(a) Explain the terms *Stokes flow* and *terminal velocity* as applied to a particle settling in a fluid. Show that, for a spherical particle immersed in a flow for which the drag coefficient C_D is *24/Re* (where *Re* is based on particle diameter), the terminal velocity - u is given by

$$u = \frac{d^2 g(\rho_s - \rho_f)}{18 \mu}$$ where p_s is the density of the particle.

(b) A gravel washing and grading plant processes gravel with a density of 2630 kg/m³. The gravel is introduced into a stream of water at 25°C which is flowing vertically upwards with a velocity of 1.0 m/s. Treating the gravel pebbles as spherical particles, determine the diameter of the largest particle which will be carried upwards by the water flow.

(c) If the water velocity is reduced to 0.5 m/s, show that particles with a diameter greater than 5.95 mm will fall downwards through the water flow.

For spherical particles, a useful empirical correlation for the drag coefficient C_D is

$$C_D = \frac{24}{R_e} + \frac{6}{1 + \sqrt{R_e}} + 0.4$$

where *Re* is the Reynolds number based on particle diameter. This correlation is applicable for the range $0.2 < Re < 10^5$

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a) For $R_e < 0.2$ the flow is called Stokes flow and Stokes showed that R = 3πd μ u hence

R = W = volume x density difference x gravity

$$R = W = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} = 3\pi d \mu u$$

ρ_s = density of the sphere material ρ_f = density of fluid   d = sphere diameter

$$u = \frac{\pi d^3 g(\rho_s - \rho_f)}{18 \pi d \mu} = \frac{d^2 g(\rho_s - \rho_f)}{18 \mu}$$

b) C_D = R/(projected area x ρu²/2) $C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\rho u^2/2)6\pi d^2/4} = \frac{4dg(\rho_s - \rho_f)}{3\rho u^2}$

$$C_D = \frac{4 \times 9.81 \times (1630 - 998)d}{3 \times 998 \times u^2} = 21.389 \frac{d}{u^2}$$

$$C_D = \frac{24}{R_e} + \frac{6}{1 + \sqrt{R_e}} + 0.4 = 21.389 \frac{d}{u^2}$$
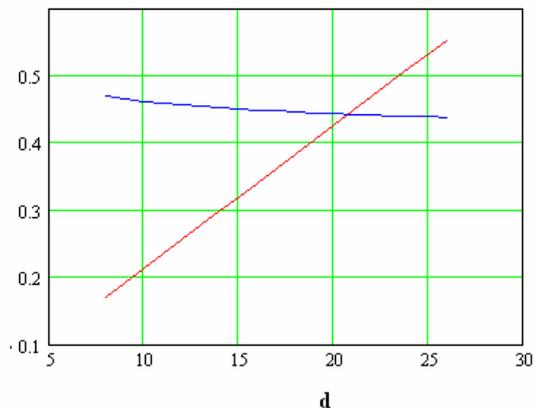
$$21.389 \frac{d}{u^2} - \frac{24}{R_e} - \frac{6}{1 + \sqrt{R_e}} = 0.4 \qquad \text{let}$$

$$21.389 \frac{d}{u^2} - \frac{24}{R_e} - \frac{6}{1 + \sqrt{R_e}} = x$$

Re = ρud/μ = 998 x 1 x d/0.89 x 10⁻³ = 1.1213 x 10⁶d

Make a table

D	0.001	0.003	0.01	0.02	0.03
Re	1121.3	3363.9	11213	22426	33639
x	-0.174	-0.045	0.156	0.387	0.608

Plot and find that when d = 0.0205 m (20.5 mm) x = 0.4

c) $u = 0.5 \text{m/s}$ $d = 5.95 \text{mm}$

$Re = \rho ud/\mu = 998 \times 0.5 \times 0.00595/0.89 \times 10^{-3} = 3336$

$$C_D = 21.389 \frac{d}{u^2} = 0.509$$

$$C_D = \frac{24}{3336} + \frac{6}{1 + \sqrt{3336}} + 0.4 = 0.509$$

Since C_D is the same, larger ones will fall.