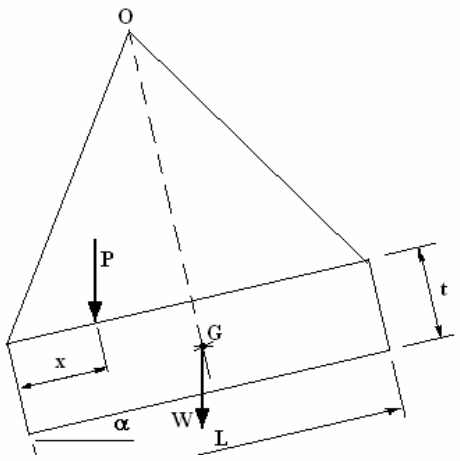
ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2004

Q 2 A uniform rigid beam of length L and weight W is suspended between two cables of equal length as shown. A force Pacts vertically downwards as shown.

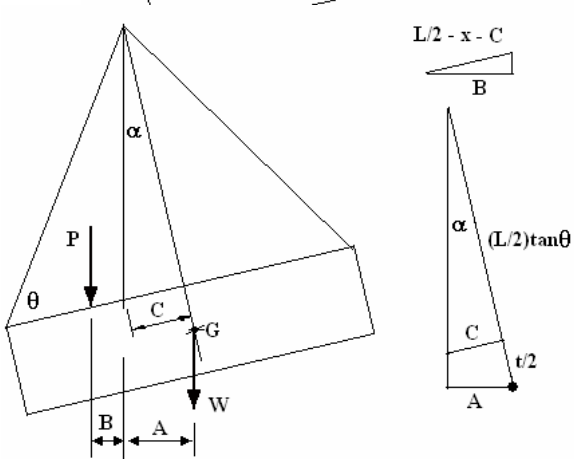


(a) determine an expression for the equilibrium angle of inclination of the beam to the horizontal.

(b) Determine an expression for the tension in each cable.

SOLUTION

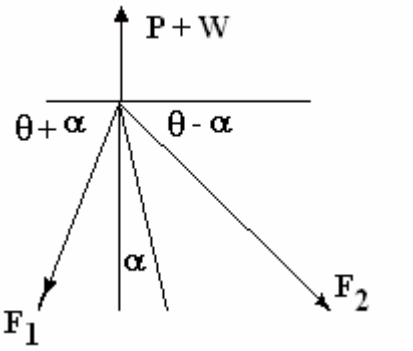
(a) These problems are largely geometry.



Moments about point O give WA = P B
With the aid of the triangles shown we find:
A = {(L/2)tanθ + (t/2)}sinα
C = (L/2)tanθ tan α
B = (L/2 – x –C)cosα
B = {L/2 – x –(L/2)tanθ tan α }cosα

$$P = \frac{WA}{B} = \frac{W\left[\frac{L}{2}\tan\theta + \frac{t}{2}\right]\sin\alpha}{\left[\frac{L}{2} - x - \frac{L}{2}\tan\theta\text{an}\theta t\right]\cos\alpha}$$

$$P = \frac{W\left[L\tan\theta + t\right]\tan\alpha}{\left[L - 2x - L\tan\theta\tan\alpha\right]} = \frac{W\left[L\tan\theta + t\right]\tan\alpha}{L(1 - \tan\theta\tan\alpha) - 2x}$$



(b) Resolve forces vertically and horizontally

$P + W = F_1\sin(\theta + \alpha) + F_2\sin(\theta - \alpha)$
$F_1\cos(\theta + \alpha) = F_2\cos(\theta - \alpha)$
$F_1 = F_2\cos(\theta - \alpha)/\cos(\theta + \alpha)$

$$P + W = \frac{F_2\cos(\theta - \alpha)}{\cos(\theta + \alpha)}\sin(\theta + \alpha) + F_2\sin(\theta - \alpha)$$

$$P + W = F_2\left[\frac{\cos(\theta - \alpha)}{\cos(\theta + \alpha)}\sin(\theta + \alpha) + \sin(\theta - \alpha)\right]$$

$$P + W = F_2\left[\cos(\theta - \alpha)\tan(\theta + \alpha) + \sin(\theta - \alpha)\right]$$

$$F_2 = \frac{P + W}{\left[\cos(\theta - \alpha)\tan(\theta + \alpha) + \sin(\theta - \alpha)\right]}$$

$$F_1 = \frac{P + W}{\left[\cos(\theta - \alpha)\tan(\theta + \alpha) + \sin(\theta - \alpha)\right]}\frac{\cos(\theta - \alpha)}{\cos(\theta + \alpha)}$$

$$F_1 = \frac{P + W}{\left[\tan(\theta + \alpha)\right]\cos(\theta + \alpha) + \tan(\theta - \alpha)}$$

No doubt we could substitute for P and simplify but this seems to require a lot of work.