ENGINEERING SCIENCE C103
EXAM SOLUTIONS 2004

Q 1 A thin spherical shell of thickness t and diameter D is subjected to internal and external pressures p_i and p_e respectively.

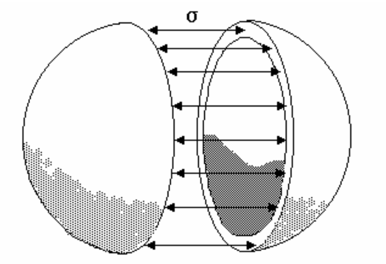
(a) Show that the circumferential strain recorded by a strain gauge attached to the outer surface is

$$\varepsilon = \frac{D}{4tE}(1-\nu)(p_i - p_e)$$

(b) A spherical pressure vessel of diameter 0.8 m made of steel of thickness 1.5 mm is to be pressure tested. Initially the vessel is filled with water at atmospheric pressure . What volume of additional water must be pumped from atmospheric pressure into the vessel to raise the gauge pressure to 10 bar? It may be assumed that the external pressure is atmospheric.

For steel E = 210 GPa and υ = 0.3 K = 2.2 GPa for water

(a) A sphere will tend to split about a diameter as shown.



The stress produced in the metal is the circumferential stress σ. Now consider the forces trying to split the sphere. The pressure difference is Δp and force due to this is

$$F = \Delta p A = \frac{\Delta p D^2}{4}$$

So long as the material holds this is balanced by the stress in the material. The force due to the stress is

$$F = \sigma \text{ multiplied by the area of the metal} = \sigma\,\pi\,Dt \qquad \sigma = \frac{\Delta p D}{4t}$$

Consider a small rectangular section of the wall of a thin walled sphere. There are two identical stresses mutually perpendicular. The circumferential strain is

$$\varepsilon = \frac{\sigma}{E} - \nu\frac{\sigma}{E} = \frac{\sigma}{E}(1-\nu) = \frac{\Delta p D}{4tE}(1-\nu)$$

$\varepsilon = \dfrac{D}{4tE}(1-\nu)(p_i - p_o)$ This is the strain on any circumference and is hence the diametral strain.

(b) The volumetric strain is 3 ε hence $\varepsilon_v = \dfrac{3D}{4tE}(1-\nu)(p_i - p_o)$

$$\varepsilon_v = \frac{3 \text{ x } 0.8}{4 \text{ x } 0.0015 \text{ x } 210 \text{ x } 10^9}(1-0.3)(10 \text{ x } 10^5) = 1.333 \text{ x } 10^{-3}$$

Initial volume of uncompressed water = $\pi D^3/6$ = 268.083 x 10^6 mm^3

Change in volume of sphere = $\Delta V = V\,\varepsilon_v$ = (π x $0.8^3/6$) x 1.333 x 10^{-3} = 357443 mm^3

Volume of compressed water inside sphere = 268.083 x 10^6 +357443 = 268.440 x 10^6 mm^3

Let the original volume of water at atmospheric pressure be V_w
Change in volume = V_w - 268.440 x 10^6
Volumetric strain = ε_{vw} = (V_w - 268.440 x 10^6)/V_w
Bulk modulus K = $\Delta p V_w/\Delta V = \Delta p V_w/(V_w$ - 268.440 x 10^6)
V_w - 268.440 x 10^6 = (Δp/K)V_w
V_w = 268.440 x 10^6/(1 – p/K) = 268.440 x 10^6/(1 – 10 x 10^5/2.2 x 10^9) = 268.562 x 10^6 mm^3
The volume of water added = V_w - 268.083 x 10^6
The volume of water added = 268.562 x 10^6 - 268.083 x 10^6 = 0.479 x 10^6 mm^3

Answer 0.479 dm^3