

MECHANICS OF SOLIDS

STATICS - NON-CONCURRENT COPLANAR FORCE SYSTEMS

This is set at the British National level 3.

On completion of this tutorial you should be able to explain and use static forces in engineering structures.

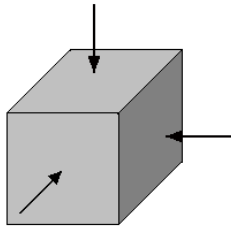
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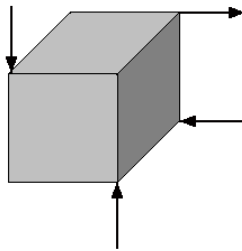
1. Introduction

This tutorial is about the kind of forces that act on a body.

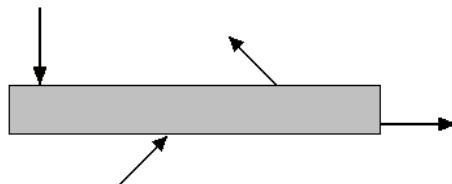
Concurrent Forces means that the forces all act at a single point like that illustrated where all the force acting on the cube pass through the centre.



Non - concurrent means that the forces do not act at a single point. In this case the cube is likely to revolve as a result of the moments created.

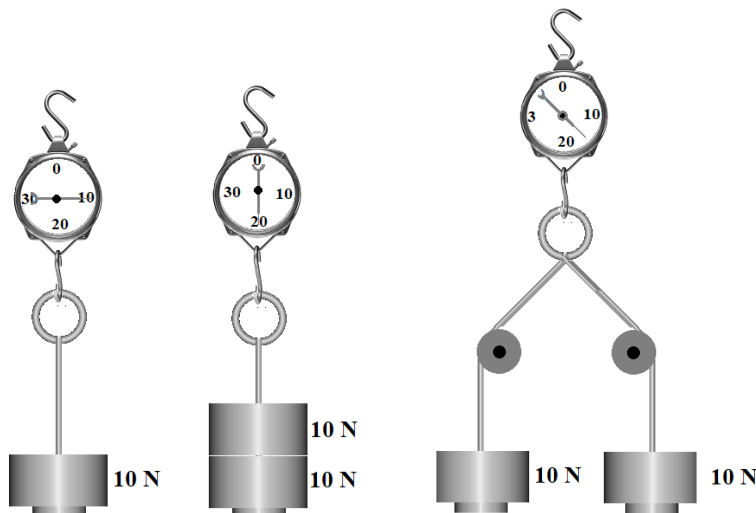


Non - concurrent coplanar means that the force act in different lines but only on one plane, in other words two dimensions only. There will be a resultant force and a resultant moment of force but only in the plane of the paper.



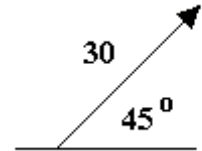
2 Force Vectors

When we use ordinary numbers we can add them, subtract them, and multiply them and so on but often there are problems where the use of ordinary numbers does not work and we need to use vectors instead. Consider the weighing systems below. When the weights hang vertically the scale indicates the true weight. This is not always the case. When two 10 N weights are hung on the hangers, the instrument reads 15 N not 20 N. This is because the two weights are no longer pulling in the same direction but in two different directions. (The pulleys take some of the force on the spindles). Clearly when the direction is important, we need a different method of adding them together. This is when we need vectors.



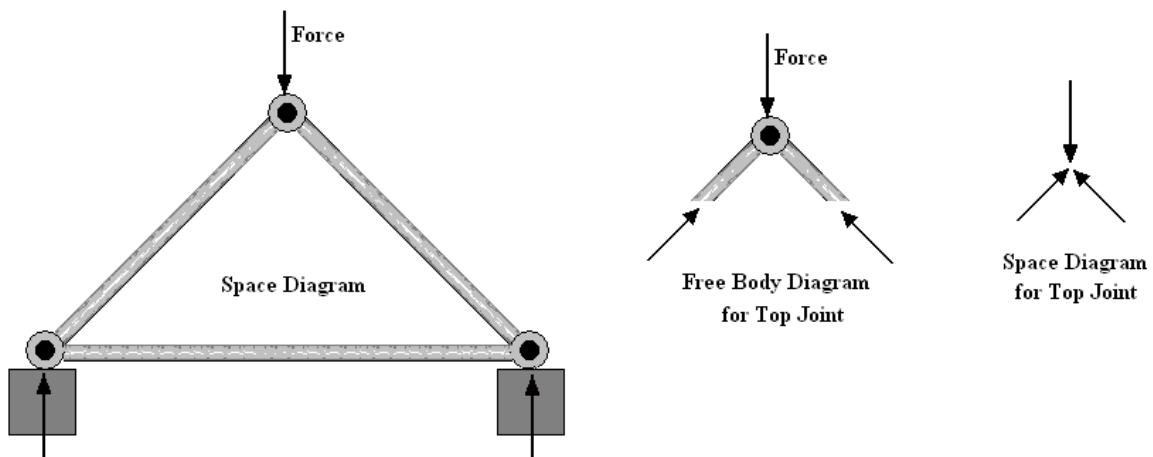
A vector may represent anything that has magnitude (size) and direction. If the quantity has magnitude and no direction, it is called a **Scalar**. Examples are temperature and density. In this module we are only concerned with **Force**.

