

MECHANICS OF SOLIDS - BEAMS

PRELIMINARY LEVEL TUTORIAL 2

REACTION FORCES FOR BEAMS

This tutorial is essential for anyone studying the group of tutorials on beams.

- Essential pre-requisite knowledge for Edexcel HNC Mechanical Principles **UNIT 21722P** outcome 2.
- Essential pre-requisite knowledge for the Engineering Council exam subject D209 Mechanics of Solids.
- Essential pre-requisite knowledge for the Engineering Council exam subject C105 Mechanical and Structural engineering.

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

On completion of this tutorial you should be able to do the following.

- Define a point load.
- Define a uniformly distributed load
- Calculate the reaction forces for beams and other structures resting on simple supports.

It is assumed that the student is already familiar with the concepts of FORCE and TURNING MOMENT

1 EQUILIBRIUM

A reaction force is the force applied to a beam or other structure when it rests against something. In this tutorial we are only going to examine the reactions to the loads acting on a beam resting on simple supports as shown in the diagram. The loads are point loads. The rests are knife edges. We consider that a knife edge pushes upwards vertically with reaction forces R_a and R_b as shown. Reactions are solved by applying two simple rules.

- A body in equilibrium has no resultant force in any direction.
- A body in equilibrium has no resultant turning moment in any direction.

Consider the beam shown below.

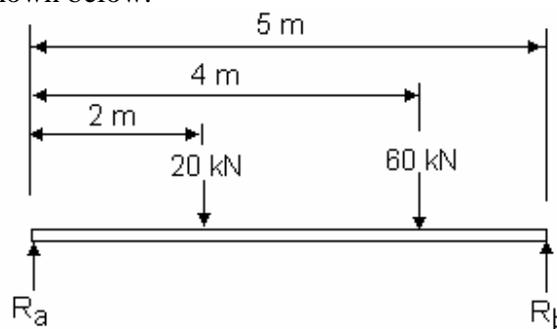


Figure 1

POINT LOADS

A point load is a load or force that acts at a single point on a structure and it is depicted by a single arrow on diagrams.

EQUILIBRIUM OF MOMENTS

The moments must add up to zero when taken about any point. If this was not the case there would be a resultant moment and the beam would be rotating about that point. In these calculations, we will use normal mathematical notation that clockwise is negative and anticlockwise is positive. It is normal to consider the moments of force about one of the reaction points. Since the reaction at that point cannot produce a moment, it is eliminated from the balance. Let's take moments about the reaction on the left.

$$(R_b \times 5) - (60 \times 4) - (20 \times 2) = 0$$

$$(R_b \times 5) = (60 \times 4) + (20 \times 2)$$

$$R_b = 280/5 = 56 \text{ kN}$$

R_a could be found by taking moments about the right end but we can also find it by balancing vertical forces.

EQUILIBRIUM OF FORCES

Since the beam is in equilibrium, the forces acting vertically must add up to zero otherwise it would accelerate in the direction of the resultant force. We will again use normal convention that up is positive and down is negative.

Adding all the vertical forces produces

$$\begin{aligned}R_a + R_b - 20 - 60 &= 0 \\R_a + R_b &= 80 \text{ kN} \\R_a + 56 &= 80 \text{ kN} \\R_a &= 80 - 56 = 24 \text{ kN}\end{aligned}$$

FRAMES

The same principles apply to frameworks resting on reactions. Consider the 45° frame work shown.

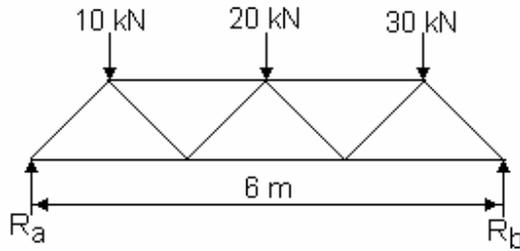


Figure 2

Since all the angle are 45° the horizontal distances between each force must be 2 m and the end forces must be 1 m from the ends.

Balancing moments about R_b we have

$$\begin{aligned}(10 \times 5) + (20 \times 3) + (30 \times 1) - (R_a \times 6) &= 0 \\50 + 60 + 30 &= 6R_a \\R_a &= 140/6 = 23.33 \text{ kN}\end{aligned}$$

Balancing vertical forces

$$\begin{aligned}R_a + R_b - 10 - 20 - 30 &= 0 \\23.33 + R_b - 10 - 20 - 30 &= 0 \\R_b &= 60 - 23.33 = 36.67 \text{ kN}\end{aligned}$$

UNIFORM LOADS

A uniform load is one which is evenly distributed along a length such as the weight of the beam or a wall built on top of a beam. It is depicted by a series of arrows as shown.

We deal with uniform loads by replacing them with an equivalent point load.

