THERMODYNAMICS

TUTORIAL 10

TURBO COMPRESSOR THEORY

This tutorial is set at QCF level 5 and 6

On completion of this tutorial you should be able to:

- Explain the principles of Turbo Compressors
- Define the parameters that define the performance of Turbo Compressors
- Explain and use Vector Diagrams to determine the power produced by flow over the compressor vanes.
- Calculate the performance of Turbo Compressors.

Contents

1. *Turbo Compressors*

Turbo air compressors come in many forms such as centrifugal and axial. Centrifugal compressors will not be covered here. You will find a good tutorial on this subject at this link.
http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node92.html

2. *Multiple Stage Axial Flow Type*

The purpose of a turbo compressor is to put energy into the air (or other gas) and raise the pressure from inlet to outlet. In essence the theory is the same as for turbines but in reverse. The moving row of blades (rotor) precedes the stationary row (stator). The stator works as a nozzle in reverse to slow the air velocity down and convert the kinetic energy into pressure. The multi stage compressor will be designed so that the absolute velocity of the air is returned to same value after each stage but the pressure and temperature will rise over each stage.

With the process being the opposite of a turbine, the blade angles are the other way with respect to the tangential direction. The size of the compressor depends on the flow rate required for the application and the number of stages depends on the overall pressure rise required. Typically the stator blades are fixed to the casing. The blade height decreases in the direction of the flow as the volume is compressed. The picture shows a typical design. Usually there is a row of inlet guide vanes before the first stage to direct the air flow at the best angle for the first moving row.

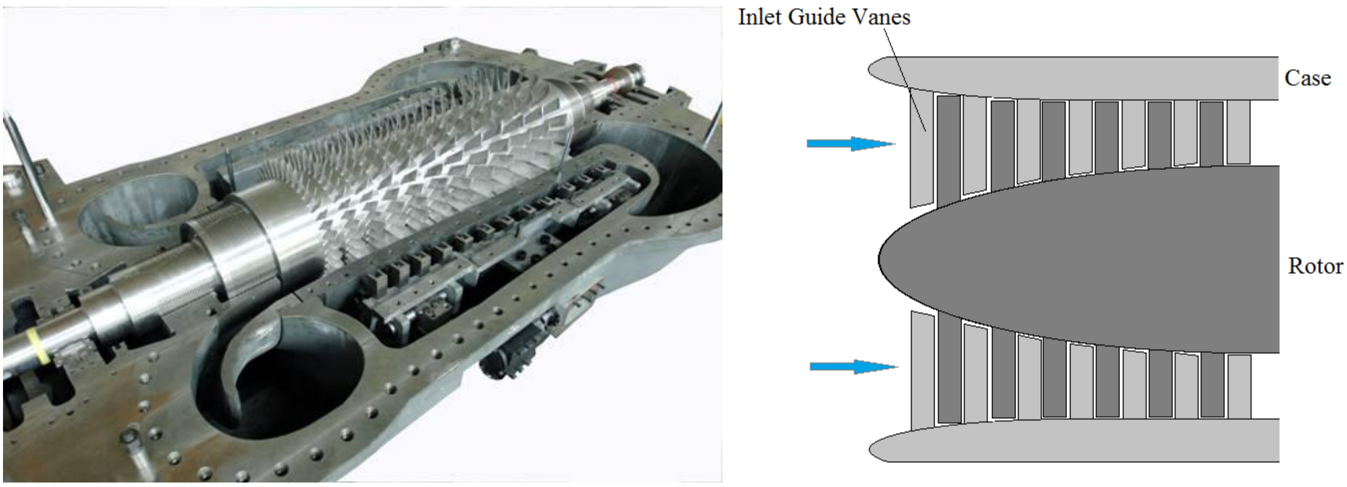


Figure 1

2.1 *Blade Vector Diagram*

Figure 2 shows the configuration and vector diagrams for the first stage.

