Q 7 Twelve $10 \mathrm{k} \Omega$ resistors are connected as shown.
Determine the voltage at each node if:-
a) a 6 V battery is connected between O and B
b) a 6 V battery is connected between O and H

a) Because of symmetry the voltage at A D and G must be 3 V

Examining node C we may deduce the current flow is as shown since $\mathrm{V}_{\mathrm{C}}$ must be less than 3 V
Working in mA , Ohms Law gives
$\mathrm{I}_{1}=\mathrm{V}_{\mathrm{C}} / 10 \mathrm{~K}=0.1 \mathrm{~V}_{\mathrm{C}}$
$\mathrm{I}_{2}=\left(3-\mathrm{V}_{\mathrm{C}}\right) / 20 \mathrm{k}=0.15-0.05 \mathrm{~V}_{\mathrm{C}}$
$\mathrm{I}_{3}=\left(3-\mathrm{V}_{\mathrm{C}}\right) / 10 \mathrm{k}=0.3-0.1 \mathrm{~V}_{\mathrm{C}}$
Kirchoff's rule gives
$\mathrm{I}_{1}=\mathrm{I}_{2}+\mathrm{I}_{3}$
$0.1 \mathrm{~V}_{\mathrm{C}}=0.15-0.05 \mathrm{~V}_{\mathrm{C}}+0.3-0.1 \mathrm{~V}_{\mathrm{C}}$
$0.25 \mathrm{~V}_{\mathrm{C}}=0.45$
$\mathrm{V}_{\mathrm{C}}=1.8 \mathrm{~V}$

$\mathrm{I}_{1}=0.18 \mathrm{~mA}$
$\mathrm{I}_{2}=0.06 \mathrm{~mA}$
$\mathrm{I}_{3}=0.12 \mathrm{~mA}$
We can easily deduce the other currents as shown Now use Ohms Law to find voltages.
$\mathrm{V}_{\mathrm{F}}=3-0.06 \times 10=2.4 \mathrm{~V}$
$\mathrm{V}_{\mathrm{E}}=6-0.18 \times 10=4.2 \mathrm{~V}$
$\mathrm{V}_{\mathrm{H}}=3+0.06 \times 10=3.6 \mathrm{~V}$

b) Again by symmetry we can deduce that $\mathrm{V}_{\mathrm{G}}, \mathrm{V}_{\mathrm{D}}$ and $\mathrm{V}_{\mathrm{A}}$ are 3 V

Again examining node C we deduce the voltage is between 3 and 0 so the currents flow as shown.
$\mathrm{I}_{1}=\mathrm{V}_{\mathrm{C}} / 10 \mathrm{~K}=0.1 \mathrm{~V}_{\mathrm{C}}$
$\mathrm{I}_{2}=\left(3-\mathrm{V}_{\mathrm{C}}\right) / 10 \mathrm{k}=0.3-0.1 \mathrm{~V}_{\mathrm{C}}$
$\mathrm{I}_{3}=\left(3-\mathrm{V}_{\mathrm{C}}\right) / 10 \mathrm{k}=0.3-0.1 \mathrm{~V}_{\mathrm{C}}$
Kirchoff's rule gives
$\mathrm{I}_{1}=\mathrm{I}_{2}+\mathrm{I}_{3}$
$0.1 \mathrm{~V}_{\mathrm{C}}=0.3-0.1 \mathrm{~V}_{\mathrm{C}}+0.3-0.1 \mathrm{~V}_{\mathrm{C}}$
$0.3 \mathrm{~V}_{\mathrm{C}}=0.6 \quad \mathrm{~V}_{\mathrm{C}}=2 \mathrm{~V}$
By symmetry $\mathrm{V}_{\mathrm{A}}=2 \mathrm{~V}$
$\mathrm{I}_{1}=0.2 \mathrm{~mA} \quad \mathrm{I}_{2}=0.1 \mathrm{~mA} \quad \mathrm{I}_{3}=0.1 \mathrm{~mA}$
Now use Ohms Law to find voltages.
$\mathrm{V}_{\mathrm{G}}=3+0.1 \times 10=4 \mathrm{~V}$
By symmetry $\mathrm{V}_{\mathrm{E}}=4 \mathrm{~V}$


