \frac{\text{Actual work output}}{980}$$

$$\text{Actual work output} = 980 \text{ x } 0.89 = 872.2 \text{ kJ/kg}$$

$$\eta_{th} = \frac{872.2}{2959} = 0.295 \text{ or } 29.5\%$$

WORKED EXAMPLE No.4

A simple gas turbine uses the Joule cycle. The pressure ratio is 6.5. The air temperature is 300 K at inlet to the compressor and 1373 K at inlet to the turbine. The adiabatic index is 1.4 throughout and the specific heat capacities may be considered constant.

i. Calculate the thermal efficiency of the ideal cycle.

ii. Calculate the thermal efficiency when the turbine and compressor has an isentropic efficiency of 90%.

iii. Sketch the cycle on a T-s diagram showing the effect of friction.

SOLUTION

IDEAL CYCLE

Compressor $\mathrm{T_2} = 300(6.5)^{0.286} = 512.4\ \mathrm{K}$

Turbine $\mathrm{T_4} = 1373(6.5)^{-0.286} = 803.8\ \mathrm{K}$

$$\eta_{\mathrm{th}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{803.8 - 300}{1373 - 512.4} = 0.415 \text{ or } 41.5\%$$

INCLUDING ISENTROPIC EFFICIENCY

Compressor $\eta_{\mathrm{is}} = 0.9 = \dfrac{512.4 - 300}{\mathrm{T_2} - 300}$

$\mathrm{T_2} = 536\ \mathrm{K}$

Turbine $\eta_{\mathrm{is}} = 0.9 = \dfrac{1373 - \mathrm{T_4}}{1373 - 803.8}$

$\mathrm{T_4} = 860.7\ \mathrm{K}$

$$\eta_{\mathrm{th}} = 1 - \frac{\mathrm{T_4} - \mathrm{T_1}}{\mathrm{T_3} - \mathrm{T_2}} = 1 - \frac{860.7 - 300}{1373 - 536} = 0.33 \text{ or } 33\%$$

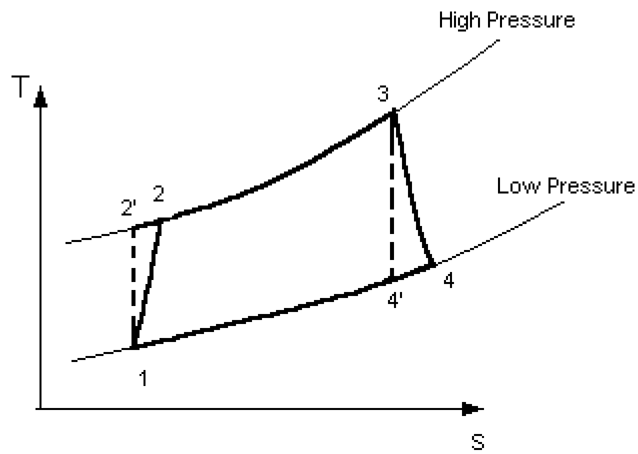

Fig. 4

SELF ASSESSMENT EXERCISE No.1

1. Steam is expanded adiabatically in a turbine from 100 bar and 600°C to 0.09 bar with an isentropic efficiency of 0.88. The mass flow rate is 40 kg/s. Calculate the power output.
   (51 mw)

2. A compressor takes in gas at 1 bar and 15°C and compresses it adiabatically to 10 bar with an isentropic efficiency of 0.89. The mass flow rate is 5 kg/s. Calculate the final temperature and the power input. c_p = 1.005 kJ/kg K γ=1.4
   (590 K and 1.51 MW)

3. A turbine is supplied with 3 kg/s of hot gas at 10 bar and 920°C. It expands adiabatically to 1 bar with an isentropic efficiency of 0.92. Calculate the final temperature and the power output. c_p = 1.005 kJ/kg K γ=-1.4
   (663 K and 1.6 MW)

4. A turbine is supplied with 7 kg/s of hot gas at 9 bar and 850°C that it expands adiabatically to 1 bar with an isentropic efficiency of 0.87. Calculate the final temperature and the power output. c_p = 1.005 kJ/kg K γ=1.4
   (667 K and 3.2 MW)

5. A simple steam power plant uses the Rankine cycle. The boiler supplies superheated steam to the turbine at 100 bar and 550°C. The condenser operates at 0.05 bar and produces saturated water. The power input to the pump is negligible.

   i. Calculate the thermal efficiency of the ideal cycle. (42.5%)

   ii. Calculate the thermal efficiency when the turbine has an isentropic efficiency of 85%. (36.1%)

6. A simple gas turbine uses the Joule cycle. The pressure ratio is 7.5. The air temperature is 288 K at inlet to the compressor and 1400 K at inlet to the turbine. The adiabatic index is 1.4 throughout and the specific heat capacities may be considered constant.

   i. Calculate the thermal efficiency of the ideal cycle. (43.8%)

   ii. Calculate the thermal efficiency when the turbine and compressor has an isentropic efficiency of 92%. (36.9%)