SOLID MECHANICS
STATICS
STRESS - PART 2

SHEAR STRESS AND STRAIN

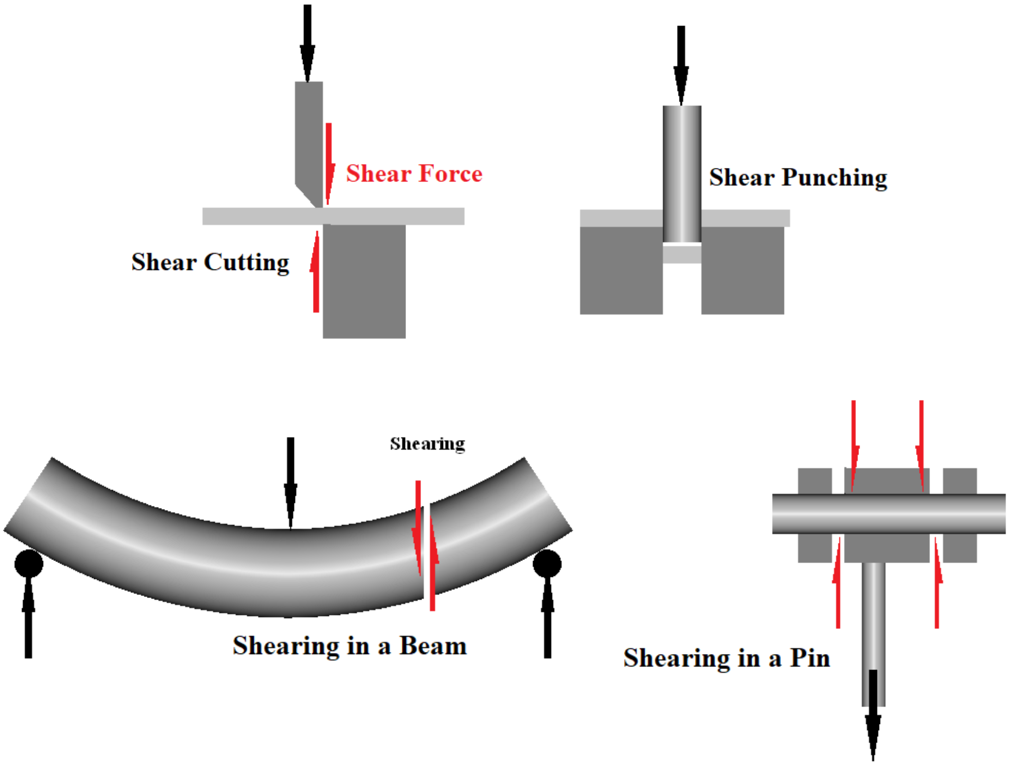
This is set at the British Edexcel National level NQF 3.

On completion of this tutorial you should be able to define shear stress and strain and the elastic constants. You will apply this to structures in shear.

Contents

1. ***Shear Stress τ***

Shear force is a force applied sideways on to the material (transversely loaded). This occurs typically in the following actions.



Shear stress is the force per unit area carrying the load. This means the cross sectional area of the material being cut, the beam and pin respectively.

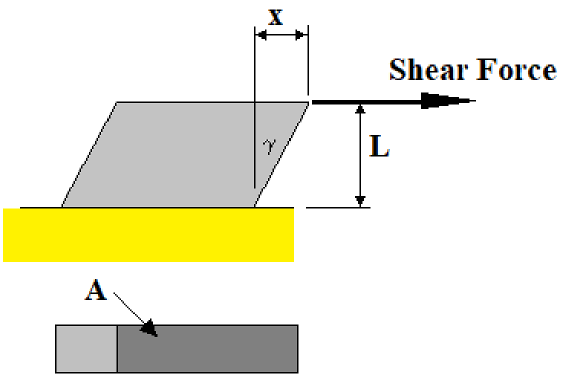
Shear stress $\tau = F/A$ The symbol τ is called Tau. **Note that A is the area being sheared.**

As with direct stress, we may define a safety factor as the ratio of the fail shear stress to the actual working stress.

The sign convention for shear force and stress is based on how it shears the materials and this is shown below. Positive is ***up*** on the left and negative is ***down*** on the left.

2. *Shear Strain γ*

In order to understand the basic theory of shearing, consider a block of material being deformed sideways as shown. (Note in reality the deformation is very small and the diagram is much exaggerated)



The shear force causes the material to deform as shown. The shear strain is defined as the ratio of the distance deformed to the height x/L.