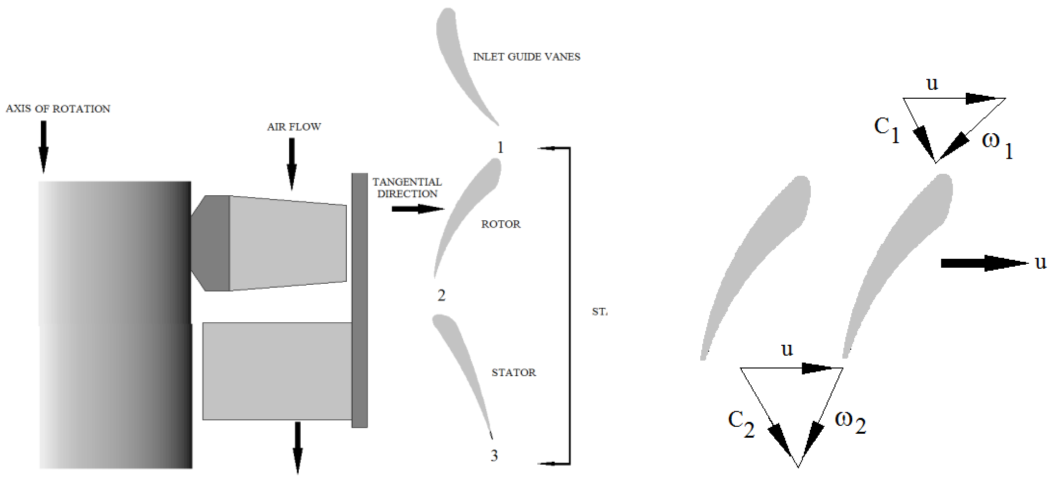


Figure 2

The combined vector diagram may be drawn as shown. It is normal to have a constant axial velocity as shown. Note that here the blade angles are shown measured relative to the tangential direction but it is common practice to use the angles relative to the axial direction so be prepared for this in other publications.

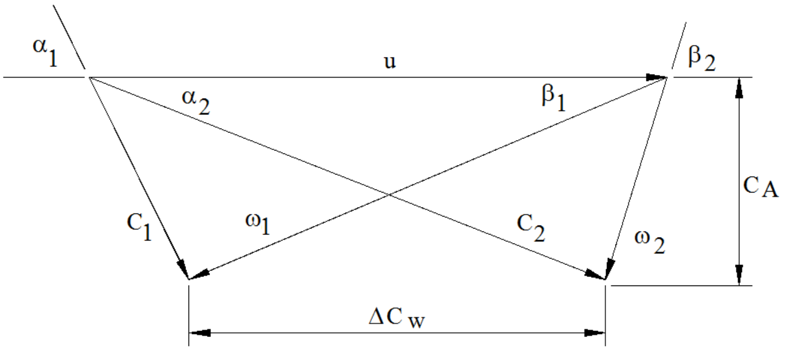


Figure 3

2.2 Degree of Reaction

The degree of reaction is given by

$$R = \frac{h_2 - h_1}{h_3 - h_1}$$

Remember the change in enthalpy over the rotor is equal to the change in relative kinetic energy.

$$h_2 - h_1 = \frac{\omega_1^2}{2} - \frac{\omega_2^2}{2} = \frac{1}{2}(\omega_1^2 - \omega_2^2)$$

Applying energy conservation to the rotor we have:

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} - P$$

P is the diagram power per unit mass flow (energy added by the rotor)

$$P = (h_2 - h_1) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right) = \frac{1}{2}(\omega_1^2 - \omega_2^2) + \frac{1}{2}\ (C_2^2 - C_1^2)$$

Applying the energy equation over the stage we have:

$$h_1 + \tfrac{C_1^2}{2} = h_3 + \tfrac{C_3^2}{2} - P \qquad P = (h_3 - h_1) + \tfrac{1}{2}(C_3^2 - C_1^2)$$

But if the velocity is returned to the same value after each stage then $C_1 = C_3$ so $P = (h_3 - h_1)$

$$R = \frac{h_2 - h_1}{h_3 - h_1} = \frac{(\omega_1^2 - \omega_2^2)}{2P} = \frac{(\omega_1^2 - \omega_2^2)}{(\omega_1^2 - \omega_2^2) + (C_2^2 - C_1^2)}$$

And again if $\alpha_2 = \beta_1$ and $\alpha_1 = \beta_2$ then $\omega_1 = C_2$ and $\omega_2 = C_1$

$$R = \frac{(\omega_1^2 - \omega_2^2)}{2(\omega_1^2 - \omega_2^2)} = \frac{1}{2}$$

This has the usual advantage of rotor and stator blades being identical angles.

