CONTROL SYSTEMS ENGINEERING D227
SOLUTIONS Q 2003

(a) Sketch the root locus of a unity feed back control system with the open loop transfer function $G(ol) = \dfrac{k}{s(s+3)(s+8)}$ For what value of k is the system stable?

(b) If a zero at s = -2 is introduced into the function of part (a), obtain the modified root locus. Describe the effect that the introduction of the zero has upon the performance.

SOLUTION (a)

(a) $G(ol) = \dfrac{k}{s(s+3)(s+8)}$ $\quad G(cl) = \dfrac{G(ol)}{G(ol)+1}$

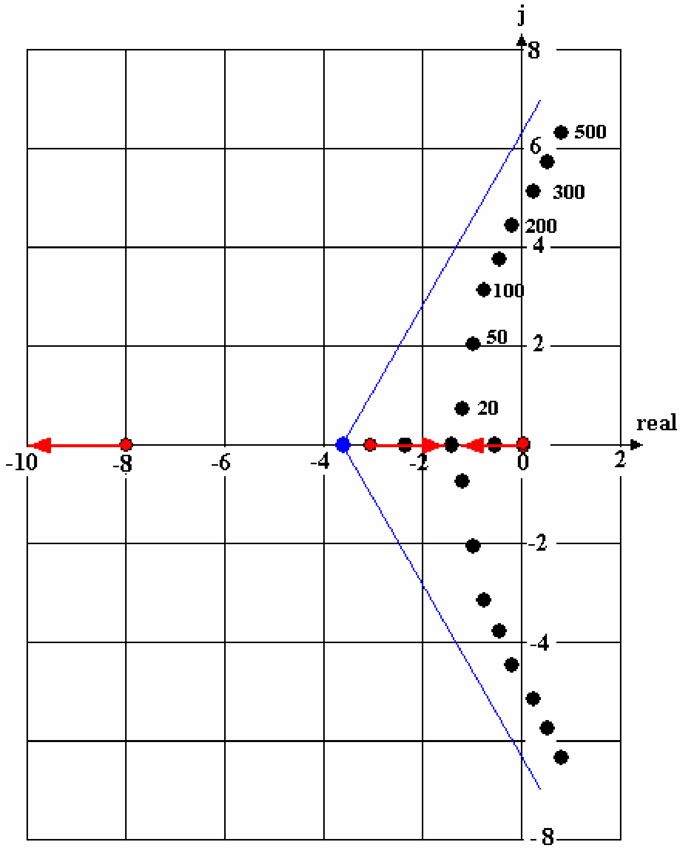
The characteristic equation is $\dfrac{k}{s(s+3)(s+8)} + 1$

There are three poles at 0, -3 and -8

For three poles the asymptotes form an angle of 60^o and -60^o to the real axis.

The intercept with the real axis is at (0 – 3 – 8)/3 = 3.67

The value of k at the point where the locus cuts the imaginary axis is found from the Routh Hurwitz criteria. The characteristic equation may be written as:-



s(s+3)(s+8) + k = 0

multiply out and $s^3 + 11s^2 + 24s^1 + ks^o$

The highest power is 3 so use the simplified test.

a = 1 b = 11 c = 24 d = k

R = c - ad/b = 24 - k/11 and this is zero when the locus cuts the imaginary axis.

24 = k/11 k = 264 Any value of k larger than this produces instability.

The asymptotes may now be drawn. One locus starts at -8 (k = 0) and moves to $-\infty$ along the real axis.

The other loci start at 0 and -3 and meet at some point before breaking away at 90^o as mirror images and eventually forming asymptotes. It would be better if we knew the break away point.

We are finding the roots of $s^3 + 11s^2 + 24s^1 + ks^o$. If a calculator that solves roots is available then the roots can be solved for various values of k and a precise plot obtained as shown.

(b) With a zero at -2 the intercept of the asymptotes is {(0 – 3 – 8) – (-2)}/2 = 4.5
One loci starts at 0 and ends at -2

The other loci start at -8 and ends at - 4.5 + j∞ one and the other starts at -3 and ends at - 4.5 - j∞
The angle of the asymptotes is 90^{o} to the real axis
The system is stable for all values of k and less oscillatory as the gain is increased.