STATICS – COMPRESSION MEMBERS – TUTORIAL 1

SELF ASSESSMENT EXERCISE No.1

Find the radius of gyration and the slenderness ratio of a strut made from 5 m length of hollow tube 50 mm outer diameter and 40 mm inner diameter.

$I = \pi(50^4)/64 - \pi(40^4)/64 = 181132 \text{ mm}^4$
$A = \pi(50^2)/4 - \pi(40^2)/4 = 706.8 \text{ mm}^2$
$k = \sqrt{\frac{I}{A}} = 16 \text{ mm}$ L = 5000 mm Slenderness Ratio = 5000/16 = 312.3 mm

SELF ASSESSMENT EXERCISE No. 2

1. A steel strut is 0.15 m diameter and 12 m long. It is built in rigidly at the bottom but completely unrestrained at the top.
   Calculate the buckling load taking E = 205 GPa.

MODE n = ½ $I = \pi D^4/64 = \pi(0.15^4)/64 = 24.85 \times 10^{-6}$

$F_e = n^2 \pi^2 EI/L^2 = ½^2 \pi^2 205 \times 10^9 \times 24.85 \times 10^{-6}/12^2 = 89.4 \times 10^3$ N

2. A steel strut has a solid circular cross section and is 8 m long. It is pinned at the top and bottom but unable to move laterally at the ends. The strut collapses under a load of 200 kN. Taking E = 205 GPa calculate the diameter of the strut.

MODE n = 1

$F_e = 200\,000 = \pi^2 EI/L^2 = \pi^2 205 \times 10^9 \times I/8^2$
$I = 200\,000 \times 8^2 / \pi^2 205 \times 10^9 = 6.326 \times 10^{-6}$
$I = 6.326 \times 10^{-6} = \pi D^4/64$ hence D = 0.1065 m

3. A shaft is made from alloy tubing 50 mm outer diameter and 30 mm inner diameter. The shaft is placed between bearings 3 m apart so that the ends are constrained to remain horizontal. The shaft also has to take a horizontal axial load. Taking E = 120 GPa determine the maximum axial load before buckling occurs.

MODE n = 2

$I = \pi(50^4)/64 - \pi(30^4)/64 = 267035 \text{ mm}^4$
$F_e = n^2 \pi^2 EI/L^2 = 2^2 \pi^2 120 \times 10^9 \times 267 \times 10^{-9}/3^2 = 140 \times 10^3$ N

4. A strut is 0.2 m diameter and 15 m long. It is pinned at both ends. Calculate Euler's critical load.
   E = 205 GPa

MODE n = 1

$I = \pi(0.2^4)/64 = 78.54 \times 10^{-6} \text{ m}^4$

$F_e = \pi^2 EI/L^2 = \pi^2 205 \times 10^9 \times 78.54 \times 10^{-6}/15^2 = 706 \times 10^3$ N

SELF ASSESSMENT EXERCISE No. 3

1.a A uniform slender elastic column of length L is pin jointed at each end and subjected to an axial compression load P.

Show that the Euler crippling load occurs when $P = \pi^2 EI/L^2$ where I is the relevant second moment of area of the column and E is the modulus of elasticity of the material. State any assumptions made.

b A straight steel rod 0.5 m long and 0.01 m diameters loaded axially until it buckles. Assuming that the ends of the rod are pin jointed, determine the Euler crippling load. Assume E = 206 GPa.

Part (a) solution is in the tutorial section 2.2 and constitutes the major part of the marks.

Mode n = 1 $I = \pi \times 0.01^4/64 = 490.87 \times 10^{-12}$ m^4

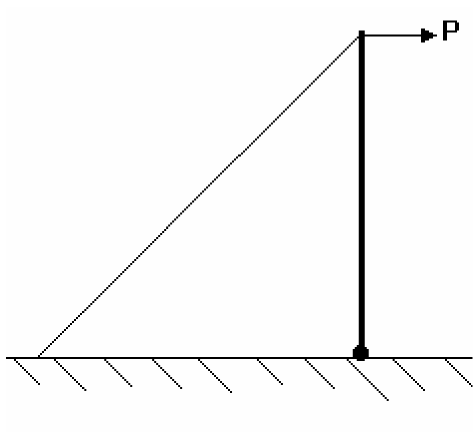
$P = \pi^2 EI/L^2 = \pi^2 \times 206 \times 10^9 \times 490.87 \times 10^{-12}/0.5^2 = 3992$ N

