ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2004

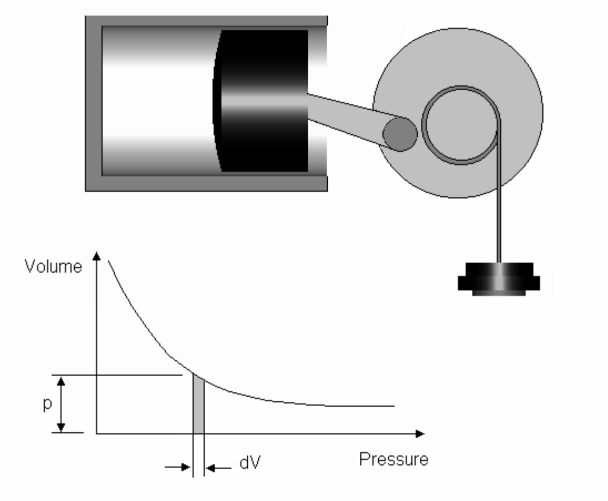
Q 5 The properties of gas follow the relationship pv = RT where p is the pressure, v the specific volume , T the temperature and R is a constant. This gas undergoes a process between states 1 and 2 such that $pv^n = C$

(a) Show that the work done by a unit mass in its surroundings is

$$\frac{p_2v_2 - p_1v_1}{n-1}$$

(b) Show that the increase in specific internal energy of the gas is $\frac{p_2v_2 - p_1v_1}{\gamma - 1}$ where γ is the ratio of the specific heats. It may be assumed that the specific heats are constant.

SOLUTION



$$W = -\int_{V_1}^{V_2} p\,dV \quad \text{but} \quad p = CV^{-n}$$

$$W = -C\int_{V_1}^{V_2} V^{-n}dV = -C\frac{\left[V_2^{-n+1} - V_1^{-n+1}\right]}{-n+1}$$

$$\text{Since } C = p_1V_1 \text{ or } p_2V_2 \qquad W = \frac{\left[p_2V_2 - p_1V_1\right]}{n-1}$$

For a unit mass V becomes v

Change in internal energy $\Delta U = M\,c_v\,(T_2 - T_1)$ or for a unit mass $\Delta u = c_v\,(T_2 - T_1)$

Substitute T = pv/R

$$\Delta u = \frac{c_v}{R}\left(p_2v_2 - p_1v_1\right)$$

$$c_p - c_v = R \quad c_p = R + c_v \qquad \frac{c_p}{c_v} = \gamma$$

$$c_v = \frac{c_p}{\gamma} = \frac{R + c_v}{\gamma} \qquad c_v\gamma = R + c_v$$

$c_v\gamma - c_v = R$

$c_v\,(\gamma - 1) = R$

$$\frac{cv}{R} = \frac{1}{\gamma - 1}$$

$$\Delta u = \frac{p_2v_2 - p_1v_1}{\gamma - 1}$$