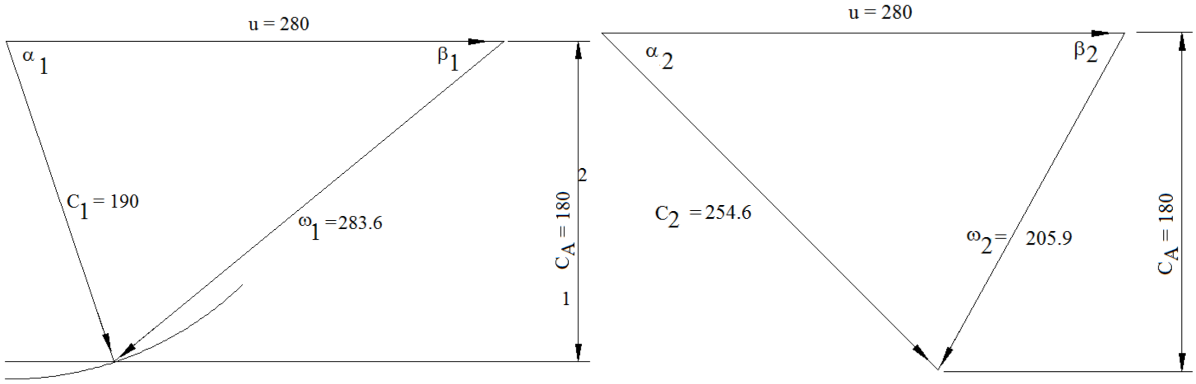
WORKED EXAMPLE No. 1

A stage of an axial flow air compressor is a moving stage followed by a stator. The axial velocity of the air is constant through the stage at 180 m/s and the tangential velocity of the blades at the mean diameter is 280 m/s. The inlet angle of the stator blades is swept back at 45°. The exit angle is swept forward. The inlet pressure and temperature are 100 kPa and 300 K respectively. The compression is ideal and adiabatic and follows the law $pV^{1.4} = C$. Draw the inlet and exit vector diagrams and determine:

i. the blade angles
ii. the diagram power for a unit mass flow rate
iii. the temperature rise over the stage taking the specific heat c_p = 1 005 J/kg K
iv. the theoretical output pressure
v. the degree of reaction

SOLUTION

In order to draw the inlet vector diagram start with u = 280 and mark off the axial velocity 180. Draw an arc for C_1 = 190 and determine the interception point with the arc and then complete the diagrams as shown. (there are two possible intercepts the one shown is swept forward). Calculate or scale the angles.



INLET OUTLET

Figure 4

$$A_1 = \sin^{-1}\left(\frac{180}{190}\right) = 71.3^{o}$$

$$\omega_1^2 = C_1^2 + u^2 - 2uC_1\cos\alpha_1 = 190^2 + 280^2 - 2 \times 280 \times 190\cos(71.3^{o}) = 80\,439.6$$

$$\omega_1 = 283.6 \text{ m/s}$$

$$\beta_1 = \sin^{-1}\left(\frac{180}{283.6}\right) = 39.4^{o}$$

In order to draw the outlet vector diagram α_2 must match the inlet angle of the stator blade so it is 45° swept forward as shown.

$$C_2 = \frac{180}{\sin 45^{o}} = 254.6 \text{ m/s}$$

$$\omega_2^2 = C_2^2 + u^2 - 2uC_2\cos\alpha_2 = 254.6^2 + 280^2 - 2 \times 280 \times 25.6\cos(45^{o}) = 4\,240.7$$

$$\omega_2 = 205.9 \text{ m/s}$$

$$\beta_1 = \sin^{-1}\left(\frac{180}{205.9}\right) = 61^{o}$$

From the combined diagram we measure or calculate ΔC_w

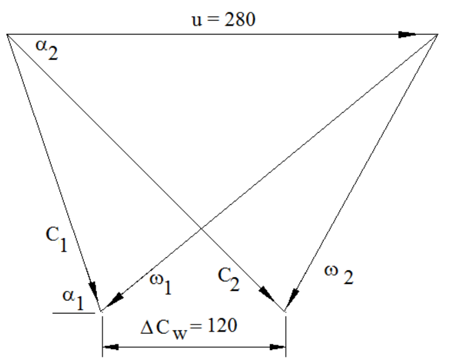


Figure 5

$\Delta C_w = C_2 \cos(\alpha_2) - C_1 \cos(\alpha_1)$

$\Delta C_w = 254.6 \cos(45^o) - 190 \cos(71.3) = 120$ m/s

D.P. = $\dot{m}\, u\, \Delta C_w = 1 \times 280 \times 120 = 33\,600$ W or 33.6 kW

