FLUID MECHANICS D209 Q2 1996

Planar irrotational flow past a Rankine body is produced by the combination of a uniform flow at velocity U in the positive x direction, a sink of strength $Q = 2\pi m$ located on the x axis at x = +a and a source of the same strength on the x axis at x = -a.

(a) Show for the above flow that the following expressions for the stream function ψ and velocity potential function ϕ apply (each term having its usual significance).
$\psi = m(\theta_1 - \theta_2) + Ur\sin\theta$
$\phi = m\ln(r_1/r_2) + Ur\cos\theta$

(b) For the case where U = 4 m/s, $Q = 2$ m²/s and a = 5 m, determine
(i) the length of the body
(ii) the width of the body.

(c) Sketch, without calculation, the variation of velocity and pressure, respectively, around the surface of the body.

A doublet is formed when an equal source and a sink are brought close together. Consider a source and sink of equal strength placed at A and B respectively. The stream function for point P relative to A and B are respectively
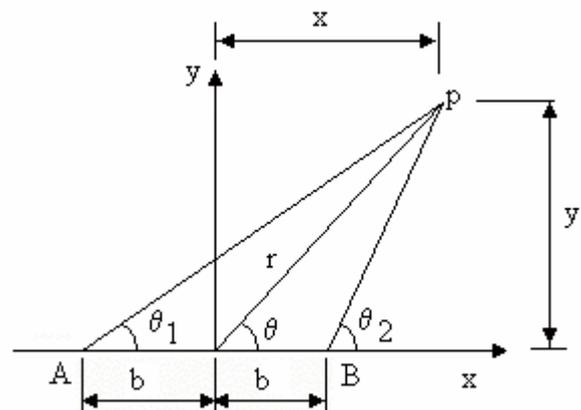
$\Psi_B = m\theta_2$ for the source

$\Psi_A = -m\theta_1$ for the sink

$\Psi_C = -Uy = -Ur\sin\theta$ Uniform flow

$\Psi_P = \Psi_B + \Psi_A + \Psi_A = m(\theta_2 - \theta_1) - Ur\sin\theta$

$\Psi_P = -m(\theta_1 - \theta_2) + Ur\sin\theta$

The minus sign depends on the sign convention used.

$\phi_{AP} = m \ln r_1$
$\phi_{BP} = m \ln r_2$
Uniform flow $\phi = Ur\cos\theta$
Combined $\phi = Ur\cos\theta + m\ln(r_1/r_2)$
$Q/2\pi r = U \quad r = 1/4\pi$
$L = 2a + 2r = 10 + 1/2\pi = 10.16$ m
$t = Q/U = 2/4 = 0.5$ m