In order to represent a force as a vector, we draw an arrow with the length proportional to the force and the direction the same as the true direction of the force. The diagram shows a vector representing 30 N at 45° .



Vector Addition and Subtraction

A diagrams showing the forces acting on a body is called the **Space Diagram**. When the space diagram applies to only part of a body showing all the force acting on it, it is called a **Free Body Diagram**. The diagram illustrates how this is applied to a triangular frame (e.g. a roof truss).



When we draw a diagram to enable us to add or subtract vectors, it is called a **Vector Diagram** as shown in the following examples.

When two forces act at a point, the total force and its true direction are found by adding them as vectors. We do not add the values of the forces.

To do this we draw the first vector (it doesn't matter which one) and then draw the second starting on the tip of the first. The new vector which starts at the tail of the first and ends at the head of the second is the resultant force vector.

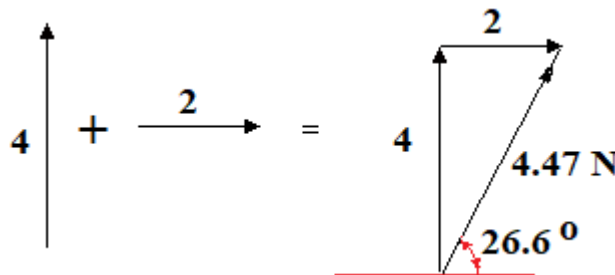
In this module, we are only dealing with forces that act on the same 2 dimensional plane and these are said to be **Coplanar**.

WORKED EXAMPLE No. 1

Determine the result of adding two coplanar forces 4 N at 90° (vertically up) and 2 N at 0° (horizontally to right).

SOLUTION

The magnitude and direction of the resultant may be found graphically by drawing it all out to scale and measuring it, or by trigonometry. In this case it is a right angle triangle so use Pythagoras.



$$R^2 = 2^2 + 4^2 = 4 + 16 = 20 \quad R = \sqrt{20} = 4.47 \text{ N}$$

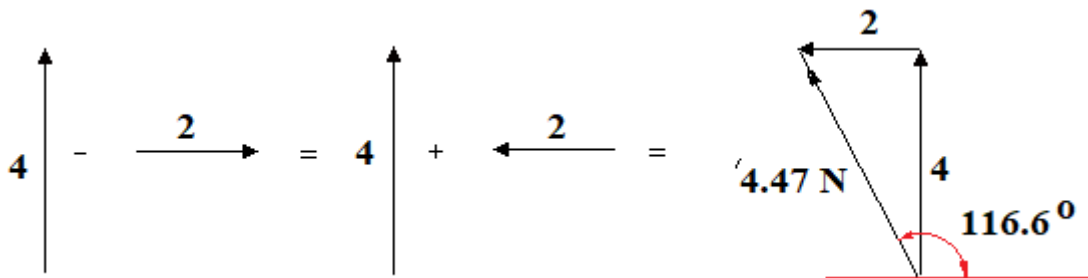
$$\tan \theta = 2/4 = 0.5 \quad \theta = 26.6^\circ$$

Note that angles are usually measured as shown, anti-clockwise being positive.

WORKED EXAMPLE No. 2

Subtract the coplanar forces of 2 at 0° from a vector of 4 at 90° .

