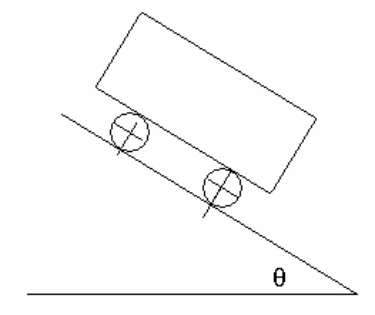
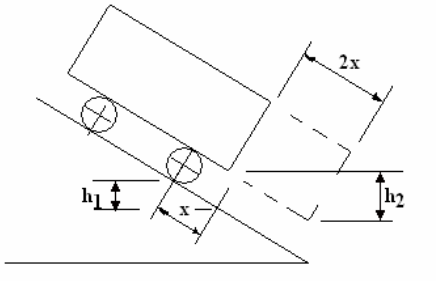
ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2005

Q 3 Two rollers of radius r, mass m and moment of inertia I support a truck of mass M as shown. Determine an expression for the acceleration of the block down the slope whilst it remains in contact with both rollers. There is no slip at the contact points.



SOLUTION



The key to the solution is to realise that the truck moves twice the distance of the roller centres so the velocity and acceleration is also double.

Let the truck move a distance 2x down the plane. The wheels move distance x
The velocity of the roller is v and the velocity of the truck is 2v
The acceleration of the roller is a and the acceleration of the truck is 2a

Potential Energy lost by the truck = $Mgh_2 = Mg\ 2x\ \sin\theta$
Potential energy lost by rollers = $2\ mgh_1 = 2mg\ x\ \sin\theta$

Kinetic Energy gained by rollers = $2\ m(v)^2/2 + 2\ I\ (v)^2/2r^2 = m\ (v)^2 + I\ (v)^2/r^2$
Kinetic Energy gained by truck = $M(2v)^2/2 = 2M(v)^2$
Equate Energies

$Mg\ 2x\ \sin\theta + 2mg\ x\ \sin\theta = 2M(v)^2 + m\ (v)^2 + I\ (v)^2/r^2$
$2x\ g\ \sin\theta\ (M + m) = (v)^2\ (2M + m + I/r^2)$

$v^2 = 2x\ g\ \sin\theta\ (M + m)/(2M + m + I/r^2)$

Distance moved $x = vt/2 \quad t = 2x/v$
$a = v/t = v^2/2x$

$$a = \frac{2xg\sin\theta(M+m)}{2M+m+I/r^2} \times \frac{1}{2x} = \frac{g\sin\theta(M+m)}{2M+m+\frac{I}{r^2}}$$

$$\text{Acceleration of truck} = 2a = \frac{g\sin\theta(M+m)}{M+\frac{m}{2}+\frac{I}{2r^2}}$$