MECHANICS OF SOLIDS

PRELIMINARY LEVEL TUTORIAL 3

PIN JOINTED FRAMES

This tutorial is essential for anyone studying the group of tutorials on beams.

- Essential pre-requisite knowledge for Edexcel HNC Mechanical Principles **UNIT 21722P** outcome 2.
- Essential pre-requisite knowledge for the Engineering Council exam subject D209 Mechanics of Solids.
- Essential pre-requisite knowledge for the Engineering Council exam subject C105 Mechanical and Structural engineering.

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

On completion of this tutorial you should be able to do the following.

- Describe pin jointed frames.
- Understand a pin joint.
- Explain the difference between a strut and a tie.
- Understand and use Bow's notation.
- Solve the forces in simple pin jointed frames.

It is assumed that the student is already familiar with the concepts of FORCE, TURNING MOMENT, POLYGON OF FORCES and REACTIONS. These should be studied before attempting this tutorial.

PIN JOINTED FRAMES

1 PIN JOINT

A pin joint allows the joined members to swivel as opposed to a rigid joint that does not. A rigid joint may be welded but a pin joint may be a bolt, a rivet or any form of swivel pin.

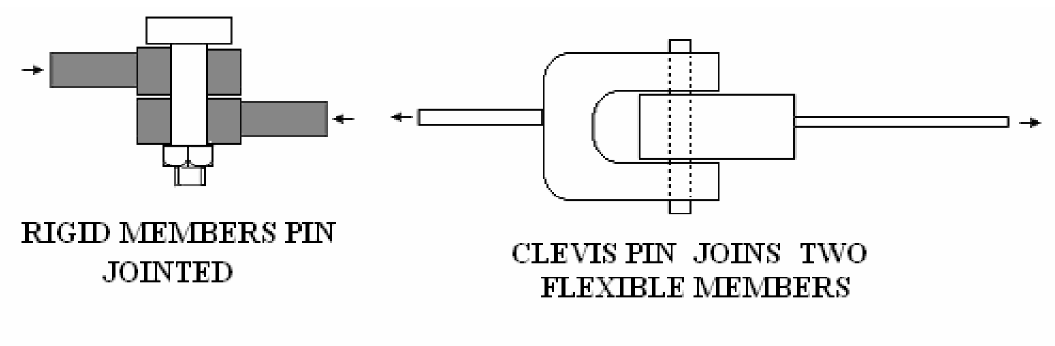

Figure 1

The important points about a pin joint are:

- The connected members are free to rotate.
- The force in the member can only pull or push along the line of the member.

2. STRUTS AND TIES

Consider a member (bar) with a pin joint at each end as shown below. A pin joint cannot transmit rotation (torque) from one to another so each can only push or pull on the joint along the direction of its length. Remember also that the force in the other end of each member also pushes or pulls and so acts in the opposite direction with equal force. A member in tension is called a TIE and is shown with arrows pointing inwards at each end. A member in compression is called a strut and is shown with arrows pointing outwards at each end.

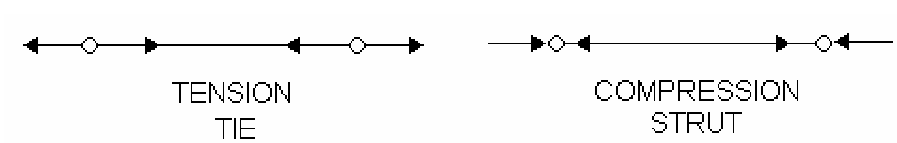

Figure 2

SELF ASSESSMENT EXERCISE No.1

Study the three members below (figure 3) and write down which are ties and which are struts.

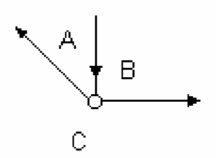

Figure 3

Member A-B is a _________________

Member B-C is a _________________

Member C-A is a _________________

Check your answers against the following.
A-B pushes on the joint so it is in compression and is hence a strut.
B-C and C-A pulls on the joint and so they are in tension and are hence ties.