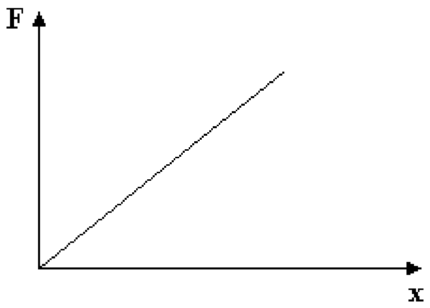
The end face rotates through an angle γ. Since this is a very small angle, it is accurate to say the distance x is the length of an arc of radius L and angle γ so that **γ = x/L**

It follows that γ is the shear strain. The symbol γ is called Gamma.

$$\gamma = \frac{x}{L}$$

3. *Modulus of Rigidity G*

If we were to conduct an experiment and measure x for various values of F, we would find that if the material is elastic, it behave like a spring and so long as we do not damage the material by using too big a force, the graph of F and x is a straight line as shown.



The gradient of the graph is constant so F/x = constant and this is the shear spring stiffness of the block in N/m.

If we divide F by the area A and x by the height L, the relationship is still a constant and we get:

$$\frac{F}{A} \div \frac{x}{L} = \frac{FL}{Ax} = \text{constant}$$

But F/A = τ and x/L = γ so:

$$\frac{\tau}{\gamma} = \text{constant}$$

This constant will have a special value for each elastic material and is called the Modulus of Rigidity with symbol G.

$$\frac{\tau}{\gamma} = G$$

4. ***Ultimate Shear Stress***

If a material is sheared beyond a certain limit it becomes permanently distorted and does not spring all the way back to its original shape. The elastic limit has been exceeded. If the material is stressed to the limit so that it parts into two (e.g. a guillotine or punch), the ultimate limit has been reached. The ultimate shear stress is τ_u and this value is used to calculate the force needed by shears and punches.

WORKED EXAMPLE No. 1

Calculate the force needed to guillotine a sheet of metal 5 mm thick and 0.8 m wide given that the ultimate shear stress is 50 MPa.

SOLUTION

The area to be cut is a rectangle 800 mm x 5 mm $\quad A = 800 \times 5 = 4\,000 \text{ mm}^2$

$$\tau = \frac{F}{A} \text{ so } F = \tau A = 40 \times 40\,000 = 200\,000 \text{ N or } 200 \text{ kN}$$

WORKED EXAMPLE No. 2

Calculate the force needed to punch a hole 30 mm diameter in a sheet of metal 3 mm thick given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be cut is the circumference x thickness

$A = \pi D t = \pi \times 30 \times 3 = 282.7 \text{ mm}^2$ The ultimate shear stress is 60 N/mm^2

$$\tau = \frac{F}{A} \text{ so } F = \tau A = 60 \times 282.7 = 16\,965 \text{ N or } 16.965 \text{ kN}$$

WORKED EXAMPLE No. 3

Calculate the force needed to shear a pin 8 mm diameter given that the ultimate shear stress is 60 MPa.

SOLUTION

The area to be cut is a circle.

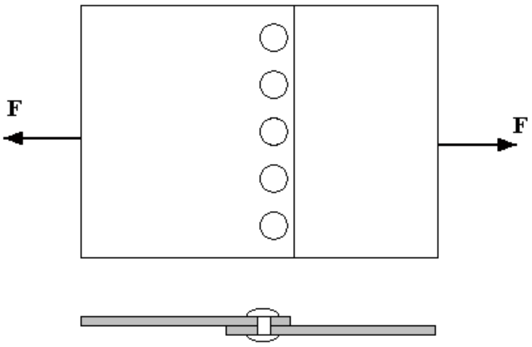
$$A = \frac{\pi D^2}{4} = \frac{\pi \times 8^2}{4} = 50.26 \text{ mm}^2$$

The ultimate shear stress is 60 N/mm^2

$$\tau = \frac{F}{A} \text{ so } F = \tau A = 60 \times 50.26 = 3\,016 \text{ N or } 3.016 \text{ kN}$$

WORKED EXAMPLE No. 4

Two sheets of steel are riveted together with 5 rivets as shown. Each rivet is 8 mm diameter. Calculate the maximum force required to break the joint in shear given that the ultimate shear stress is 60 MPa and a safety factor of 1.5 is to be used..



