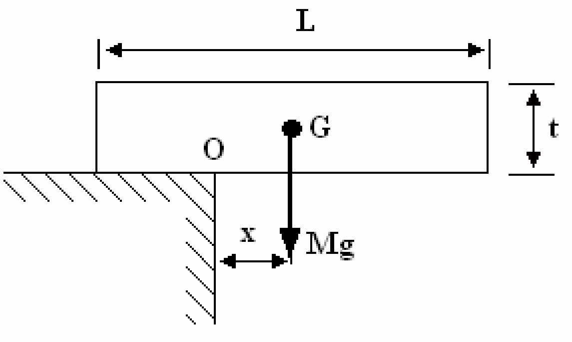
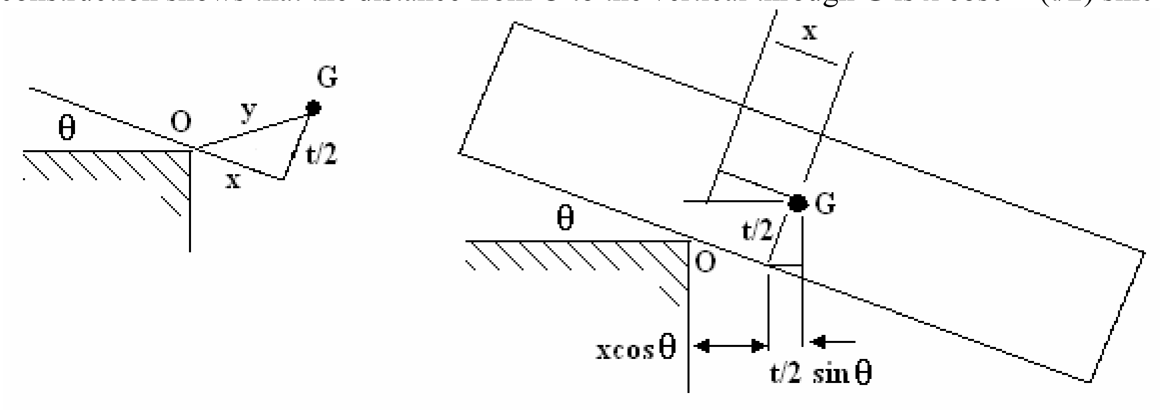
Q 4 The uniform block of mass M is released from the position shown. If no slip occurs between the block and the support at point O, show that the angular acceleration of the block is given by $\alpha = \frac{6g(2x\cos\theta + t\sin\theta)}{L^2 + 12x^2 + 4t^2}$ where θ is the angle between the block upper surface and the horizontal.

The moment of inertia about the centre of gravity is $I_G = \frac{M(L^2 + t^2)}{12}$



SOLUTION

The construction shows that the distance from O to the vertical through G is x cosθ + (t/2) sinθ



The torque acting on the body is T = Mg { x cosθ + (t/2) sinθ } = (Mg/2){ 2x cosθ + t sinθ }

From Newton's second Law T = I α

$I_G = \frac{M(L^2 + t^2)}{12}$ We need I_O and to get this we use the parallel axis theorem.

$I_O = I_G + My^2$ where y is the distance between the parallel axis. This is obtained from Pythagoras.

$y = \sqrt{\left(\frac{t}{2}\right)^2 + x^2}$ hence $I_O = \frac{M(L^2 + t^2)}{12} + M\left(\left(\frac{t}{2}\right)^2 + x^2\right)$

$$I_O = M\left[\frac{(L^2 + t^2)}{12} + \frac{t^2}{4} + x^2\right]$$

$$\alpha = \frac{Mg\{2x\cos\theta + t\sin\theta\}}{2M\left[\frac{(L^2 + t^2)}{12} + \frac{t^2}{4} + x^2\right]} = \frac{g\{2x\cos\theta + t\sin\theta\}}{\left[\frac{(L^2 + t^2)}{6} + \frac{t^2}{2} + 2x^2\right]} = \frac{6g\{2x\cos\theta + t\sin\theta\}}{[L^2 + t^2 + 3t^2 + 12x^2]}$$

$$\alpha = \frac{6g\{2x\cos\theta + t\sin\theta\}}{[L^2 + 4t^2 + 12x^2]}$$