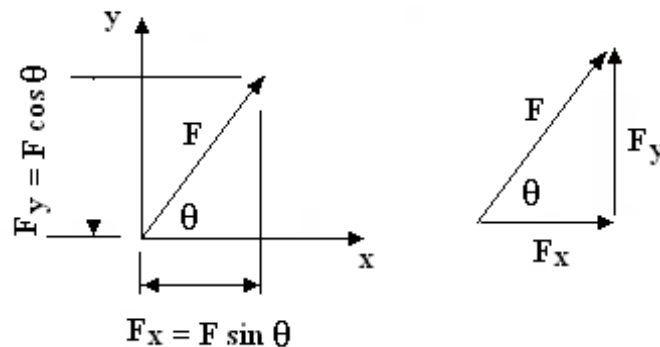
SOLUTION



The resultant is 4.47 N at 116.6°

Resolution

This is particularly useful when the forces are not vertical and horizontal. Consider a vector of magnitude F at angle θ as shown. Note that θ is measured anticlockwise from the positive x axis.



The vertical component of F is called F_y and may be found from trigonometry as $F_y = F \sin \theta$

The horizontal component is called F_x and may be found from trigonometry as $F_x = F \cos \theta$

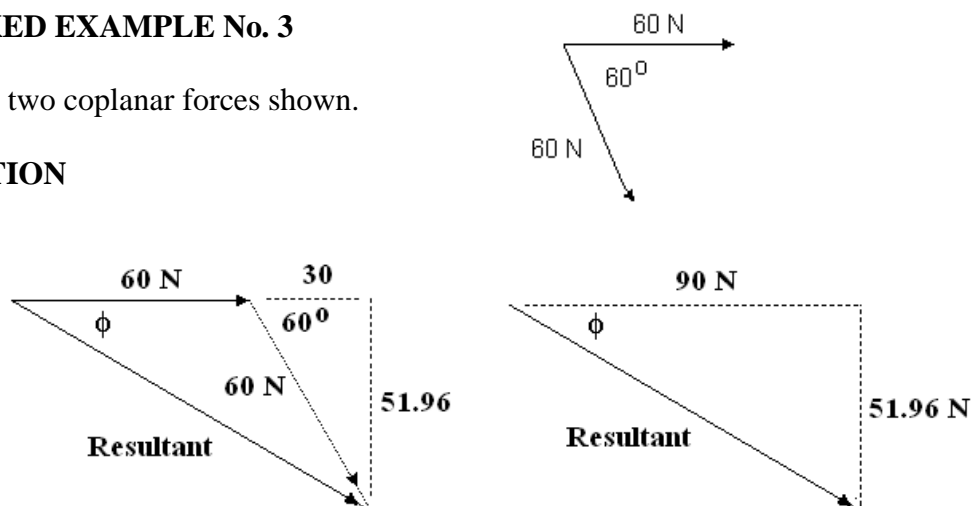
We can also use Pythagoras' theorem to give $F = (F_x^2 + F_y^2)^{1/2}$

If we add the vertical and horizontal components we get back to the original vector as shown.

WORKED EXAMPLE No. 3

Add the two coplanar forces shown.

SOLUTION



Draw the vectors as shown (not to scale here). Measure the resultant and the angle. You should get 103.9 N and 30° .

Alternatively resolve the vectors vertically and horizontally and add them to get the coordinates of the resultant. Note the angle is shown clockwise so it is -60° .

Resolving the first gives 60 N horizontal and 0 N vertical. Resolving the other we get:

$$\text{Vertical} = 60 \sin (-60^\circ) = -51.96 \text{ N} \quad \text{Horizontal} = 60 \cos (-60^\circ) = 30 \text{ N}$$

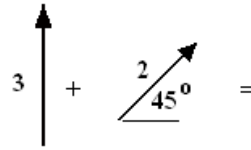
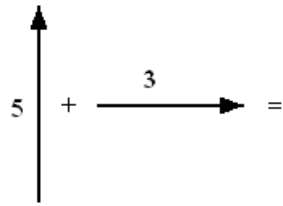
$$\text{Total vertical} = 51.96$$

$$\text{Total horizontal} = 90 \text{ N}$$

$$\text{Resultant} = \sqrt{(90^2 + 51.96^2)} = 103.9 \text{ N} \quad \phi = \tan^{-1} 51.96/90 = 30^\circ$$

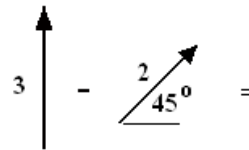
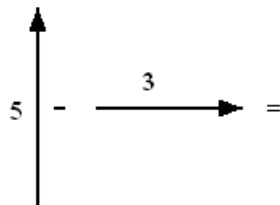
SELF ASSESSMENT EXERCISE No. 1

1. Determine the result of the vector additions shown.



(Answers 5.831 at 59° and 4.64 at 72.2°.)

