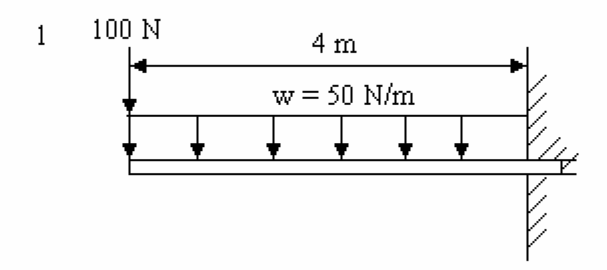
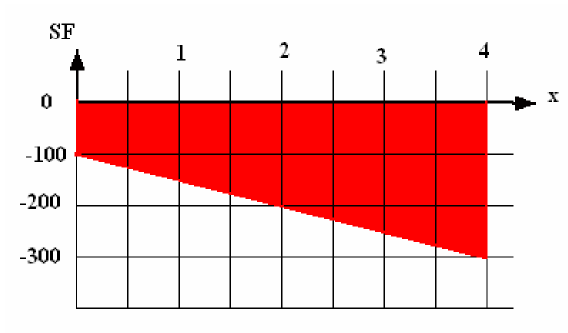
BEAM TUTORIAL 2 SOLUTIONS

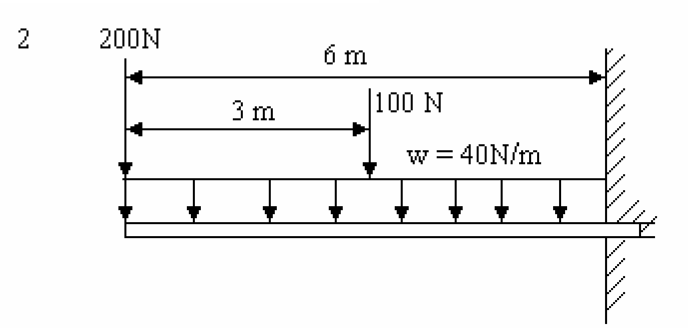
SAE 1

Draw the shear force diagram for the cases below and determine the greatest shear force..



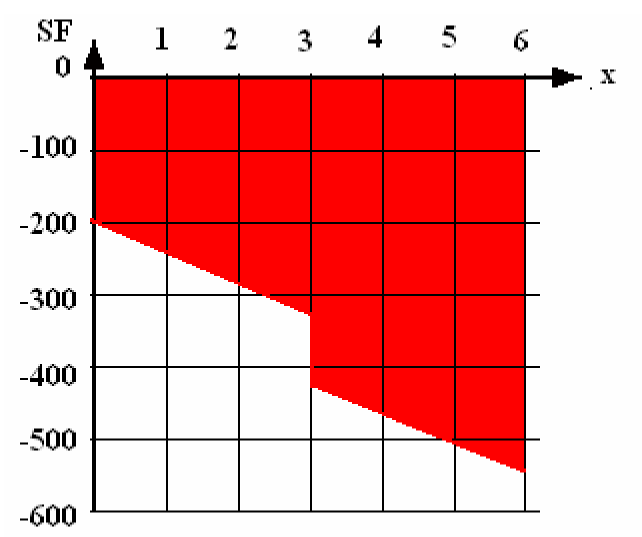
SOLUTION





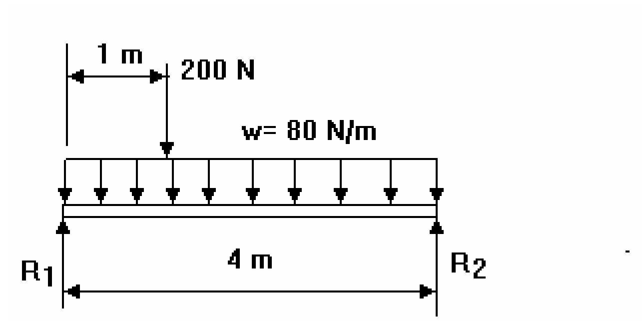
x = 0 SF = 0/-200
x = 1 SF = -240
x = 2 SF = -280
x = 3 SF = -320/-420
x = 4 SF = -460
x = 5 SF = -500
x = 6 SF = -540

(Answer -540 N)



SAE 2

1. A beam is loaded as shown below. Calculate the reactions and draw the shear force diagram. **(Answers 310 N and 210 N)**