SOLUTION

The area to be sheared in each rivet is the circular area

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4} = 50.26 \text{ mm}^2$$

The total area is $5 \times 50.26 = 251.3$ mm^2

The ultimate shear stress is 60 N/mm^2

Working stress = 60÷safety factor = 60/1.5 = 40 N/mm^2

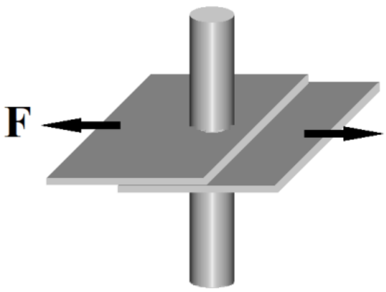
$$\tau = \frac{F}{A} \qquad F = \tau \times A = 40 \times 251.3 = 10\,052 \text{ N or } 10.052 \text{ kN}$$

SELF ASSESSMENT EXERCISE No. 1

1. A guillotine must shear a sheet of metal 0.6 m wide and 3 mm thick. The ultimate shear stress is 45 MPa. Calculate the force required. (Answer 81 kN)

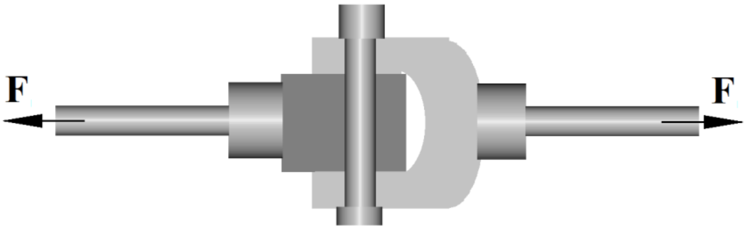
2. A punch must cut a hole 30 mm diameter in a sheet of steel 2 mm thick. The ultimate shear stress is 55 MPa. Calculate the force required. (Answer 10.37 kN)

3. Two strips of metal are pinned together as shown with a rod 10 mm diameter. The ultimate shear stress for the rod is 60 MPa. Determine the maximum force required to break the pin. (Answer 4.71 kN)



5. ***Double Shear***

Consider a pin joint with a support on both sides as shown. This is called a ***Clevis*** and ***Clevis Pin***. The shear force is resisted on both sides so the area to be sheared is twice the cross sectional area of the pin so it takes twice as much force to break the pin as for a case of single shear. Double shear arrangements doubles the maximum force allowed in the pin.



Another way of looking at this is that half the force is taken by each cross section. The area sheared is twice the cross section of the pin

WORKED EXAMPLE No. 5

A pin is used to attach a clevis to a rope. The force in the rope will be a maximum of 60 kN. The ultimate shear stress allowed in the pin is 120 MPa. Calculate the diameter of a suitable pin if the safety factor must be 3.0 based on the ultimate value.

SOLUTION

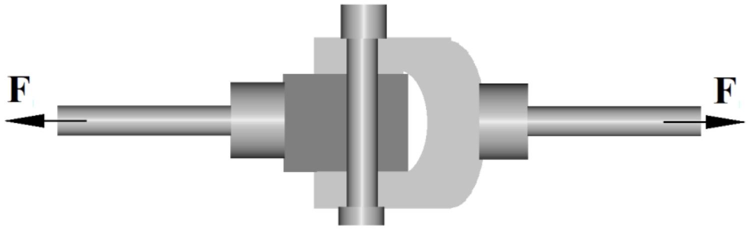
The working stress = 120/3 = 40 MPa. The pin is in double shear so the shear stress is:

$$\tau = \frac{F}{2A} \quad A = \frac{F}{2\tau} = \frac{60\ 000}{1 \times 40 \times 10^6} = 750\ \text{mm}^2$$

$$A = 750\ \text{mm}^2 = \frac{\pi d^2}{4} \qquad d = \sqrt{\frac{4 \times 750}{\pi}} = 30.9\ \text{mm}$$

SELF ASSESSMENT EXERCISE No. 2

1. A clevis pin joint as shown is used to joint two steel rods in a structure. The pin is 8 mm diameter. The ultimate shear stress for the pin material is 80 MPa. Determine the maximum force that can be exerted using a safety factor of 2.0. (Answer 4.02 kN)



2. The steel tie rods used in question 1 are 20 mm diameter. The steel yield stress is 160 MPa. Based on the answer to question 1, determine the safety factor for the rods based on yield stress. (Answer 12.5)

3. Three metal plates are joined together with steel M8 screws as shown along one edge. Find the root diameter of M8 screws and a typical ultimate shear stress for steel screws. Based on a safety factor of 1.5, calculate the number of screws needed withstand a shearing force of 40kN. (Answer 14)