This is power into the air.
D.P. = $\dot{m}\, c_p\, \Delta T$ hence $\Delta T = 33\,600/1005 = 33.4$ K

$p_1 V_1^{1.4} = p_2 V_2^{1.4}$ so $p_2/p_1 = (V_1/V_2)^{1.4}$

Since pV = mRT then

$$\frac{p_2}{p_1} = \left(\frac{mRT_1 p_2}{mRT_2 p_1}\right)^{1.4} = \left(\frac{T_1 p_2}{T_2 p_1}\right)^{1.4} = \left(\frac{T_1}{T_2}\right)^{1.4}\left(\frac{p_2}{p_1}\right)^{1.4}$$

$$\left(\frac{p_2}{p_1}\right) - \left(\frac{p_2}{p_1}\right)^{1.4} = \left(\frac{p_2}{p_1}\right)^{-0.4} = \left(\frac{T_1}{T_2}\right)^{1.4} = \left(\frac{300}{333.4}\right)^{1.4} = 0.8626$$

$$\left(\frac{p_2}{100}\right)^{-0.4} = 0.8626 \qquad \frac{p_2}{100} = 1.447 \qquad p_2 = 144.7 \text{ kPa}$$

$$R = \frac{(\omega_1^2 - \omega_2^2)}{(\omega_1^2 - \omega_2^2) + (C_2^2 - C_1^2)} = \frac{(283.6^2 - 205.9^2)}{(283.6^2 - 205.9^2) + (254.6^2 - 190^2)} = 0.57$$

SELF ASSESSMENT EXERCISE No. 1

All the stages of an axial flow air compressor are to have 50% reaction and the air is to enter the rotor in an axial direction at 60 m/s. The blade velocity is 80 m/s at the mean diameter. The inlet pressure and temperature are 100 kPa and 300 K respectively. The compression is ideal and adiabatic and follows the law $pV^{1.4} = C$. Draw the inlet and exit vector diagrams and determine:

i. the blade angles ($\alpha_2 = \beta_1 = 90^o$ $\alpha_1 = \beta_1 = 36.87^o$)
ii. the diagram power for a unit mass flow rate (6.4 kW)
iii. the temperature rise over the first stage taking the specific heat $c_p = 1005$ J/kg K (6.37 K)
iv. the theoretical pressure ratio after the first stage (107.6 kPa)

Sketch the blade configuration.

3. *Polytropic or Small Stage Efficiency*

This is an alternative way of approaching isentropic efficiency. In this method, the compression is supposed to be made up of many stages, each raising the pressure a small amount. The theory applies to any type of compressor.

For an adiabatic gas compression the law of compression $pV^{\gamma} = C$ and the gas law $pV/T = C$ may be combined to give

$$\frac{T}{p^{1-1/\gamma}} = C$$

and this can be expressed in differential form

$$dT = C\left(\frac{\gamma-1}{\gamma}\right)p^{-1/\gamma}dp$$

Divide by p

$$\frac{dT}{p} = C\left(\frac{\gamma-1}{\gamma}\right)p^{-1/\gamma}\frac{dp}{p}$$

$$\frac{dT}{p^{1-1/\gamma}} = C\left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} \quad \text{but } \frac{T}{C} = p^{1-1/\gamma}$$

Differentiate

$$\frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p}$$

For an isentropic compression, let the final temperature be designated T_2' and the change in temperature be $\Delta T'$.