3. BOW'S NOTATION

When several members are pinned together and the joint is in total equilibrium (not moving), the resultant force must be zero. This means that if we add up all the forces as vectors, they must form a closed polygon. If one or even two of these forces is unknown, then it must be the vector, which closes the polygon.

Consider three members joined by a pin as shown in figure 4. Only one of these forces is known. Bow's notation helps us to identify and label each member and draw the Polygon or (in this case) the triangle of forces. The process is as follows.

1. Label the spaces between each member. This is why the diagram is called a **SPACE DIAGRAM.**

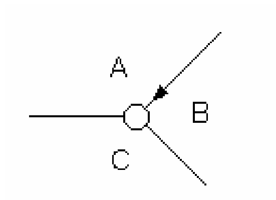

Figure 4

2. Starting at any space, say A, identify each member by moving clockwise around the joint so the first becomes a-b, the next b-c and the last c-a (in this case only).

3. Draw the known vector a-b. We know that the next vector b-c starts at b but we do not know its length. Draw a 'c' line from 'b' in the direction of member b-c. We know that when all the vectors are added, they must form a closed triangle so c-a must end at 'a'. Draw a 'c' line through 'a' in the direction of member c-a. Where the two 'c' lines cross must be point 'c'.

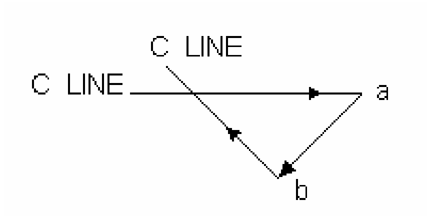

Figure 5

4. Finally, transfer the arrows back to the space diagram in the same direction as on the triangle of forces (figure 6). If they push onto the pin joint, the member must be in compression and so is a strut. If the arrow pulls on the joint, the member must be in tension and so is a tie.

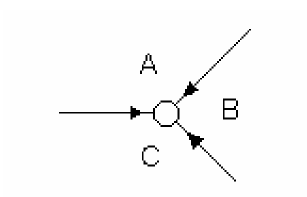

Figure 6

Now let's apply Bow's notation to a simple problem in order to solve the unknown forces.

WORKED EXAMPLE No.1

A strut is held vertical as shown by two guy ropes. The maximum allowable compressive force in the strut is 20 kN. Calculate the forces in each rope. Note that ropes can only be in tension and exert a pull. They cannot push.

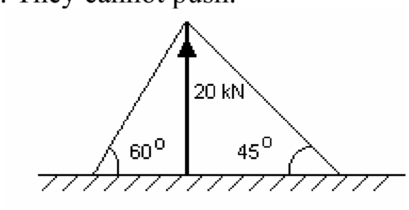

Figure 7a

SOLUTION

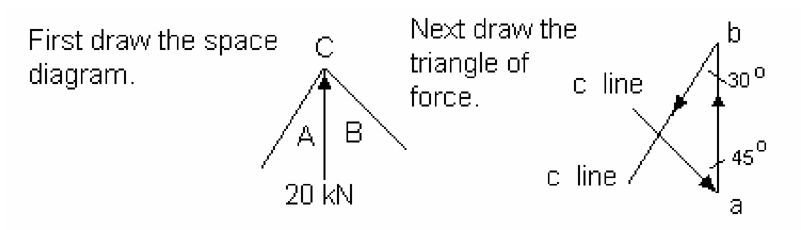

Figure 7b and c

The forces in the ropes are b-c and c-a. These may be found by scaling or by trigonometry.

$bc/\sin 45^\circ = 20/\sin 105^\circ$ $\quad$ $ca/\sin 30^\circ = 20/\sin 105^\circ$

bc= 14.64 kN (Tension) $\quad$ ca = 10.35 kN (Tension)

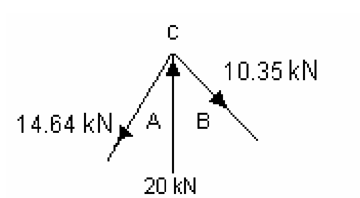

Figure 7d

SELF ASSESSMENT EXERCISE No.2

1. The strut is held upright by a rope and a horizontal force P acts on the top as shown.

   The maximum compressive force allowed in the strut is 5 kN. Determine the maximum allowable force P and the corresponding force in the rope.