2.a The Euler buckling load P for a slender strut of length L and second moment of area I, pin jointed at each end , is given by $P = \pi^2 EI/L^2$

E is the modulus of elasticity of the material. Using this expression without proof, obtain the formula for the Euler buckling load when the strut is

i. fixed (built in) at each end.

ii. fixed at one end and pin jointed at the other.



b) A vertical pole 6 m long, pinned at the lower end and supported by a wire at the upper end. The pole consists of a tube 50 mm outside diameter and 40 mm inside diameter and the wire has an effective diameter of 6 mm. What is the maximum load P that this system can withstand before failure occurs?

For steel assume that the modulus of elasticity E is 206 GPa and for the wire assume that the ultimate stress is 480 MPa.

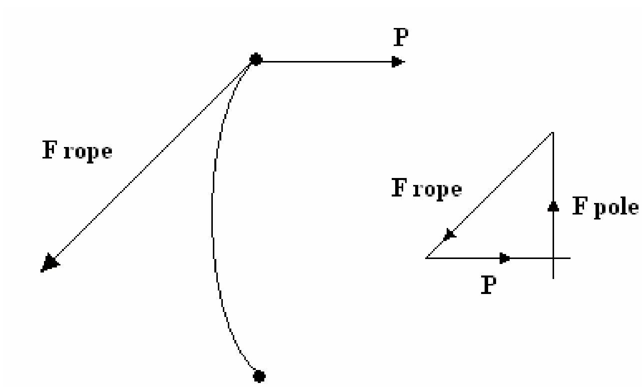
Part (a) derivation is in the tutorial.

Buckling load

$F_e = \pi^2 EI/L^2$

$F_e = \pi^2 \; 206 \times 10^9 \times 181.1 \times 10^{-9}/6^2$

$F_e = 10230 \times 10^3$ N



From triangle of forces

F pole = P F rope = 0.707 P

Stress in rope = 480 MPa hence the breaking force is

$F = \sigma A = 480 \times 10^6 \times \pi \times 0.006^2/4 = 13570$ N

If the rope breaks P = 13570 x 0.707 = 9590 N

Hence the rope breaks before the pole buckles.

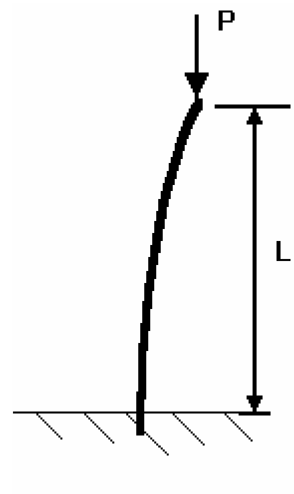
3.a A uniform slender strut of length L which is clamped at one end and free at the other is subjected to an axial compression load P as shown in fig.12. Show that according to EULER'S theory, the strut will buckle when $P = (\pi/2L)^2 EI$ where I is the minimum second moment of area of the strut and E is the modulus of elasticity for the material.

b A straight steel rod 9 mm diameter is rigidly built into a foundation, the free end protruding 0.5 m normal to the foundation. An axial load is applied to the free end of the rod which deflects as shown in fig.12.
Determine the following.

i. Euler's buckling load. **(636 N)**

ii. The deflection of the free end of the rod when the total compressive stress reaches the elastic limit. **(32.6 mm)**

For steel assume E = 200GPa and the stress at the elastic limit is 300MPa.



Part (a) derivation is in the tutorial.

Mode $n = \frac{1}{2}$
$I = \pi(0.009^4)/64 = 322 \times 10^{-12}\ m^4$
$A = \pi(0.009^2)/4 = 63.6 \times 10^{-6}\ m^2$

Buckling load
$P = n^2\pi^2\ EI/L^2 = (\frac{1}{2})^2\ \pi^2\ 200 \times 10^9 \times 322 \times 10^{-12} /2^2 = 636\ N$

Stress $\sigma = 300 \times 10^6 = F/A + My/I$
$M = FX \qquad y = 0.0045\ m$
$300 \times 10^6 = 636/63.6 \times 10^{-6} + 636X \times 0.0045/322 \times 10^{-12} = 10 \times 10^6 + 8.889 \times 10^9\ X$
$290 \times 10^6 = 8.889 \times 10^9\ X$
$X = 0.0326\ m$