A uniform load on a beam is shown below. It has a value of 2.5 kN per metre.

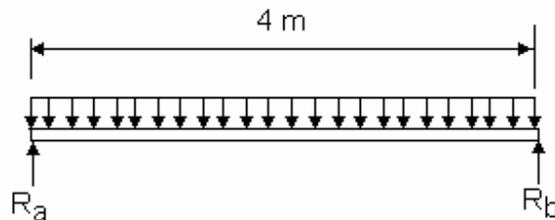


Figure 3

If the load is 2.5 kN for each metre of length then the total load is $4 \times 2.5 = 10$ kN

This load is equivalent to a single point load at the middle of 10 kN. It should be apparent that in this case, the reactions must be equal to half the total load so are both 5 kN acting up. Let's do the calculation anyway. Taking moments about R_b we have

$$\begin{aligned}(10 \times 2) - (R_a \times 4) &= 0 \\ 4R_a &= 20 \\ R_a &= 5 \text{ kN}\end{aligned}$$

Balancing vertical forces

$$\begin{aligned}R_a + R_b - 10 &= 0 \\ 5 + R_b - 10 &= 0 \\ R_b &= 5 \text{ kN}\end{aligned}$$

Now consider a distributed load of 2.5 kN/m over part of the beam only.

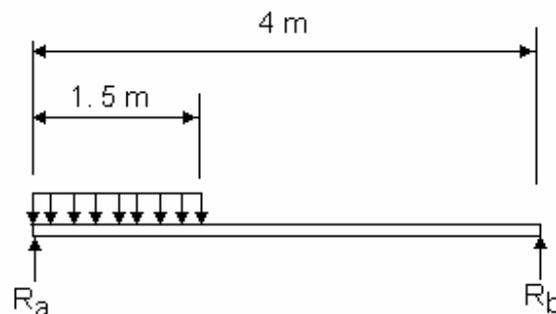


Figure 4

The total load is now $1.5 \times 2.5 = 3.75$ kN

This acts at the middle of the length 0.75 m from the end. The equivalent point load would be as shown.

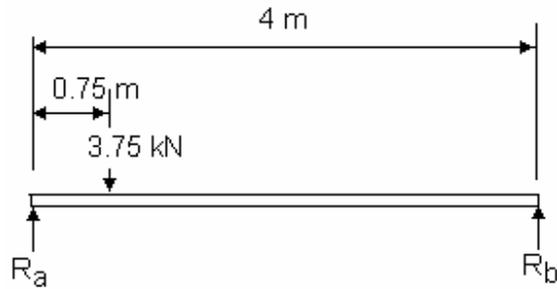


Figure 5

Balancing moments about R_a we have

$$(R_b \times 4) - (3.75 \times 0.75) = 0$$
$$R_b = 2.8125/4 = 0.703 \text{ kN}$$

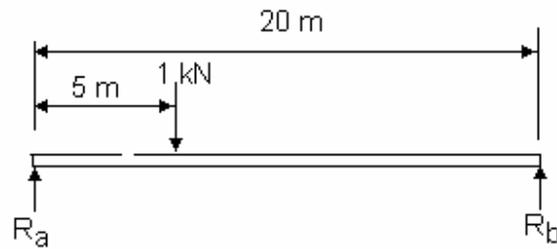
Balancing vertical forces

$$R_b + R_a = 3.75$$
$$R_a = 3.75 - 0.703 = 3.05 \text{ kN}$$

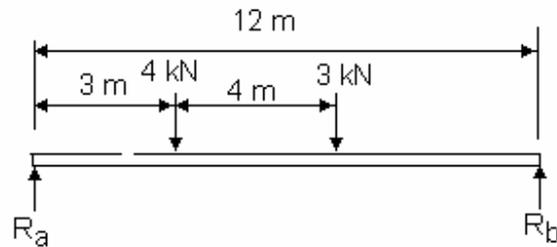
SELF ASSESSMENT EXERCISE No.1

Solve the reactions for each case below.

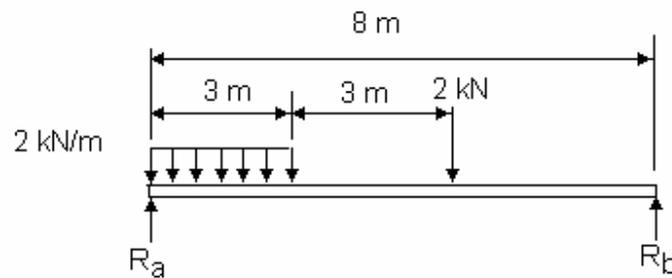
1. (Answers 0.75 and 0.25 kN)



2. (Answers 4.25 and 2.75 kN)



3. (Answers 5.375 and 2.625 kN)



4. (Answers 5 kN and 7 kN)