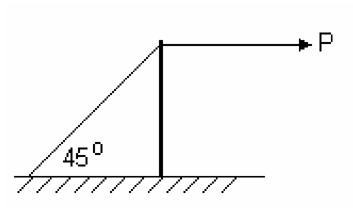

Figure 8

(Ans. 7.07 kN in rope and P= 5 kN)

4. SOLVING THE FORCES IN PIN JOINTED FRAMES

Let's now apply our knowledge to unknown forces in latticework frames. Here are some examples of lattice work frames.

Roof trusses.
Electric Pylons.
Cranes.
Bridges

Many of these structures are riveted and not entirely free to rotate at the joint but the theory of pin jointed frames seems to work quite well for them. We will apply Bow's notation to each joint in turn and so solve the forces in each member. By transferring the direction back from the polygon to the framework diagram, it can be deduced which are struts and which are ties. Knowing this, the force direction is determined at the other end of the member and this is needed to solve the other pin joints.

You learned earlier that a strut is a member in compression and a tie is a member in tension. They are drawn with the internal forces shown as follows.

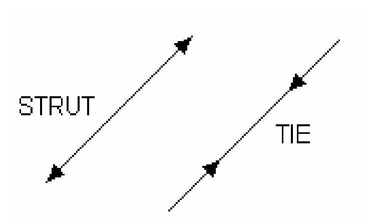

Figure 9

The following worked example show's you how solve a basic problem.

WORKED EXAMPLE No.2

Solve the forces and the reactions for the frame shown on figure 10a

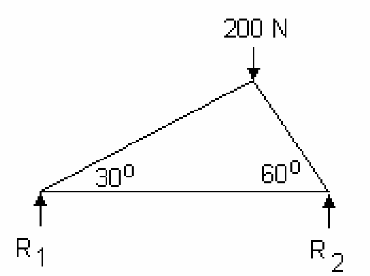

Figure 10a

SOLUTION

First draw the space diagram and label the spaces (figure 10b).

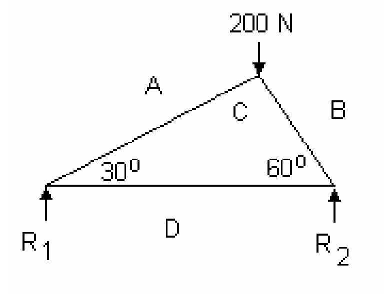

Figure 10b

Next solve the joint with the known force (figure 10 c).

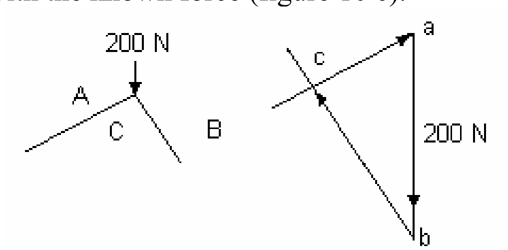

Figure 10c

By scaling or use of trigonometry b-c = 173 N (strut) c-a = 100 N (strut)

Next solve the other joint (figure 10d).

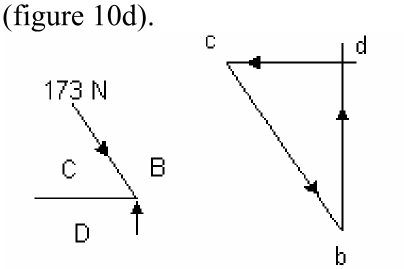

Figure10d

By scaling or trigonometry b-d =R_1 = 150 N. c-d = 86.5 N (Tie)

R_2 may easily be deduced since the total upwards force is 200N then R_2 must be 200-150 = 50N. The solution for the other joint is not really needed but here it is (figure 10e).

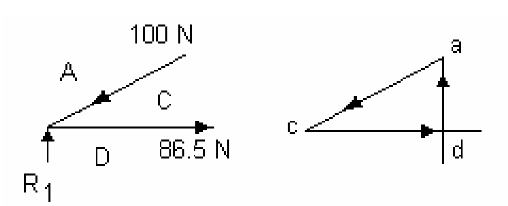

Figure 10 e

From this a-d = R_2 = 50 N as expected.

