

## STATICS – COMPLEX STRESS - PRESSURE VESSELS

### SELF ASSESSMENT EXERCISE No.1

1. A thin walled cylinder is 80 mm mean diameter with a wall 1 mm thick. Calculate the longitudinal and circumferential stresses when the inside pressure is 500 kPa larger than on the outside.

$$\sigma_C = \frac{pD}{2t} = \frac{500 \times 10^3 \times 90 \times 10^{-3}}{2 \times 0.001} = 20 \times 10^6 \text{ N/m}^2$$

$$\sigma_L = \frac{pD}{4t} = \frac{500 \times 10^3 \times 90 \times 10^{-3}}{4 \times 0.001} = 10 \times 10^6 \text{ N/m}^2$$

2. Calculate the wall thickness required for a thin walled cylinder which must withstand a pressure difference of 1.5 MPa between the inside and outside. The mean diameter is 200 mm and the stress must not exceed 60 MPa.

$$t = \frac{pD}{4\sigma_C} = \frac{1.5 \times 10^6 \times 200 \times 10^{-3}}{4 \times 1.5 \times 10^6} = 2.5 \times 10^{-3} \text{ m}$$

3. Calculate the stress in a thin walled sphere 100 mm mean diameter with a wall 2 mm thick when the outside pressure is 2 MPa greater than the inside. (Answer -25 MPa).

$$\sigma = \frac{pD}{4t} = \frac{-2 \times 10^6 \times 100 \times 10^{-3}}{4 \times 0.002} = -25 \times 10^6 \text{ N/m}^2 \text{ (compressive)}$$

### SELF ASSESSMENT EXERCISE No. 2

1. A cylinder is 200 mm mean diameter and 1 m long with a wall 2.5 mm thick. It has an inside pressure 2 MPa greater than the outside pressure. Calculate the change in diameter and change in volume. Take  $E = 180 \text{ GPa}$  and  $\nu = 0.3$

$$\epsilon_C = \frac{pD(2-\nu)}{4tE} = \frac{2 \times 10^6 \times 200 \times 10^{-3}(1-2 \times 0.3)}{4 \times 0.002 \times 180 \times 10^9} = 377.87 \times 10^{-6}$$

$$\epsilon_C = \frac{\Delta D}{D} = \frac{\Delta D}{200} = 377.87 \times 10^{-6} \quad \Delta D = 0.075 \text{ mm}$$

$$\epsilon_v = 2\epsilon_C + \epsilon_L = 844 \times 10^{-6} \text{ or } \frac{pD(5-4\nu)}{4tE} = 844 \times 10^{-6}$$

$$\text{Volume} = (\pi \times 200^2/4) \times 1000 = 31.416 \times 10^6 \text{ mm}^3$$

$$\Delta V = \epsilon_v V = 26529 \text{ mm}^3 \quad \text{Final Volume} = 31.416 \times 10^6 + 26529 = 31.442 \times 10^6 \text{ mm}^3$$

2. A sphere is 50 mm mean diameter with a wall 0.5 mm thick. It has an inside pressure 0.5 MPa greater than the outside pressure. Calculate the change in diameter and change in volume.

Take  $E = 212 \text{ GPa}$  and  $\nu = 0.25$

(Answers 0.0022 mm and 8.68 mm<sup>3</sup>)

$$\epsilon_C = \frac{pD(1-\nu)}{4tE} = \frac{0.5 \times 10^6 \times 50 \times 10^{-3}(1-0.25)}{4 \times 0.0005 \times 212 \times 10^9} = 44.22 \times 10^{-6}$$

$$\epsilon_C = \frac{\Delta D}{D} = \frac{\Delta D}{50} = 44.22 \times 10^{-6} \quad \Delta D = 0.0022 \text{ mm}$$

$$\epsilon_v = 3\epsilon_C = 132.6$$

$$\text{Volume} = (\pi \times 50^3/6) = 64450 \text{ mm}^3 \quad \Delta V = \epsilon_v V = 8.68 \text{ mm}^3$$