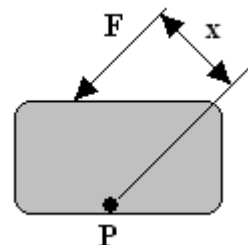
2. Find the resultant vector for the cases shown.



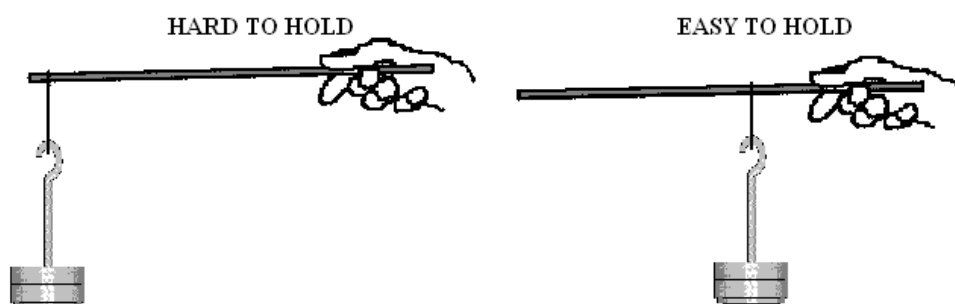
(Answers 5.83 at 121° and 4.64 at 107.8°)

3. Moments of Force

A moment of a force is the result of multiplying the force by the distance from a given point. The distance must be measured at 90° to the force. The diagram shows a force acting on a body. The moment of the force about point P is $F \times x$. The basic units are N m. Often the moment of a force is called Torque but this is more likely to be used in connection with rotating bodies. The distance x is called the moment arm.



In order to demonstrate the effect of the moment arm, try suspending a kg mass from a stick and hold it out horizontally. If the mass is suspended from the far end of the stick, it is very difficult to hold up. If the mass is suspended from the end close to the hand, it is easy to hold up. The weight is the same in both cases but the larger the moment, the harder it is to hold it up.



WORKED EXAMPLE No. 4

If the mass of 1 kg is held 0.6 m from the hand, what is the moment of force exerted by the hand on the stick?

SOLUTION

1 kg exerts a gravitational force downwards of 9.81 N. This acts 0.6 m measured horizontally from the hand. The moment of force is hence $9.81 \times 0.6 = 5.89$ N m (anticlockwise). In order to hold the stick horizontally, the hand must exert a moment of 5.89 N m clockwise on the stick.

4. Static Equilibrium

Any body that does not move has no resulting force or moment acting on it and it is said to have **Static Equilibrium**. This must mean that:

- All forces in any given direction add up to zero.
- All the turning moments about a given point must add up to zero.

If we only use Cartesian coordinates it follows that:

- All the vertical forces upwards (+ve) must equal all the vertical forces downwards (-ve). In other words $\Sigma F_x = 0$
- All the horizontal forces to the right (+ve) must equal all the horizontal forces to the left (-ve) In other words $\Sigma F_y = 0$
- All the clockwise turning moments (+ve) must equal all the anticlockwise turning moments (-ve). In other words $\Sigma M = 0$

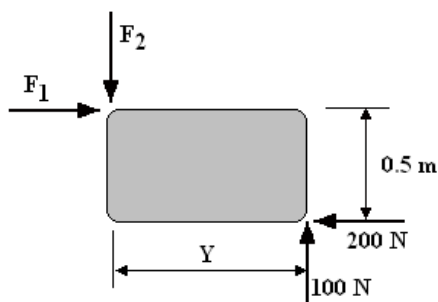
This is known as **D'Alembert's Principle**.

It is of interest to note that in maths anti-clockwise is normally positive and the use of the opposite convention in some mechanical problems can cause confusion so bear this in mind.

Another important point is that if a body is in static equilibrium, all the force must be concurrent, i.e. they must all act through a common single point.

WORKED EXAMPLE No. 5

Examine the diagram. Apply the rules and determine the values of F_1 and F_2 . Also determine the distance Y.



SOLUTION

Balancing horizontal forces we have: $F_1 - 200 = 0$ hence $F_1 = 200$ N

Balancing vertical forces we have: $100 - F_2 = 0$ hence $F_2 = 200$ N.

Now take turning moments about the bottom right hand corner we deduce the distance Y.