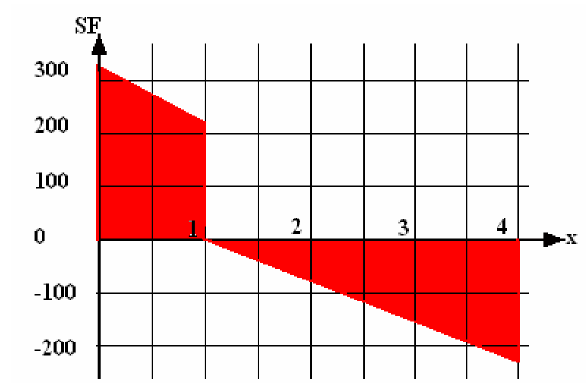


Moments about left end gives

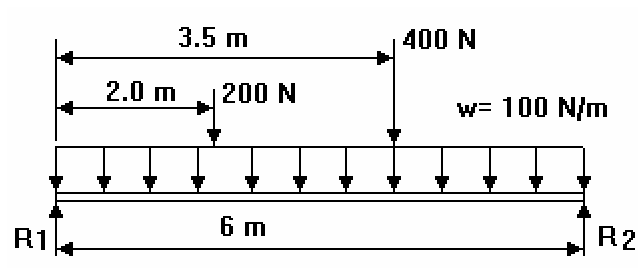
(200 x 1) + (80 x 4 x 2) = 4R_2 R_2 = 210 N

R_1 = 520 - 210 = 310 N

x	0	1	2	3	4
SF	0/310	230/30	-50	-130	-210/0



2. A beam is loaded as shown below. Calculate the reactions and draw the shear force diagram. (Answers 600 N and 600 N)



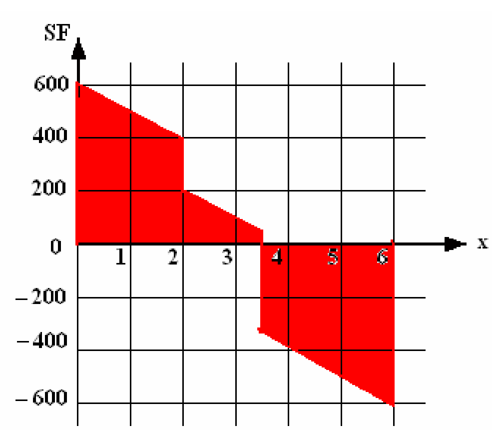
Moments about left end gives

(200 x 2) + (400 x 3.5) = 6R_2 R_2 = 600 N

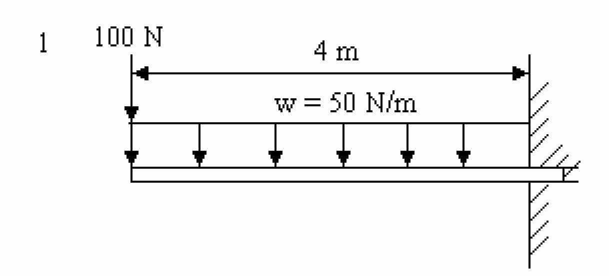
R_1 = 200 + 400 + (6 x 100) - 600 = 600 N

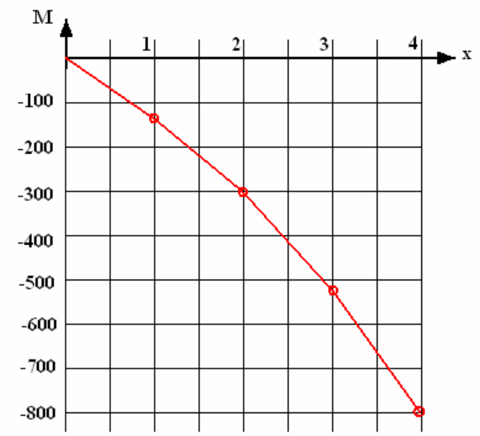
x	0	1	2	3	3.5
SF	0/600	500	400/200	100	50/-350

x	4	5	6
SF	-400	-500	-600



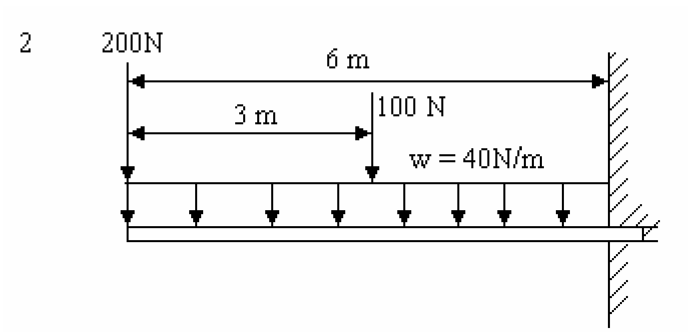
Draw the bending moment diagram for the cases below and determine the greatest bending moment.



