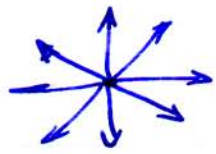
Q2 2001

a) SOURCE – FLOW EMERGING FROM A VERTICAL LINE IN LONG AND SPREADING OUT RADIALLY IN ALL DIRECTION

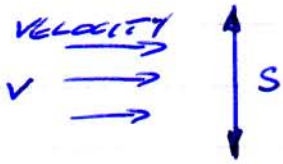


FLOW EMERGING = M × 1

SINK – OPPOSITE OF A SOURCE

DOUBLET – SOURCE AND SINK COMBINED AS THE GET VERY NEAR TO THE SAME POINT

STREAM FUNCTION ψ



THE FLUX ACROSS A LINE

$\psi = V \times S$ or $V\,ds$ if not const.

SINCE NO FLOW CROSSES A STREAM LINE CONSTANT VALUES OF ψ REPRESENT THE STREAM LINE

Q2 2001 CONT

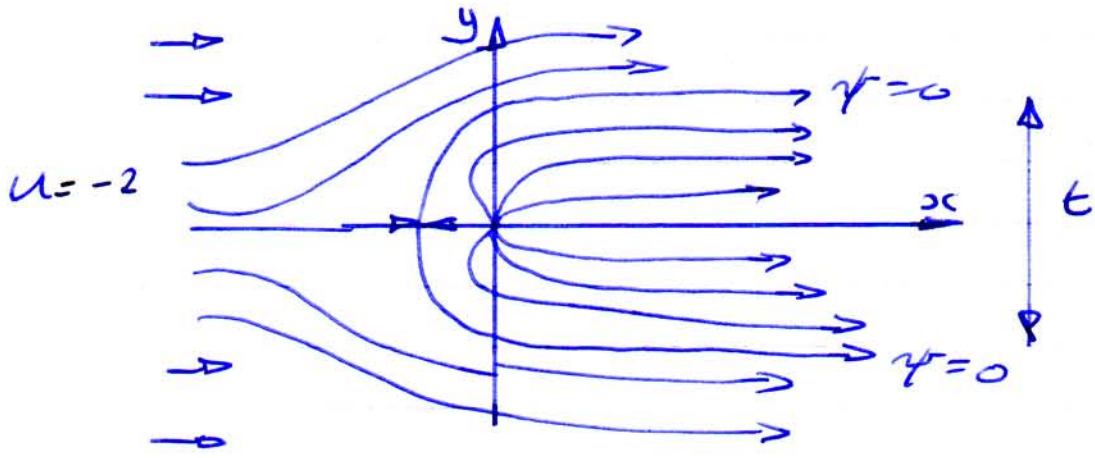
b) UNIFORM FLOW $\psi_1 = -Uy$

SOURCE $\psi_2 = \frac{m}{2\pi}\theta$

COMBINED FLOW $\psi = -Uy + \frac{m\theta}{2\pi}$

$y = r\sin\theta$ $\psi = -Ur\sin\theta + \frac{m\theta}{2\pi}$

c)



$\psi = 0$ IS THE DIVIDING SL

AT LARGE x FLOW IS UNIFORM

$0 = -Uy + \frac{m\theta}{2\pi}$ $y = \frac{m\theta}{2\pi U}$

THE FLUX BETWEEN $y=0$ AND THE DIVIDING S.L. IS HALF OF THE TOTAL EMERGING FROM THE SOURCE

PUT $\theta = \pi$ (180°)

$\psi_2 = \frac{m}{2\pi}\cdot\pi = \frac{m}{2}$ $\psi_1 = -Uy$

$\psi = 0 = -Uy + \frac{m}{2}$ $y = \frac{m}{2U}$

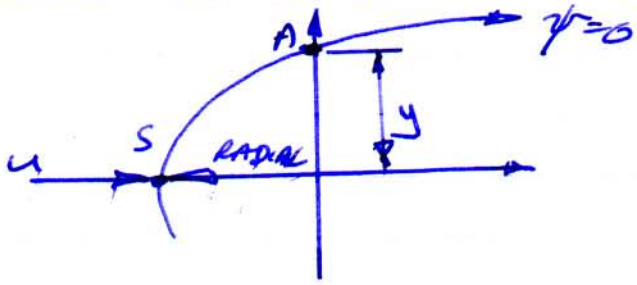
$t = 2y = \frac{2m}{2U} = \frac{m}{U} = \frac{8}{2} = 4m$

AT THE STAGNATION POINT U = RADIAL VELOCITY

RADIAL VELOCITY FROM SOURCE = $\frac{M}{2\pi r}$

$U = \frac{M}{2\pi r}$ $\quad r = \frac{M}{2\pi U} = \frac{8}{2\pi \times 2} = \frac{2}{\pi}$

DISTANCE TO STAGNATION POINT = $2/\pi$ m

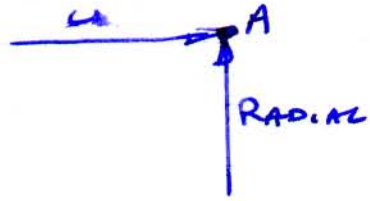


BERNOULLI BETWEEN S AND A

$\frac{\rho V_s^2}{2} + p_s = \frac{\rho V_A^2}{2} + p_A$ $\quad V_s = 0$ (STAGNATION)

$$p_s - p_A = \frac{\rho V_A^2}{2}$$

AT A WE HAVE TWO VELOCITIES



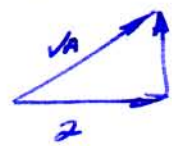
RADIAL VELOCITY $= M/2\pi r$

AT A $\psi = 0 = -Uy + \frac{m\theta}{2\pi}$ $\quad \theta = \pi/2$

$$y = \frac{m\theta}{2\pi U} = \frac{8 \times \pi/2}{2\pi \times 2 \times 2} = 1\text{ m}$$

RADIAL VELOCITY $= 8/2\pi r = 4/\pi$ m/s

TRUE VELOCITY AT 'A'



$V_A^2 = 2^2 + (4/\pi)^2 = 5.62$

$p_s - p_A = \frac{\rho V_A^2}{2} = \frac{800 \times 5.62}{2} = \underline{\underline{2248\ N/m^2}}$