- 3a. A thin walled cylinder of mean diameter  $D$  and length  $L$  has a wall thickness of  $t$ . It is subjected to an internal pressure of  $p$ . Show that the change in length  $\Delta L$  and change in diameter  $\Delta D$  are

$$\Delta L = (pDL/4tE)(1 - 2\nu) \quad \text{and} \quad \Delta D = (pD^2/4tE)(2 - \nu)$$

The derivation is given in the tutorial

- b. A steel cylinder 2 m long and 0.5 m mean diameter has a wall 8 mm thick. It is filled and pressurised with water to a pressure of 3 MPa gauge. The outside is atmosphere. For steel  $E = 210$  GPa and  $\nu = 0.3$ . For water  $K = 2.9$  GPa.

Calculate the following.

- The maximum stress.
- The increase in volume of the cylinder.
- The volume of water at atmospheric pressure required.

$$\sigma_c = \frac{pD}{2t} = \frac{3 \times 10^6 \times 0.5}{2 \times 0.008} = 93.75 \times 10^6 \text{ N/m}^2$$

$$\epsilon_v = \frac{pD(5 - 4\nu)}{4tE} = \frac{3 \times 10^6 \times 0.5 \times (5 - 4 \times 0.3)}{4 \times 0.008 \times 210 \times 10^9} = 848.2 \times 10^{-6}$$

$$\text{Initial volume of water} = \pi(500)^2(2000)/4 = 392699100 \text{ mm}^3$$

$$\Delta V = \epsilon_v V = 848.2 \times 10^{-6} \times 392699100 = 333087 \text{ mm}^3$$

$$\text{Volume of compressed water} = 392699100 + 333087 = 393032187 \text{ mm}^3$$

$$K = pV/\Delta V \quad V = k\Delta V/p = 2.9 \times 10^9 \times \Delta V/(3 \times 10^6) = 966.7\Delta V$$

$$966.7\Delta V = 393032187 + \Delta V$$

$$\Delta V = 406992 \text{ mm}^3$$

$$\text{Volume of uncompressed water} = 393032187 + \Delta V(\text{water}) = 393439179 \text{ mm}^3$$

$$\text{Volume of water added to the vessel is } 393439179 - 392699100 = 740079 \text{ mm}^3$$

- 4a. A thin walled sphere of mean diameter  $D$  has a wall thickness of  $t$ . It is subjected to an internal pressure of  $p$ . Show that the change in volume  $\Delta V$  and change in diameter  $\Delta D$  are  $\Delta V = (3pDV/4tE)(1 - \nu)$  where  $V$  is the initial volume.

The derivation is given in the tutorial

- b. A steel sphere 2m mean diameter has a wall 20 mm thick. It is filled and pressurised with water so that the stress in steel 200 MPa. The outside is atmosphere. For steel  $E = 206$  GPa and  $\nu = 0.3$ . For water  $K = 2.1$  GPa. Calculate the following.

- The gauge pressure

- The volume water required.

$$\sigma = \frac{pD}{4t} \quad p = \frac{4t\sigma}{D} = \frac{4 \times 0.02 \times 200 \times 10^6}{2} = 8 \times 10^6 \text{ N/m}^2$$

$$\delta V = \frac{3pD(1 - \nu)}{4tE} V = \frac{3 \times 8 \times 10^6 \times 2 \times (1 - 0.3)}{4 \times 0.02 \times 206 \times 10^9} = 2.03 \times 10^{-3} V$$

$$\text{Initial volume of water} = \pi D^3/6 = 4.18888 \times 10^9 \text{ mm}^3$$

$$\delta V = 8.5403 \times 10^6 \text{ mm}^3$$

$$\text{Final Volume} = 4.18888 \times 10^9 + 8.5403 \times 10^6 = 4.1973 \times 10^9 \text{ mm}^3$$