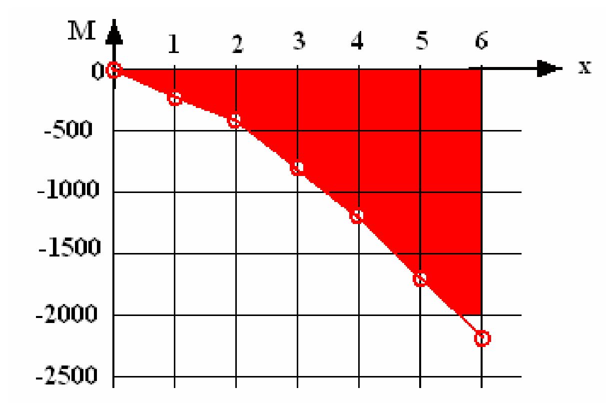


(Answer 800 Nm hogging)

x	0	1	2	3	4
M Nm	0	(-100x1) – (50 x1) = -125	-(100 x2)-(50x2x1) =-300	-100-(50x3x1.5) =-525	-(100x4)-(50x4x2) =-800



(Answer 2220 Nm hogging)

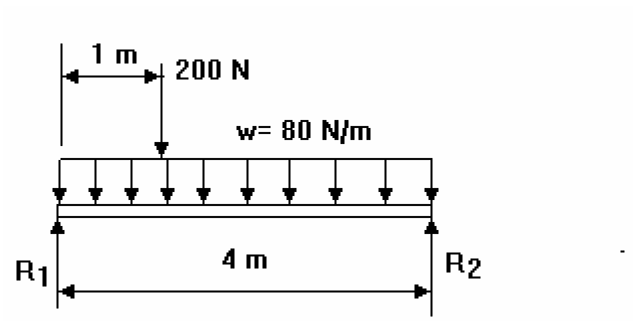


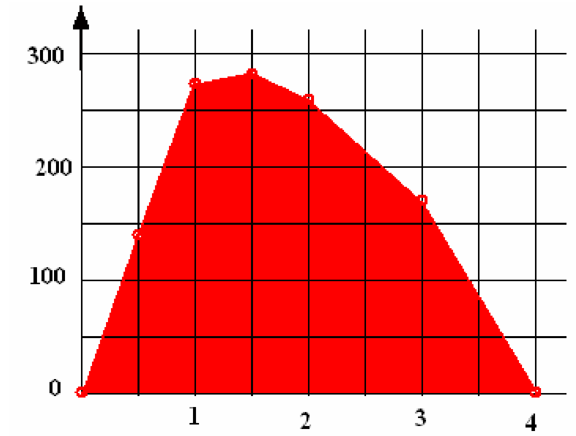
x	0	1	2	3	4
BM	0	-200x1-40x0.5 =-220	-200x2-80x1 =-480	-200x3 – 120x1.5 =-780	-200x4-100x1-160x2 =-1220

x	5	6
BM	-200x5-100x2-200x2.5 =-1700	-200x6-100x3-240x3 =-2220

SAE 4

1. A beam is loaded as shown below. Calculate the reactions and draw the Bending Moment diagram. Determine the maximum bending moment. **(Answers 310 N. 210 N and 275 Mm)**



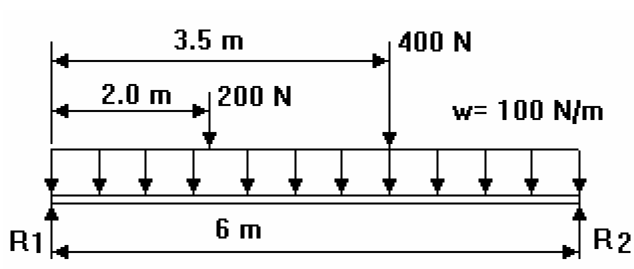


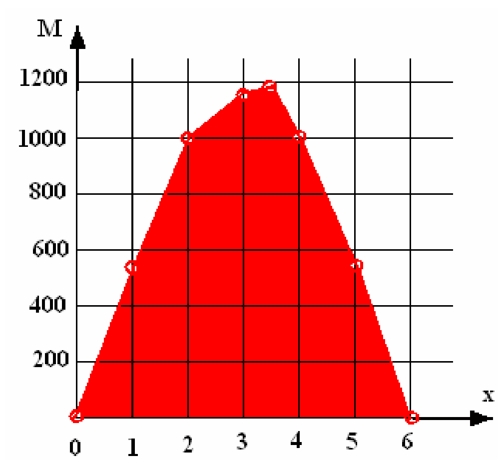
First part is Q1 SAE 1 so only draw BM diagram.

x	0	0.5	1	1.5	2
BM Nm	0	310 x 0.5 -80x0.5^2/2 = 145	310x1-80x1^2/2 = 270	310x1.5-200x 0.5-80x1.5^2/2 = 275	310x2-200x 1-80x2^2/2 = 260

x	3	4
BM	310 x 3-200 x 2- 80x3^2/2 =170	0

2. A beam is loaded as shown below. Calculate the reactions and draw the bending moment diagram. Determine the greatest ending moment. **(Answers 600 N, 600 N and 1188 Nm)**





