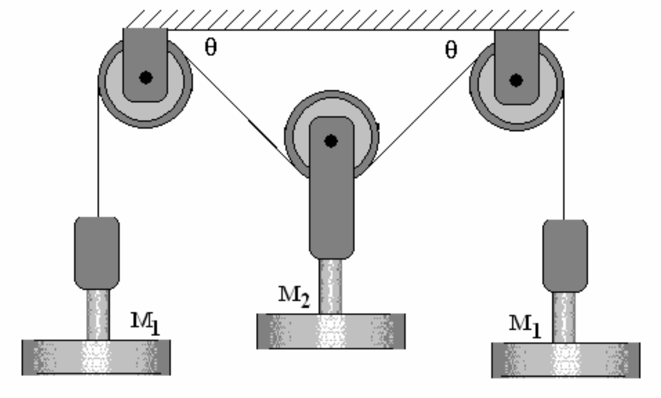
ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2004

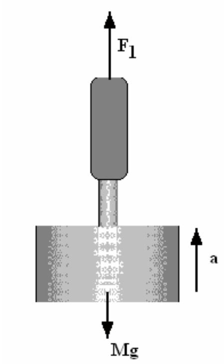
Q 3 Each pulleys has a radius r and moment of inertia I. The mass of each of the suspended components is M and in the case of the centre one, this includes the mass of the pulley. If the centre one is released show that the initial downward acceleration is

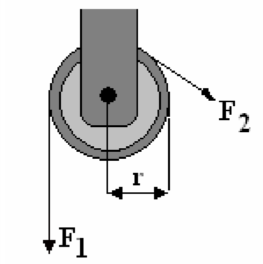
$$\frac{(1-2\sin\theta)Mg}{M+2\left(M+\frac{I}{r^2}\right)\sin^2\alpha}$$



Determine the angle θ when the system is in static equilibrium. Assume that the system is symmetrical at all times and that the centre mass only moves in a vertical direction. The radius r is small compared to the distances between the pulleys. Friction should be ignored.

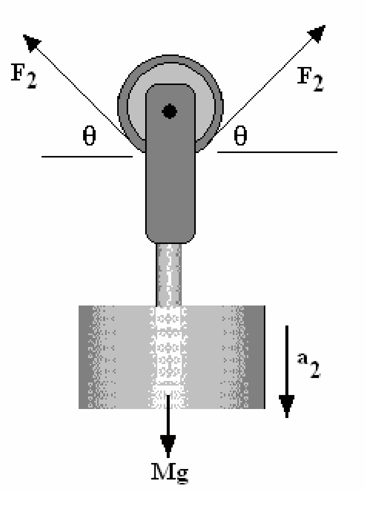
SOLUTION





Consider the free body diagram for one of the side masses.
$F_1 = M(g + a_1)$
Consider the pulley.
Torque = $(F_2 - F_1)r = I\alpha$
$\alpha = a_1/r$
$F_2 = (Ia_1/r^2) + F_1 = (Ia_1/r^2) + M(g + a_1)$

Now consider the middle mass and pulley.



$2F_2\sin\theta = M(g-a_2)$

$$2\left[\frac{Ia_1}{r^2}+Ma_1+Mg\right]\sin\theta = Mg - Ma_2$$

$$\left[\frac{Ia_1}{r^2}+Ma_1+Mg\right]\sin\theta = \frac{Mg}{2}-\frac{Ma_2}{2}$$

$$a_1\left[\frac{I}{r^2}+M\right]\sin\theta = \frac{Mg}{2}-\frac{Ma_2}{2} - M\,g\sin\theta$$

$$2a_1\left[\frac{I}{r^2}+M\right]\sin\theta = Mg(1\text{-}2\sin\theta) - Ma_2$$

The ropes on each side are accelerating in their line of action at a_1
It follows that $a_1 = a_2 \sin\theta$
(so if $\theta = 90^\circ$ $a_1 = a_2$ and if $\theta = 0$ then $a_2 = \infty$)

$$2a_2\left[\frac{I}{r^2}+M\right]\sin^2\theta = Mg(1\text{-}2\sin\theta) - Ma_2$$

$$Ma_2 + 2a_2\left[\frac{I}{r^2}+M\right]\sin^2\theta = Mg(1\text{-}2\sin\theta) \qquad a_2\left\{M+2\left[\frac{I}{r^2}+M\right]\sin^2\theta\right\} = Mg(1\text{-}2\sin\theta)$$

$$a_2 = \frac{Mg(1\text{-}2\sin\theta)}{\left\{M+2\left[\frac{I}{r^2}+M\right]\sin^2\theta\right\}}$$

For static equilibrium $a_2 = 0$ hence $\sin\theta = ½$ $\theta = 30^\circ$