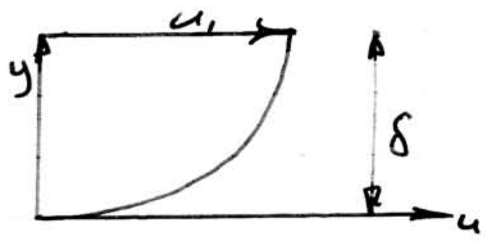
Q3 2001



3 i) $\frac{u}{u_1} = \sin Ay$

BOUNDARY CONDITION $u = u_1$ @ $y = \delta$

$\frac{u_1}{u_1} = \sin A\delta = 1$ $\quad A\delta = \pi/2$

$A = \pi/2\delta$

$\frac{u}{u_1} = \sin \frac{\pi y}{2\delta}$

ii) $\theta = \int_0^\delta \frac{u}{u_1}\left(1 - \frac{u}{u_1}\right)dy = \int_0^\delta \left[\frac{u}{u_1} - \left(\frac{u}{u_1}\right)^2\right]dy$

$\theta = \int_0^\delta \left[\sin \frac{\pi y}{2\delta} - \sin^2 \frac{\pi y}{2\delta}\right]dy$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$ — TRIG IDENTITY

$\theta = \int_0^\delta \left[\sin \frac{\pi y}{2\delta} - \frac{1}{2} + \frac{1}{2}\cos \frac{\pi y}{\delta}\right] dy$

$\theta = \left[-\frac{2\delta}{\pi}\cos \frac{\pi y}{2\delta} - \frac{y}{2} + \frac{\delta}{2\pi}\sin \frac{\pi y}{\delta}\right]_0^\delta$

$\theta = \left[0 - \frac{\delta}{2} + 0\right] - \left[-\frac{2\delta}{\pi} - 0 + 0\right]$

$\theta = -0.5\delta + 0.6366\delta = 0.137\delta$

Q3 2001

iii WALL SHEAR STRESS $\tau_0 = \left(\mu \frac{du}{dy}\right)_{y=0}$

$$\tau_0 = \mu u_1 \frac{d\left(\sin \frac{\pi y}{2\delta}\right)}{dy} = \mu u_1 \cos \frac{\pi y}{2\delta} \times \frac{\pi}{2\delta} \quad @ y=0$$

$$\tau_0 = \frac{\mu u_1 \pi}{2\delta} \cos 0 = \frac{\mu u_1 \pi}{2\delta} \quad ---(1)$$

$$C_f = \frac{2\tau_0}{\rho u_1^2} \quad ---(2) \qquad \text{DEFINITION}$$

$$C_f = \frac{2 d\theta}{dx} . \quad \text{AND } \theta = 0.137\delta$$

$$C_f = \frac{2 d(.137\delta)}{dx} = 2 \times 0.137 \frac{d\delta}{dx} \quad ---(3)$$

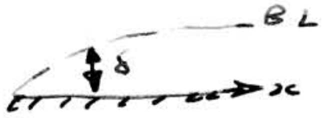
(2) → (3) $$\frac{2\tau_0}{\rho u_1^2} = 0.274 \frac{d\delta}{dx}$$

$$\tau_0 = \rho u_1^2 \times .137 \frac{d\delta}{dx} \quad ---(4)$$

(1) = (4) $$\frac{\mu u_1 \pi}{2\delta} = 0.137 \rho u_1^2 \frac{d\delta}{dx}$$

$$\frac{\mu \pi}{2 \times 0.137 \rho u_1} x = \frac{\delta^2}{2} + C \qquad C = \text{CONST of INT.}$$

@ $x=0$ $\delta=0$ $\therefore C=0$



$$\delta / x = \left(\frac{2\pi}{2 \times .137}\right)^{1/2} \left(\frac{\mu}{\rho u_1 x}\right)^{1/2}$$

$$Re_x = \frac{\rho u_1 x}{\mu}$$

μ = dynamic viscosity
ρ = density
$\mu/\rho = \nu$ = KINEMATIC VISC

$$\delta / x = \frac{4.79}{Re_x^{1/2}}$$

$$C_f = \frac{2\tau_0}{\rho u_1^2} \qquad \tau_0 = \frac{\mu u_1 \pi}{2\delta}$$

$$C_f = \frac{2\mu u_1 \pi}{\rho u_1^2 2\delta} = \frac{\mu \pi}{\rho u_1 \delta} = \frac{\mu \pi x}{\rho u_1 \delta x}$$

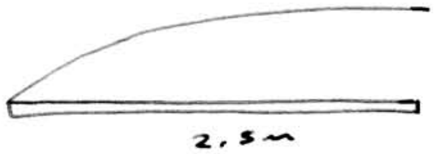
$$Re_x = \frac{\rho u_1 x}{\mu} \qquad C_f = \frac{1}{Re_x}\frac{\pi x}{\delta}$$

SUBSTITUTE $\delta/x = 4.79\, Re_x^{-1/2}$

$$C_f = \frac{1}{Re_x} \times \pi \times \frac{Re_x^{1/2}}{4.79}$$

$$C_f = \frac{\pi}{4.79} Re_x^{-1/2} = \underline{\underline{0.65\, Re_x^{-1/2}}}$$

b)



$x = 2.5\,m$

$\nu = 10^{-4}\, m^2/s$

$u_1 = 5\, m/s$

$$\frac{\delta}{x} = 4.79\, Re_x^{-1/2}$$

$$Re_x = \frac{\rho u_1 x}{\mu} = \frac{u_1 x}{\nu} = \frac{5 \times 2.5}{10^{-4}} = 12.5 \times 10^4$$

$$\delta = 2.5 \times 4.79 \times (12.5 \times 10^4)^{-1/2}$$

$\delta = 0.03387\, m \qquad$ **33**.87 mm