$$\text{Volume of uncompressed water} = K \Delta V/p = 2.1 \times 10^9 \times \Delta V/(8 \times 10^6)$$

$$4.1973 \times 10^9 = 2.1 \times 10^9 \times \Delta V/(8 \times 10^6) - \Delta V = 261.5 \Delta V$$

$$\Delta V(\text{water}) = 16.0509 \times 10^3 \text{ mm}^3$$

$$V + \Delta V = 4.1973 \times 10^9 + 16.0509 \times 10^3 = 4.2134 \times 10^9 \text{ mm}^3$$

$$\text{Volume added to vessel} = 4.2134 \times 10^9 - 4.18888 \times 10^9 = 24.55 \times 10^6 \text{ mm}^3$$

### SELF ASSESSMENT EXERCISE No.3

1. A thick cylinder has an outer diameter of 150 mm and an inner diameter of 50 mm. The OUTSIDE is pressurised to 200 bar greater than the inside.

Calculate the following.

- The circumferential stress on the inside layer.
- The circumferential stress on the outside layer.

The boundary conditions are

Inner surface  $r_i = 25 \text{ mm}$   $\sigma_R = 0 \text{ MPa}$  (compressive)

Outer surface  $r_o = 75 \text{ mm}$   $\sigma_R = 200 \times 10^5 \text{ Pa}$  (compressive)

Substituting into Lamé's equation we have

$$\sigma_R = 0 = a - b/r^2 = a - b/0.025^2 = a - 1600b \quad a = 1600b$$

$$\sigma_R = -20 \times 10^6 = a - b/r^2 = a - b/0.075^2 = a - 177.8 b$$

$$-20 \times 10^6 = a - 177.8 b = 1600b - 177.8 b = 1422.2b$$

$$b = -20 \times 10^6 / 1422.2 = -14062.5$$

$$a = 1600b = -22.5 \times 10^6$$

Now solve the circumferential stress on the inside.  $\sigma_C = a + b/r_i^2 = -45 \text{ MPa}$

Now solve the circumferential stress on the outside.  $\sigma_C = a + b/r_o^2 = -25 \text{ MPa}$

2. A thick cylinder has an outside diameter of 100 mm and an inside diameter of 60 mm. It is pressurised until internally until the outer layer has a circumferential stress of 300 MPa.

Calculate the pressure difference between the inside and outside. (266.6 MPa)

The boundary conditions are

Inner surface  $r_i = 30 \text{ mm}$   $\sigma_R = -p_i$

Outer surface  $r_o = 50 \text{ mm}$   $\sigma_R = -p_o$  and take this as zero and solve  $p_i$  relative to it.

Substituting into Lamé's equation we have

$$\sigma_R = -p_o = 0 = a - b/r_o^2 \quad a = b/r_o^2 = 400 b$$

Outer surface  $r_o = 50 \text{ mm}$   $\sigma_C = 300 \times 10^6$

Substituting into Lamé's equation we have

$$\sigma_C = 300 \times 10^6 = a + b/r_o^2 = 400b + 400b = 800b$$

$$b = 375000 \text{ hence } a = 150 \times 10^6$$

On the inner surface  $r_i = 30 \text{ mm}$

$$\sigma_R = -p_i = a - b/r_i^2 = 150 \times 10^6 - 375000/0.03^2$$

$$p_i = 266.7 \text{ MPa}$$

The pressure difference is hence 266.7 MPa

3. A thick cylinder is 100 mm outer diameter and 50 mm inner diameter. It is pressurised to 112 MPa gauge on the inside. Calculate the following.

