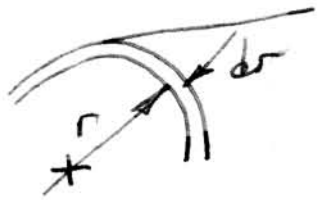
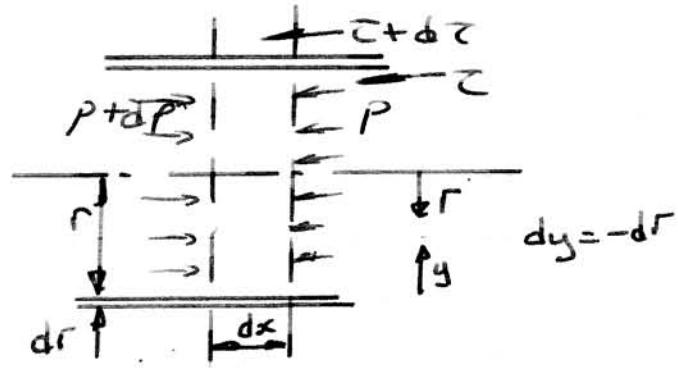
i) FORCE BALANCE ON AN ELEMENTAL CYLINDER





PRESSURE FORCE

$$= dp\left[\pi(r+dr)^2 - \pi r^2\right]$$

SHEARING $= (\tau + d\tau)(2\pi(r+dr))\,dx$
$\quad - \tau \times 2\pi r\, dx$

EQUATE AND SIMPLIFY IGNORE 2nd ORDER

$$\frac{dp}{dx} = \frac{\tau}{r} + \frac{d\tau}{dr}$$

NEWTONIAN FLUID $\tau = \mu\frac{du}{dy} = -\mu\frac{du}{dr}$

$$\frac{dp}{dx} = -\frac{\mu}{r}\frac{du}{dr} + \frac{d\left(-\mu\frac{du}{dr}\right)}{dr}$$

$$\frac{r\,dp}{dx} = -\mu\frac{du}{dr} - \mu r\frac{d\left(\frac{du}{dr}\right)}{dr}$$

$$r\frac{dp}{dx} = -\mu\frac{du}{dr} - \mu r\frac{\left(\frac{du}{dr}\right)}{dr}$$

$$r\frac{dp}{dx} = -\mu\frac{du}{dr}\left\{1 + r\frac{du}{dr}\right\}$$

− SIGN SHOWS $\frac{dp}{dx}$ IS NEGATIVE

ie DERIVATION ASSUMES p decreases with x

ii) FULL DERIVATION SEE T~~1~~203-pdf page 22

$$-\frac{r}{\mu}\frac{dp}{dx} = \frac{du}{dr} + r\frac{d^2u}{dr^2} = \frac{d\left(r\frac{du}{dr}\right)}{dr}$$

INTEGRATE $\quad \frac{r\,du}{dr} = -\frac{r^2}{2\mu}\frac{dp}{dx} + A$

$$\frac{du}{dr} = -\frac{r}{2\mu}\frac{dp}{dx} + \frac{A}{r}$$

INTEGRATE $\quad u = -\frac{r^2}{4\mu}\frac{dp}{dx} + A\ln r + B$

BOUNDARY CONDITIONS FOR AN ANNULUS

$u = 0$ @ $r = R_1$ AND $r = R_2$

HENCE GO ON TO SHOW

$$A = \frac{1}{4\mu}\frac{dp}{dx}\frac{\{R_1^2 - R_2^2\}}{\ln R_1/R_2}$$

$$B = \frac{1}{4\mu}\frac{dp}{dx}\left[R_1^2 - \left\{\frac{R_2^2 - R_1^2}{\ln R_2/R_1}\right\}\ln R_1\right]$$

AND HENCE $\quad u = \frac{1}{4\mu}\frac{dp}{dx}\left[\frac{(R_2^2 - R_1^2)}{\ln R_2/R_1}\ln\frac{r}{R_1} + R_1^2 - r^2\right]$

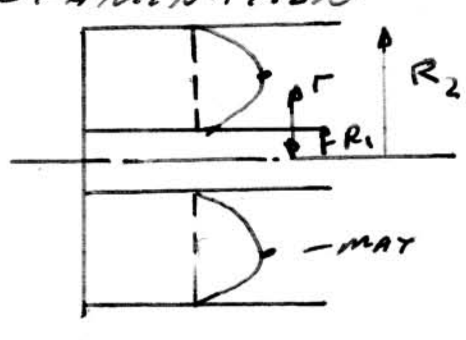
$$u = \frac{1}{4\mu}\frac{dp}{dx}\left[a\ln r/R_1 + R_1^2 - r^2\right]$$

$$= -\frac{1}{4\mu}\frac{dp}{dx}\left[r^2 - R_1^2 - a\ln R_1/r\right]$$

FULL DERIVATION WOULD TAKE FAR TO MUCH TIME IN AN EXAMINATION

FORMULAE YIELDS SAME RESULTS AS THE ONE IN THE QUESTION

VELOCITY DISTRIBUTION



Q4 2001

FOR MAX VELOCITY $\frac{du}{dr} = 0$

iii)

DIFF

$$\frac{du}{dr} = \frac{1}{4\mu}\frac{dP}{dx}\left[\frac{a}{r} - 2r\right] = 0$$

$$\frac{a}{r} = 2r \qquad \frac{a}{2} = r^2 \qquad r = \sqrt{\frac{a}{2}}$$

iv

$R_1 = 0.1m \quad R_2 = 0.2m \quad \mu = 0.29\ Ns/m^2$

$$a = \frac{R_2^2 - R_1^2}{\ln R_2/R_1} = \frac{0.2^2 - 0.1^2}{\ln 0.2/0.1} = 0.04328$$

$$r = \sqrt{\frac{0.04328}{2}} = 0.147m$$

$dP/dx = 400\ N/m^3$

$$u = \frac{1}{4 \times .29} \times 400\left[0.04328 \ln\frac{.147}{.1} + .1^2 - .147^2\right]$$

$$= \frac{400}{4 \times .29} \times \text{~~.026674~~}\ 0.005065 = 1.747 m/s$$