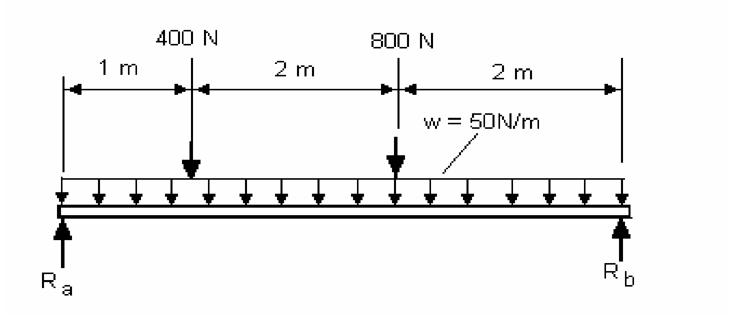
First part is SAE2 Q2 so just do the BM diagram.

x	0	1	2	3	3.5
BM Nm	0	600 x 1 -100x1^2/2 = 550	600 x 2 -100x2^2/2 = 1000	600 x 3 -100x3^2/2 -200 x1 = 1150	600 x 3.5 -100x3.5^2/2 -200 x1.5 = 1187.5

x	4	5	6
BM	600 x 4 -100x4^2/2 -200 x2 – 400 x 0.5 = 1000	600 x 5 -100x5^2/2 -200 x3 – 400 x 1.5 = 550	0

SAE 5

Question 1
Draw the SF diagram for the beams below. Determine the greatest bending moment and the position at which it occurs.



Formulae for use in MCAD F1 = 400 F2 = 800 A = 1 B = 3 L = 5

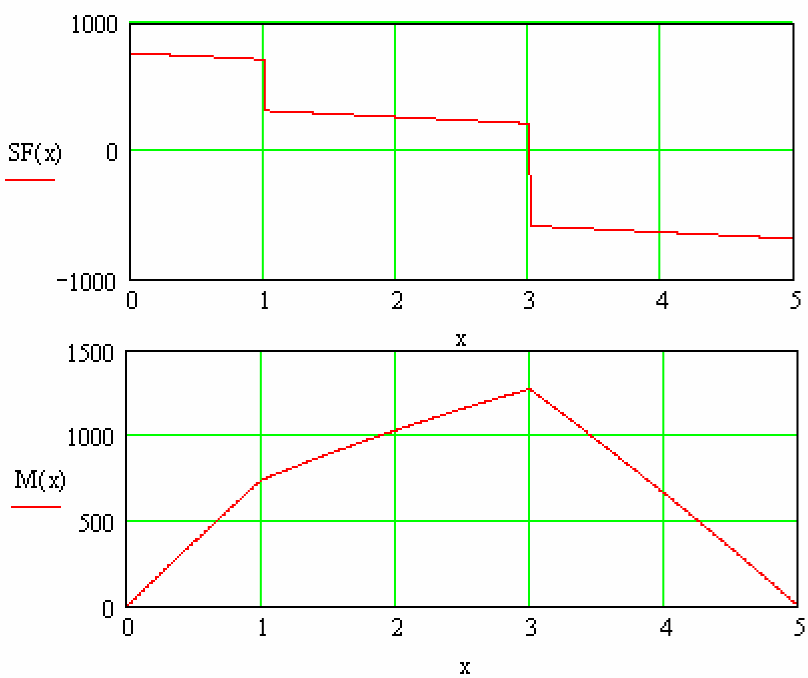
$Rt := F1 + F2 + w \cdot L \qquad Rt = 1.45 \times 10^3$

$$Rb := \frac{F1 \cdot A + F2 \cdot B + \frac{w \cdot L^2}{2}}{L} \qquad Rb = 685 \qquad Ra := Rt - Rb \qquad Ra = 765$$

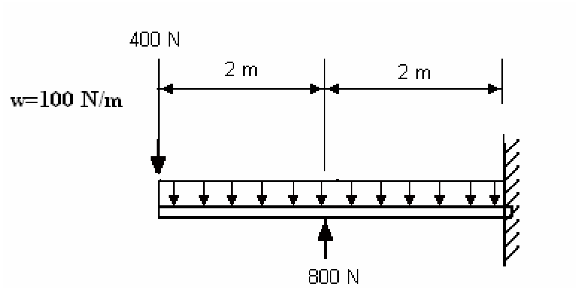
$x := 0, 0.01 .. 5$

$SF(x) := Ra - w \cdot x - F1 \cdot (x > 1) - F2 \cdot (x > 3)$

$$M(x) := Ra \cdot x - \frac{w \cdot x^2}{2} - F1 \cdot (x - 1) \cdot (x > 1) - F2 \cdot (x - 3) \cdot (x > 3)$$



SAE 5 Question 2.