Remember that the clockwise and anticlockwise moments must add up to zero and that the forces acting through the point do not exert any moment.

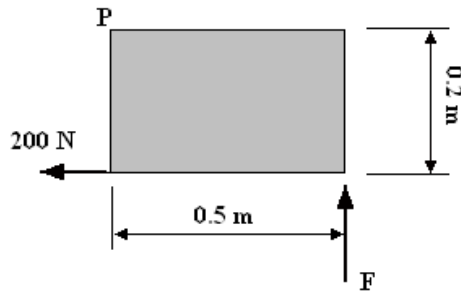
C.W. moment is $F_1 \times 0.5 = 200 \times 0.5 = 100$ Nm

A.C.W. moment is $F_2 y = -100 y$

Add and equate to zero. $100 - 100y = 0$ so $y = 1$ m

WORKED EXAMPLE No. 6

Find the value of force F which just prevents the body shown from revolving about point P .
Go on to find the force (magnitude and direction) which keeps the body in equilibrium.
Where should this force be applied?



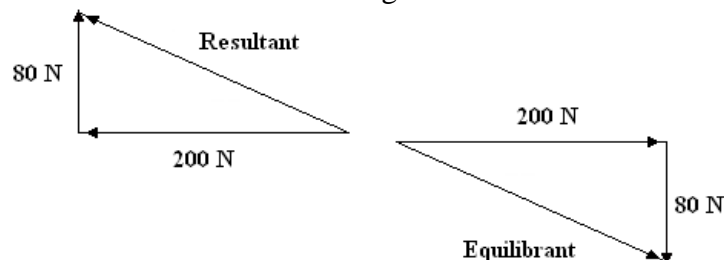
SOLUTION

The clockwise moment about P is $200 \times 0.2 = 40\text{ Nm}$

The anticlockwise moment is $-0.5 F$

For equilibrium $40 - 0.5F = 0$ so $F = 40/0.5 = 80\text{ N}$ (up)

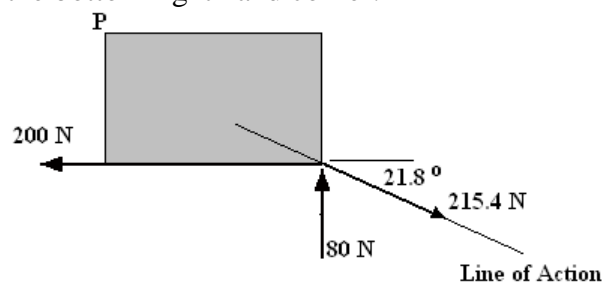
The resultant force must be the result of adding the vertical and horizontal forces.



The force needed to keep the body in equilibrium must be equal and opposite of the resultant.
In other words the body needs 80 N down to cancel the 80 N up and 200 N to the right to balance the 200 N to the left.

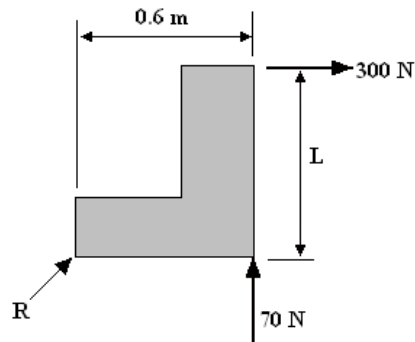
The required force must be found by vector addition as shown. By scaling or by trigonometry the equilibrant force is 215.4 N at 21.8° to the horizontal.

If the body is in static equilibrium when the equilibrant is applied, then all the force must be concurrent so they must pass through a common point. It follows that the line of action of this force must be through the bottom right hand corner.



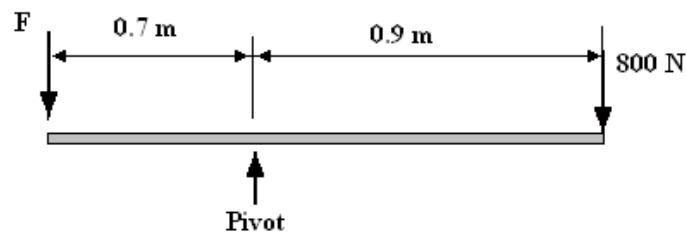
SELF ASSESSMENT EXERCISE No. 2

1. The body shown in the diagram is in total equilibrium. Calculate the length L and the force R .



(Answers 0.14 m , 308 N at 13.1°)

2. Find the value of the force F that holds the lever shown in equilibrium.



(Answer 1028.6 N)