When you are proficient at this work, you may find it convenient to draw all the solutions together as one diagram. For this example we would have figure 10f.

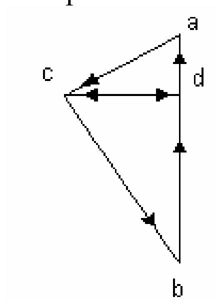

Figure 10f

SELF ASSESSMENT EXERCISE No.3

Solve the forces in the frameworks below and determine which are ties and which are struts.

1.

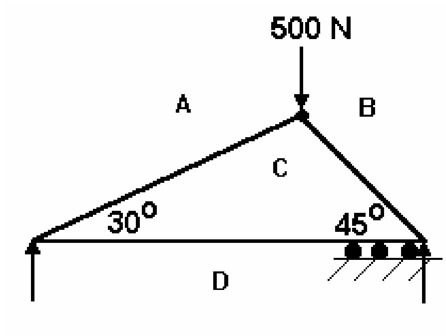

Figure 11

Note that the rollers ensure R_2 is vertical.

(Ans. b-c 450 N strut, c-a 370 N strut, d-c 317 N tie, R_1=183 N R_2= 317 N)

2.

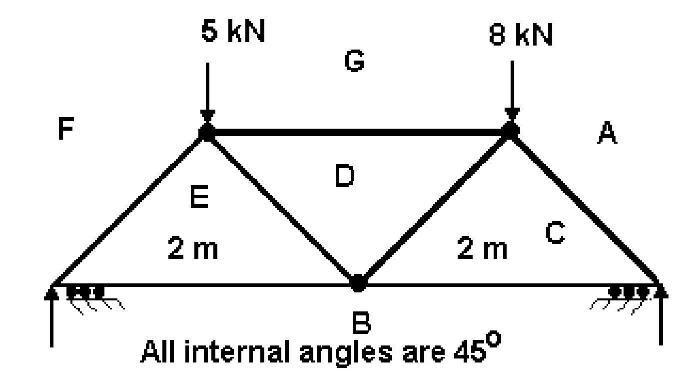

Figure 12

Answers
R_1 = 5.75 kN, R_2=7.25 kN.
b-c 7.25 kN tie, a-c 10.25 kN strut, d-g 6.5 kN strut, c-d 1 kN strut.
e-f 8.2 kN strut, d-e 1kN tie, e-b 5.7 kN tie)

3. The diagram below shows a plane pin-jointed framework subjected to vertical loads of 15 kN and 30 kN. The frame is supported by a pin joint at the left-hand end and by rollers at the right hand end. Solve all the forces and reactions in the frame. The lengths of the vertical and horizontal members are 3 m. (note the spaces are not normally labelled for you).

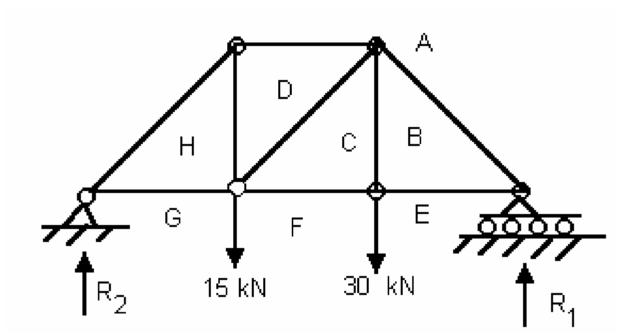

Figure 13

Answers

R_1 = 25 kN R_2 = 20 kN

b-a = 35.5 kN (strut)
e-b = 25 kN (tie)
h-g = 20 kN (tie)
h-a = 29 kN (strut)
f-c = 25 kN (tie)
c-d = 7.07 kN (strut)
d-a = 20 kN (strut)
h-d = 20 kN (tie)
c-b = 30 kN 9tie)

Tip for solution. Start by finding the reactions and then solving one of the bottom corners where only 3 forces act.

FINAL NOTE

There are other more mathematical ways to solve the forces in frames and this is covered in the more advanced tutorials on statics.