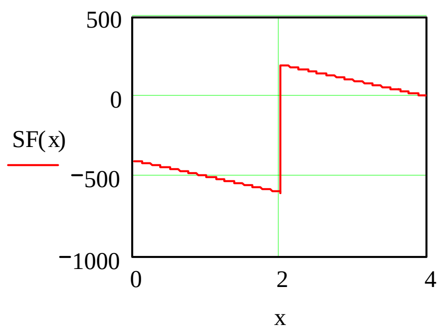


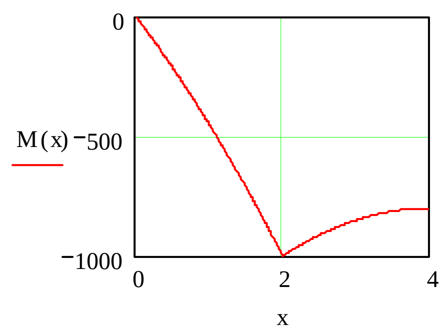
F1=-400 F2=800 A=2   L = 4   w=100

$F1 := -400$ $\quad F2 := 800$ $\quad A := 2$ $\quad L := 4$ $\quad w := 100$

$x := 0, 0.01..4 \quad SF(x) := F1 - w \cdot x + F2 \cdot (x > 2)$

$$M(x) := F1 \cdot x - \frac{w \cdot x^2}{2} + F2 \cdot (x - 2) \cdot (x > 2)$$



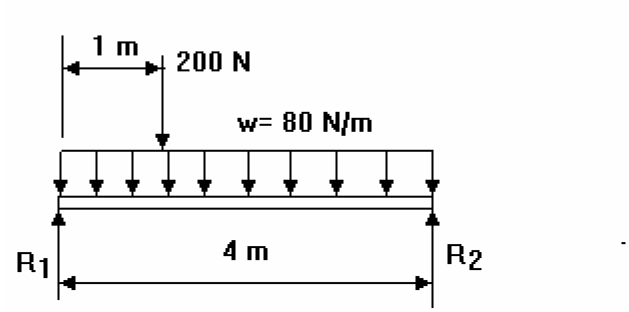


SAE 6

1. A beam is loaded as shown below. The beam has a second moment of area about its centroid of 5 x 10^{-6} m^4 and the distance to the edge from the centroid is 50 mm. Draw the bending moment diagram and determine

   i) The maximum bending moment. **(Answer 275 Nm)**

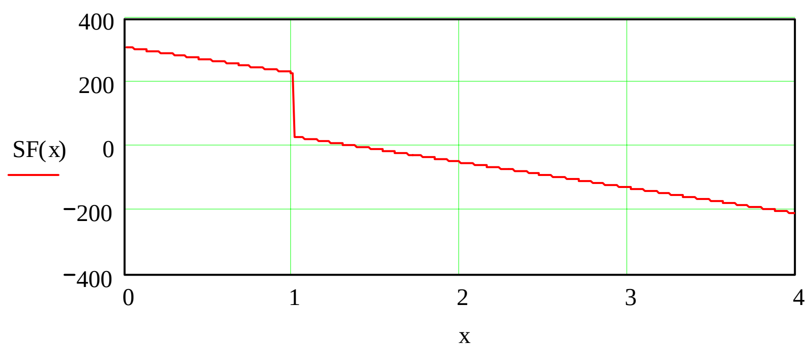
   ii) The maximum bending stress. **(Answer 2.75 MPa)**

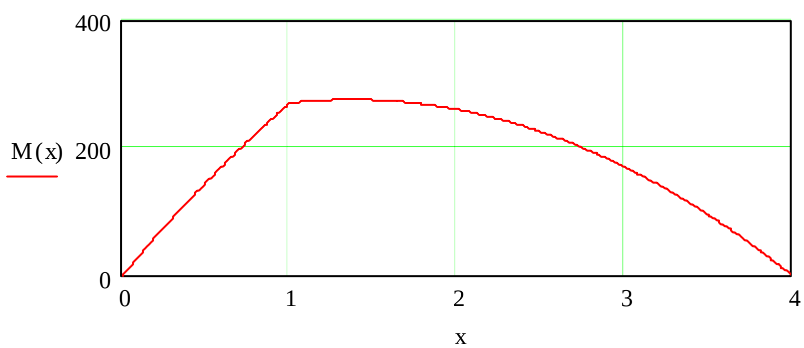


A = 1 L = 4   w = 80 F1=200   Rt=F1 + wL   Rt=520

$$Rb := \frac{F1 \cdot A + \frac{w \cdot L^2}{2}}{L} \qquad Rb = 210 \qquad Ra := Rt - Rb \qquad Ra = 310$$

$$x := 0, 0.01 .. L \qquad SF(x) := Ra - w \cdot x - F1 \cdot (x > A) \qquad M(x) := Ra \cdot x - \frac{w \cdot x^2}{2} - F1 \cdot (x - 1) \cdot (x > 1)$$





