ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2004

Q6

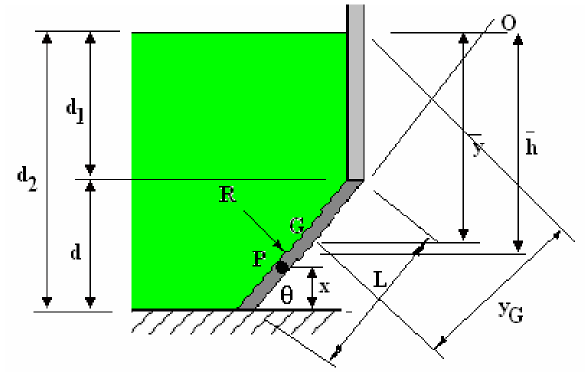
The hydraulic gate shown is designed to open by turning about a hinge at P when the water level exceeds depth d_2

a) Show that the resultant torque on the on the gate is given by

$$\frac{\rho g B d}{6\sin^2\theta}\left[(2x-d)3d_2+(2d-3x)d\right]$$

where ρ is the density and B the width of the gate.

(b) Determine the depth of water d_2 at which the gate opens if x = 2 m, B = 10m and d = 5 m.



$R = \rho g A \bar{y} \qquad A = B \text{ x } L$

$L = (d_2 - d_1)/\sin\theta$

$A = B\,(d_2 - d_1)/\sin\theta$

$\bar{y} = (d_2 + d_1)/2$

$R = (\rho g B)\,(d_2 - d_1)\,(d_2 + d_1)/2\sin\theta$

$R = (\rho g B)\,(d_2^2 + d_1^2)/2\sin\theta$

$$y_{cp} = \frac{I_O}{Ay_G}$$

Using the parallel axis theorem $I_O = I_G + Ay_G^2$

$I_G = BL^3/12 \qquad y_G = (d_2 + d_1)/2\sin\theta$

$$I_O = \frac{B}{12}(L)^3 + Ay_G{}^2$$

$$y_{cp} = \frac{I_O}{Ay_G} = \frac{\frac{B}{12}(L)^3 + Ay_G{}^2}{Ay_G}$$

$$y_{cp} = \frac{\bar{h}}{\sin\theta} = \frac{I_O}{Ay_G} = \frac{\frac{B}{12}(L)^3 + A\left\{\frac{\bar{y}}{\sin\theta}\right\}^2}{A\left\{\frac{\bar{y}}{\sin\theta}\right\}}$$

$$\bar{h} = \frac{\left[\frac{B}{12}(L)^3 + A\left\{\frac{\bar{y}}{\sin\theta}\right\}^2\right]\sin^2\theta}{A\bar{y}}$$

$$\bar{h} = \left[\frac{B}{12}\frac{(L)^3}{A\bar{y}} + \frac{A}{A\bar{y}}\left\{\frac{\bar{y}}{\sin\theta}\right\}^2\right]\sin^2\theta$$

$$\bar{h} = \frac{B}{12A\bar{y}}(L)^3\sin^2\theta + \bar{y} \qquad \bar{h} = \frac{B}{12BL\bar{y}}(L)^3\sin^2\theta + \bar{y} \qquad \bar{h} = \frac{(L)^2\sin^2\theta}{12\bar{y}} + \bar{y}$$

$$L\sin\theta = d \qquad \bar{h} = \frac{2d^2}{12(d_2+d_1)} + \frac{(d_2+d_1)}{2} \qquad \bar{h} = \frac{d^2}{6(d_2+d_1)} + \frac{(d_2+d_1)}{2}$$

$$d_1 = d_2 - d \qquad \bar{h} = \frac{d^2}{6(2d_2-d)} + \frac{(2d_2-d)}{2}$$

The vertical distance from the hinge to the centre of pressure is $\bar{h} - (d_2 - x) = \bar{h} - d_2 + x$

The moment arm is hence $(\bar{h} - d_2 + x)/\sin\theta$

$M = R\,(\bar{h} - d_2 + x)/\sin\theta$

$$M=\frac{\rho gB(d_2-d_1)(d_2+d_1)}{2\sin^2\theta}\left\{\frac{d^2}{6(2d_2-d)}+\frac{(2d_2-d)}{2}-d_2+x\right\}$$

$$M=\frac{\rho gB(d)(2d_2-d)}{2\sin^2\theta}\left\{\frac{d^2}{6(2d_2-d)}+\frac{(2d_2-d)}{2}-d_2+x\right\}$$

$$M=\frac{\rho gB(d)}{2\sin^2\theta}\left\{\frac{d^2}{6}+\frac{4d_2^2+d^2-4dd_2}{2}-d_2(2d_2-d)+x(2d_2-d)\right\}$$

$$M=\frac{\rho gB(d)}{2\sin^2\theta}\left\{\frac{d^2}{6}+\frac{4d_2^2+d^2-4dd_2}{2}-2d_2^2+dd_2+2d_2x-dx\right\}$$

$$M=\frac{\rho gB(d)}{12\sin^2\theta}\left\{d^2+12d_2^2+3d^2-12dd_2-12d_2^2+6dd_2+12d_2x-6dx\right\}$$

$$M=\frac{\rho gB(d)}{12\sin^2\theta}\left\{4d^2-6dd_2+12d_2x-6dx\right\}$$

$$M=\frac{\rho gB(d)}{6\sin^2\theta}\left\{2d^2-3dd_2+6d_2x-3dx\right\}$$

$$M=\frac{\rho gB(d)}{6\sin^2\theta}\left\{(2x-d)3d_2+(2d-3x)d\right\}$$

(b) The gate opens when M =0

$(2x-d)3d_2+(2d-3x)d=0$

$(2x-d)3d_2=-(2d-3x)d$

x = 2 m, B = 10m and d = 5 m.

$(4-5)3d_2=-(10-6)5=-20$

$d_2=20/3=6.67$ m