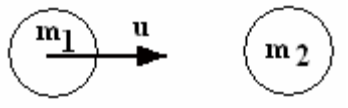
ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2005

Q 4 A particle of mass m1 has a velocity u towards a stationary second particle of mass m2 as shown. Determine expressions for the velocities of each particle following their collisions. The coefficient of restitution is e.



The total momentum before a collision is equal to the total momentum after the collision.

After collision the velocities change to v_1 and v_2 respectively.

The initial momentum = $m_1 u_1 + m_2 (0) = m_1 u_1$

The Final momentum = $m_1 v_1 + m_2 v_2$

By the law of conservation of momentum we have $m_1u_1 = m_1v_1 + m_2v_2$

$$v_2 = \frac{m_1(u_1 - v_1)}{m_2} \qquad v_1 = u_1 - \frac{m_2(v_2)}{m_1}$$

$$e = -\frac{v_1 - v_2}{u_1 - u_2} \quad \text{but } u_2 = 0 \text{ so } e = -\frac{v_1 - v_2}{u_1}.$$

$v_1 = v_2 - eu_1$ $\qquad$ $v_2 = v_1 + eu_1$

$$v_1 = \frac{m_1(u_1 - v_1)}{m_2} - eu_1 = \frac{m_1(u_1)}{m_2} - \frac{m_1(v_1)}{m_2} - eu_1$$

$$v_1\left(1 + \frac{m_1}{m_2}\right) = \frac{m_1(u_1)}{m_2} - eu_1$$

$$v_1 = \frac{\frac{m_1(u_1)}{m_2} - eu_1}{\left(1 + \frac{m_1}{m_2}\right)} = \frac{(m_1 - em_2)u_1}{(m_2 + m_2)}$$

$$v_2 = u_1 - \frac{m_2(v_2)}{m_1} + eu_1 = u_1(1+e) - \frac{m_2(v_2)}{m_1}$$

$$v_2\left(1 + \frac{m_2}{m_1}\right) = u_1(1+e)$$

$$v_2 = \frac{u_1(1+e)}{1 + \frac{m_2}{m_1}}$$

$$v_2 = \frac{u_1 m_1(1+e)}{m_1 + m_2}$$