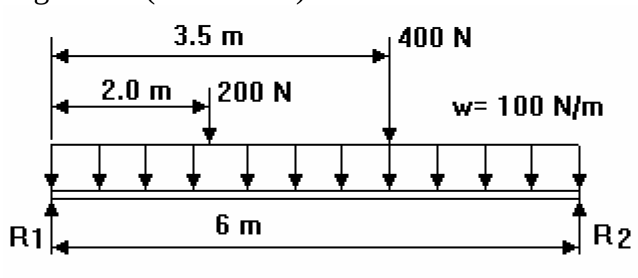
The SF diagram tells us the maximum BM is at x = 1 m and this is 275 Nm

σ = My/I = 275 x 0.05/5 x 10^{-6}= 2.75 MPa

2. A beam is loaded as shown below. The beam has a second moment of area about its centroid of 12 x 10^{-6} m^4 and the distance to the edge from the centroid is 70 mm. Draw the bending moment diagram and determine

i) The maximum bending moment. (1187.5 Nm)

ii) The maximum bending stress. (6.927 MPa)

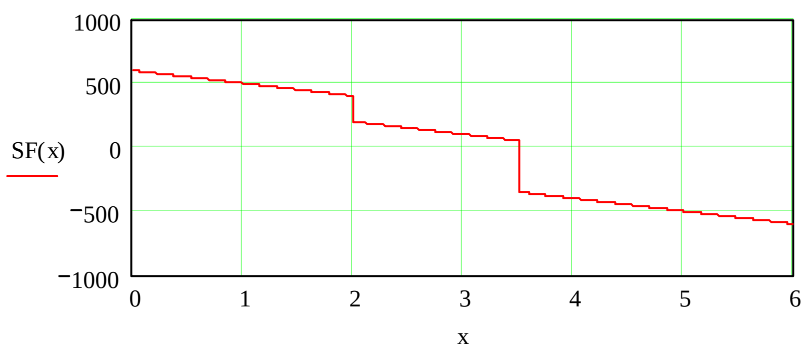


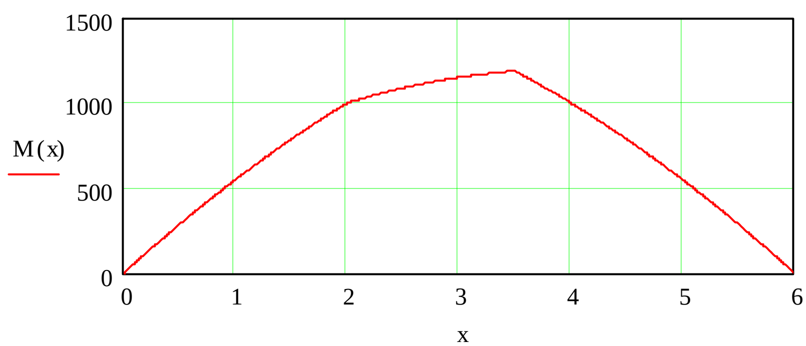
$Rt := F1 + F2 + w \cdot L \qquad Rt = 1.2 \times 10^3 \qquad Rb := \dfrac{F1 \cdot A + F2 \cdot B + \dfrac{w \cdot L^2}{2}}{L} \qquad Rb = 600$

$Ra := Rt - Rb \qquad Ra = 600$

$x := 0, 0.01 .. L$

$SF(x) := Ra - w \cdot x - F1 \cdot (x > A) - F2 \cdot (x > B) \qquad M(x) := Ra \cdot x - \dfrac{w \cdot x^2}{2} - F1 \cdot (x - A) \cdot (x > A) - F2 \cdot (x - B) \cdot (x > B)$





From the SF diagram the maximum BM occurs at x = 3.5 m and the max moment is 1187.5 Nm

$\sigma = My/I = 1187.5 \times 0.07/12 \times 10^{-6} = 6.927$ MPa

3. A light alloy tube of 10 cm outer diameter and 7.5 cm inner diameter rests horizontally on simple supports 4 m apart. A concentrated load of 500 N is applied to the tube midway between the supports.

Sketch diagrams of shear force and bending moments due to the applied load.

Determine the maximum bending moment and the corresponding maximum stress.