- The circumferential stress on the outside layer
- The circumferential stress on the inside layer
- The longitudinal stress
- The circumferential strain in the outside layer
- The circumferential strain in the inside layer
- The change in the inner diameter
- The change in the outer diameter

Take  $E = 205 \text{ GPa}$  and  $\nu = 0.27$

The boundary conditions are

Inner surface  $r_i = 25 \text{ mm}$   $\sigma_R = -112 \text{ MPa}$  (compressive)

Outer surface  $r_o = 50 \text{ mm}$   $\sigma_R = 0$

Substituting into Lamé's equation we have

$$\sigma_R = 0 = a - b/r_o^2 = a - b/0.05^2 = a - 400b \quad a = 400b$$

$$\sigma_R = -112 \times 10^6 = a - b/r_i^2 = a - b/0.025^2 = a - 1600b$$

$$-112 \times 10^6 = 400b - 1600b = -1200b$$

$$b = 112 \times 10^6 / 1200 = 93333.3$$

$$a = 400b = 37.33 \times 10^6$$

Now solve the circumferential stress on the inside.  $\sigma_C = a + b/r_i^2 = 186.7 \text{ MPa}$

Now solve the circumferential stress on the outside.  $\sigma_C = a + b/r_o^2 = 74.64 \text{ MPa}$

The longitudinal stress is Force/Area

The force is  $p \times \pi r_i^2 = 219.9 \text{ kN}$

Wall section area =  $\pi(r_o^2 - r_i^2) = 5.89 \times 10^{-3} \text{ m}^2$

$\sigma_L = F/A = 37.33 \text{ MPa}$

Circumferential strain at outside

$$\epsilon_C = (1/E)\{\sigma_C - \nu(\sigma_R + \sigma_L)\}$$

$$\epsilon_C = (1/205 \times 10^9)\{74.64 - 0.27(0 + 37.33)\} \times 10^6 = 315 \mu\epsilon$$

$$\Delta D = D_o \epsilon_C = 100 \times 315 \times 10^{-6} = 0.031 \text{ mm}$$

Circumferential strain at inside

$$(1/205 \times 10^9)\{186.7 - 0.27(-112 + 37.33)\} \times 10^6 = 1.009 \mu\epsilon$$

$$\Delta D = D_i \epsilon_C = 50 \times 1.009 \times 10^{-6} = 0.05 \text{ mm}$$

4. A shaft has a diameter of 45.08 mm and is an interference fit with a sleeve 60 mm outer diameter, 45 mm inner diameter and 80 mm long. Calculate the force needed to slide the sleeve on the shaft if the coefficient of friction is 0.25. The elastic properties for both parts are the same with  $E = 200$  GPa and Poisson's ratio = 0.3

Calculate the change in radius of the shaft and sleeve at the inside.

$$p = \frac{\delta}{\frac{R}{E} \left[ \frac{R^2 + R_o^2}{R_o^2 - R^2} + 2\nu - 1 \right]}$$

$$p = \frac{0.08}{\frac{45.08}{200 \times 10^9} \left[ \frac{45.08^2 + 60^2}{60^2 - 45.08^2} + (2 \times 0.3) - 1 \right]} = 111.2 \times 10^6 \text{ N/m}^2$$

$$\text{Surface Area} = 2\pi RL = 2\pi \times 0.04508 \times 0.08 = 0.0226 \text{ m}^2$$

$$\text{Normal Force} = N = pA = 2.519 \text{ MN}$$

$$\text{Friction force} = \mu N = 0.25 \times 2.519 = 0.63 \text{ MN}$$

$$\text{For the sleeve } \Delta R = \frac{pR}{E} \left( \frac{R^2 + R_o^2}{R_o^2 - R^2} + \nu \right)$$

$$\Delta R = \frac{112 \times 10^6 \times 0.04508}{200 \times 10^9} \left( \frac{45.08^2 + 60^2}{60^2 - 45.08^2} + 0.3 \right) = 97.5 \times 10^{-6} \text{ m}$$

$$\text{For the shaft } \Delta R = \frac{pR}{E} (-1 + \nu)$$

$$\Delta R = \frac{112 \times 10^6 \times 0.04508}{200 \times 10^9} (-1 + 0.3) = -17.5 \times 10^{-6} \text{ m}$$

Check  $\delta = 97.5 - 17.5 = 80 \mu\text{m}$  which was the original interference fit.