D203 FLUID MECHANICS
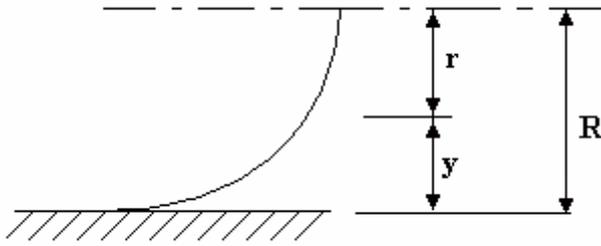
Q4 1999

For an elementary ring $dQ = 2\pi r \, dr \, u = 2\pi r \, dr \, u_1(y/R)^{1/n}$

$Y + r = R \quad r = R - y \qquad dr = - dy$

$dQ = 2\pi r \, u_1 \, (R - y)(y/R)^{1/n}$

$$dQ = \frac{-2\pi u_1}{R^{1/n}} \int_R^0 \left(Ry^{1/n} - y^{1+1/n} \right) dy$$

$$Q = \frac{-2\pi u_1}{R^{1/n}} \left[\frac{Ry^{1+1/n}}{1 + 1/n} - \frac{y^{2+1/n}}{2 + 1/n} \right]_R^0$$

$$Q = \frac{2\pi u_1}{R^{1/n}} R^{2+1/n} \left[\frac{n}{n + 1} - \frac{n}{2n + 1} \right]$$

$$Q = 2\pi u_1 R^2 n \left[\frac{(2n + 1) - (n + 1)}{(n + 1)(2n + 1)} \right]$$

$u_m = Q/\pi R^2$

$$u_m = 2u_1 n \left[\frac{n}{(n + 1)(2n + 1)} \right]$$

$$\frac{u_m}{u_1} = \frac{2n^2}{(n + 1)(n + 2)}$$

Part C
If $u/u_1 = (y/R)^{1/n}$ $\qquad$ $\log u/u_1 = (1/n)\log(y/R)$

Plot