(Answers 500 Nm and 7.45 MPa)

SF(x)

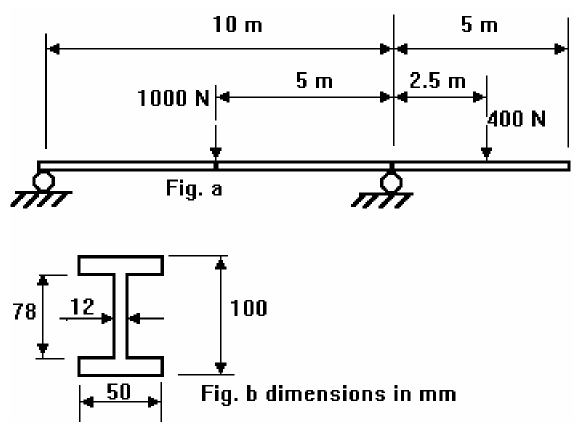
x
x

M(x)

x

This is a symmetrical system so Ra = Rb = 250 N
The maximum BM is at the middle M = 500 Nm
$I = \pi(0.1^4 - 0.075^4)/64 = 3.356 \times 10^{-6}\ m^4$
$\sigma = My/I = 500 \times 0.05/3.356 \times 10^{-6} = 7.45$ MPa

4. Part of a test rig consists of a 15 m long elastic beam which is simply supported at one end and rests on a frictionless roller located 5 m from the other end, as shown. The beam has a uniformly distributed load of 150 N/m due to its own weight and is subjected to concentrated loads of 1000 N and 400 N as shown.



(i) Calculate the reactions. **(Answers 962.5 N and 2687.5 N)**

(ii) Sketch the shear force and bending moment diagrams and determine the magnitude and location of the maximum values of the shear force and bending moment.
(Answers SF 1537.5 N 10 m from left end, BM 2 937.5 Nm 5 m from left end).

(ii) Determine the maximum bending stress in the beam if the section is that shown in and with bending about the neutral axis x-x. **(Answer 55.13 MPa)**

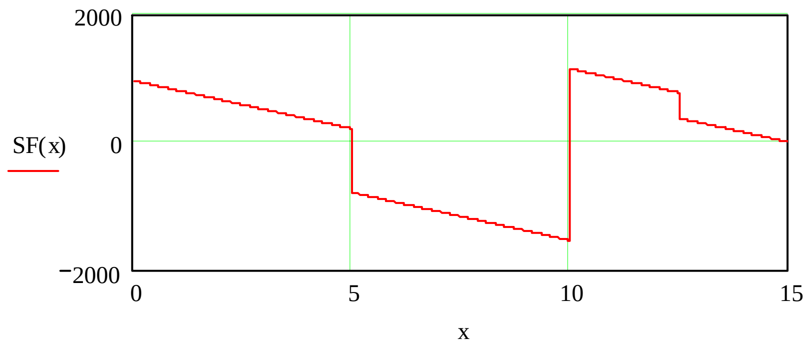
A=5 B = 10 C = 12.5 L = 15 w = 150 F1 = 1000 F2 = 400

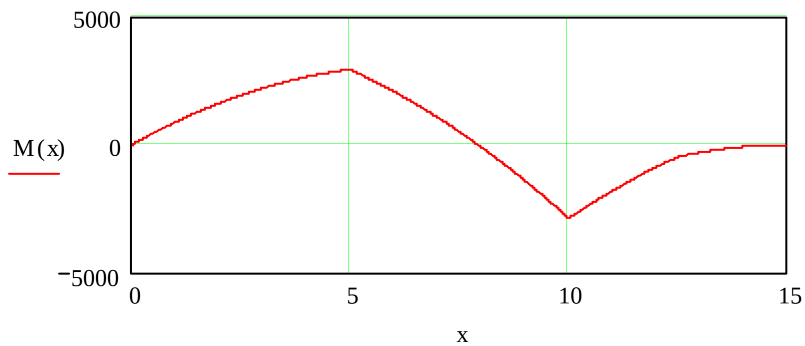
$Rt := F1 + F2 + w \cdot L \qquad Rt = 3.65 \times 10^{3}$

$$Rb := \frac{F1 \cdot A + F2 \cdot C + \frac{w \cdot L^2}{2}}{B} \qquad Rb = 2.688 \times 10^{3} \qquad Ra := Rt - Rb \qquad Ra = 962.5$$

$x := 0, 0.01 .. L \qquad SF(x) := Ra - w \cdot x - F1 \cdot (x > A) - F2 \cdot (x > C) + Rb \cdot (x > 10)$

$$M(x) := Ra \cdot x - \frac{w \cdot x^2}{2} - F1 \cdot (x - A) \cdot (x > A) - F2 \cdot (x - C) \cdot (x > C) + Rb \cdot (x - B) \cdot (x > B)$$