$$\frac{dT'}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} \ldots\ldots(1)$$

$$\eta_{is} = \left(\frac{T_2' - T_1}{T_2 - T_1}\right)$$

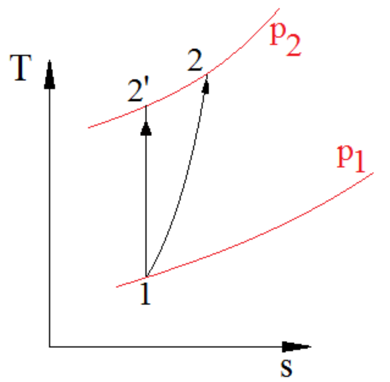


Figure 6

Let the change be infinitesimally small such that

$$T_2 = T_1 + dT \quad \text{and} \quad T_2' = T_1 + dT'$$

$$\text{The Polytropic Efficiency is defined as } \eta_\infty = \frac{dT'}{dT}$$

If we think of the compression as being made up of many tiny steps each with the same value of η_∞.

$$dT' = \eta_\infty dT \qquad \frac{dT'}{T} = \eta_\infty\frac{dT}{T}$$

Integrate

$$\left(\frac{\gamma-1}{\gamma}\right)\int_{p_1}^{p_2}\frac{dp}{p} = \eta_\infty\int_{T_1}^{T_2}\frac{dT}{T} \qquad \left(\frac{\gamma-1}{\gamma}\right)\ln\frac{p_2}{p_1} = \eta_\infty\ln\frac{T_2}{T_1}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_2}{T_1}\right)^{\eta_\infty} \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\,\eta_\infty}}$$

T_1 is the starting temperature and T_2 is the final temperature. The isentropic efficiency is

$$\eta_{is} = \frac{T_2' - T_1}{T_2 - T_1}$$

Adiabatic Process

$$\frac{T_2'}{T_1} = r^{\frac{\gamma-1}{\gamma}} \quad T_2' = T_1 r^{\frac{\gamma-1}{\gamma}}$$

Polytropic Process

$$\frac{T_2'}{T_1} = r^{\frac{n-1}{n}} \quad T_2' = T_1 r^{\frac{n-1}{n}} = T_1 r^{\frac{\gamma-1}{\gamma\eta_\infty}}$$

Substitute

$$\eta_{is} = \frac{T_1 r^{\frac{\gamma-1}{\gamma}} - T_1}{T_1 r^{\frac{\gamma-1}{\gamma\eta_\infty}} - T_1} = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1}$$

Compare

$$\frac{T_2'}{T_1} = r^{\frac{n-1}{n}} \text{ with } \frac{T_2'}{T_1} = r^{\frac{\gamma-1}{\gamma\eta_\infty}} \text{ and } \eta_\infty = 1 \text{ as expected for an isentropic compression}$$

This theory may be applied to expansions as well as compressions. It may also be applied to expansions in nozzles. In steam work, it is more usual to use the ***Reheat Factor***, which is based on the same principle.

WORKED EXAMPLE 2

A compressor draws in air at 15°C and 0.3 bar. The air is compressed to 1.6 bar with a polytropic efficiency of 0.86. Determine the temperature and the isentropic efficiency. Take $\gamma = 1.4$

SOLUTION

$$T_2 = T_1 r^{\frac{\gamma-1}{\gamma\eta_\infty}} = 288\left(\frac{1.6}{0.3}\right)^{\frac{1.4-1}{1.4\times0.86}} = 2.88 \times (5.33)^{0.332} = 502 \text{ K}$$

$$T_2' = T_1 r^{\frac{\gamma-1}{\gamma}} = 2.88 \times (5.33)^{0.286} = 464.5\text{K}$$

$$\eta_{is} = \frac{464.5 - 288}{502 - 288} = 0.825$$

SELF ASSESSMENT EXERCISE No. 2

1. Show that for any compression process the overall efficiency is given by

$$\eta_{is} = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1}$$

Where η_∞ is the polytropic efficiency.

Determine the index of compression for a gas with an adiabatic index of 1.4 and a polytropic efficiency of 0.9. (1.465)

Determine the overall efficiency when the pressure compression ratio is 4/1 and 8/1. (0.879 and 0.866)

2. A compressor draws in air at 223.3 K temperature and 0.265 bar pressure. The compression ratio is 6. The polytropic efficiency is 0.86. Determine the temperature after compression. Take $\gamma = 1.4$ (405 K)