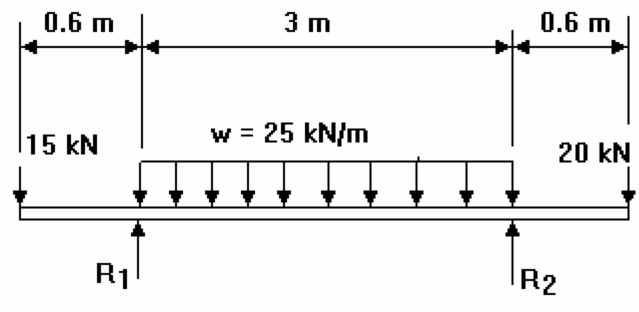


The maximum SF is 1537.4 N at x = 10 m The max BM is 2937.5 Nm at x = 5 m

$I = 50 \times 100^3/12 - 38 \times 78^3/12 = 2.664 \times 10^6$ mm^4

$\sigma = My/I = 2937.5 \times 0.05/2.664 \times 10^{-6} = 55.13$ MPa

5. A horizontal beam 4.2 m long, resting on two simple supports 3.0 m apart, carries a uniformly distributed load of 25 kN/m between the supports with concentrated loads of 15 kN and 20 kN at the ends, as shown.
Assuming the weight of the beam is negligible determine the reactions R_1 and R_2 at the supports.
(Answers 51.5 kN and 58.5 N)
Sketch diagrams of shear force and bending moments and indicate the point of maximum bending moment.
State the greatest bending moment and shear force. **(Answers -38.5 kN and 18 kNm)**
The beam has a uniform rectangular cross-section, the depth being equal to 1.5 times the width. Determine the size of the section required to limit the maximum bending stress to 375 MN/m^2.
(Answers 50 mm wide and 75.6 mm deep)



A = 0.6 B = 3.6 L = 4.2 w = 25 F1 = 15 F2 = 20

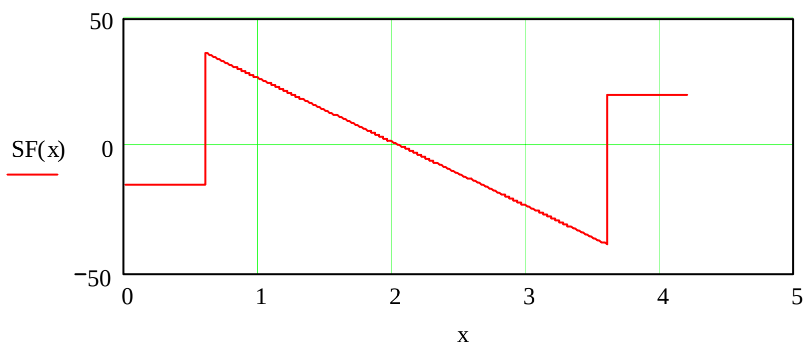
$Rt := F1 + F2 + w\cdot(B - A) \quad Rt = 110$

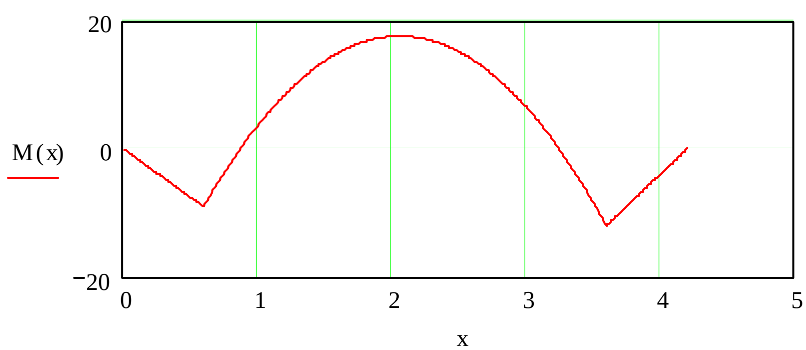
$$Rb := \frac{-F1\cdot A + F2\cdot(L - A) + \frac{w\cdot(B-A)^2}{2}}{B - A}$$

$Rb = 58.5 \quad Ra := Rt - Rb \quad Ra = 51.5 \qquad x := 0, 0.01..L$

$$SF(x) := -F1 + Ra\cdot(x > A) - w\cdot(x - A)\cdot(x > A) + Rb\cdot(x > B) + w\cdot(x - B)\cdot(x > B)$$

$$M(x) := -F1\cdot x + Ra\cdot(x - A)\cdot(x > A) - w\cdot\frac{(x-A)^2}{2}\cdot(x > A) + w\cdot\frac{(x-B)^2}{2}\cdot(x > B) + Rb\cdot(x - B)\cdot(x > B)$$





$\sigma = My/I \quad I = BD^3/12 = B(1.5B)^3/12 \qquad y = 1.5B/2$
$375 \times 10^6 = 18000 \times (1.5B/2)/\{B(1.5B)^3/12\}$ hence B = 0.05 m and D = 0.0756 m