FLUID MECHANICS D209 Q11 1996

It is required to pump water at a rate of 0.0160 m³/s against a total head of 30.5 m. Four geometrically similar pumps, whose sizes are 100 mm, 125 mm, 225 mm and 300 mm, are available.

The characteristics of the 100 mm size pump, tested at 150 rad/s, are tabulated below.

Discharge	0	0.0076	0.0151	0.0226	0.0302	m³/s
Head	43.9	46.1	43.9	34.2	14.6	m
Efficiency	0	48	66	66	45	%

(a) Determine which pump should be used, and the speed at which it should be driven, so that maximum possible efficiency is obtained.

(b) If, temporarily, only the 125 mm pump is available, determine the speed of operation and the input power from the motor, necessary to satisfy the head and discharge requirements.

By plotting the data for the 100 mm pump we can determine that the optimal point (for max efficiency) is when Q = 0.0188 m³/s and H = 40 m. The peak efficiency is 68%
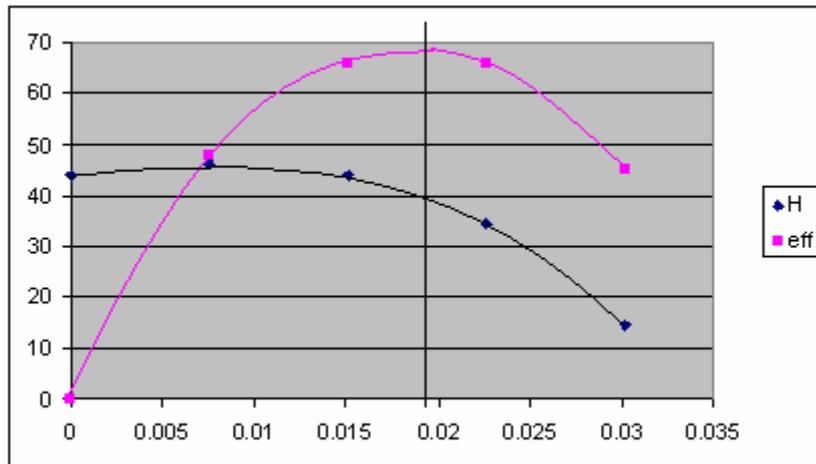
For the 100 mm pump H = 40 m   Q = 0.0188 m³/s   N = 150 rad/s

$$Ns = \left(\frac{N_1 Q_1^{1/2}}{H^{3/4}}\right) = \frac{150 \times 0.0188^{1/2}}{40^{3/4}} = 1.293 \text{ rad/s} \quad (12.34 \text{ rev/min})$$

For the required condition

$$1.293 = \frac{N \times 0.016^{1/2}}{30.5^{3/4}} \text{ Hence } N = 131 \text{ rad/s } (1251 \text{ rev/min})$$

For the optimal size, remember that condition (1) is the optimal condition of the pump and condition (2) is the actual operating conditions.
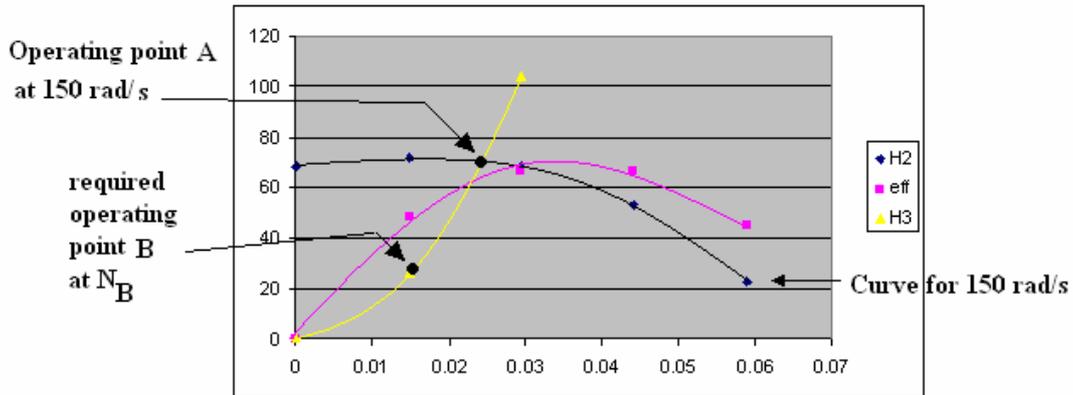
Equating Flow Coefficients we get $\frac{D_2}{D_1} = \left(\frac{Q_2 N_1}{Q_1 N_2}\right)^{1/3} = \left(\frac{0.016 \times 150}{0.0188 \times 131}\right)^{1/3} = 1$

Equating head coefficients we get we get $\frac{D_2}{D_1} = \frac{N_1}{N_2}\sqrt{\frac{H_2}{H_1}} = \frac{150}{131}\sqrt{\frac{30.5}{40}} = 1$

The 100 mm seems to be the best.

(b) 125 mm pump at the same speed

The larger pump must slower to obtain the same flow. First calculate the corresponding flow and head for the 100 mm pump.

For the same Flow coefficient $Q_2 = 0.016 = Q_1\left(\dfrac{D_2}{D_1}\right)^3 = Q_1\left(\dfrac{125}{100}\right)^3 = 1.953Q_1 = 1.953 \times Q_1$

For the same Head coefficient $H_2 = 40 = H_1\left(\dfrac{D_2}{D_1}\right)^2 = H_1\left(\dfrac{125}{100}\right)^2 = 1.562H_1$

Q_1	0	0.0076	0.0151	0.0226	0.0302	m^3/s
H_1	43.9	46.1	43.9	34.2	14.6	m
Efficiency	0	48	66	66	45	%
Q_2	0	0.0148	0.0295	0.0441	0.059	
H_2	68.6	72	68.6	53.4	22.8	

Plotting H_2 and Q_2 gives the curve shown. It is assumed that the efficiency is unchanged.

As can be seen we cannot obtain the required operating point at 150 rad/s.
For the same flow coefficient between at two different speeds

$$\dfrac{Q_B}{N_B D_B{}^3} = \dfrac{Q_A}{N_A D_A{}^3} \qquad Q_B = Q_A \dfrac{N_B}{N_A}$$

For the same Head Coefficient at two different speeds

$$\dfrac{g\,H_A}{N_A{}^2 D_A{}^2} = \dfrac{g\,H_A}{N_A{}^2 D_A{}^2} \qquad H_B = H_A \dfrac{N_B{}^2}{N_A{}^2} = H_A \dfrac{N_B{}^2}{N_A{}^2}$$

Substitute $\dfrac{N_B}{N_A} = \dfrac{Q_B}{Q_A}$ to eliminate the speed

$$H_B = H_A\left(\dfrac{Q_B}{Q_A}\right)^2 \qquad H_A = H_B\left(\dfrac{Q_A}{Q_B}\right)^2 \qquad \text{Where A and B correspond to different speeds.}$$

For the case in hand let B be the values at the new speed and A the values at 150 rad/s

$$H_A = 30.5\left(\dfrac{Q_A}{0.016}\right)^2 = 119141 Q_A^2$$

Calculate the flows at the new speed for the 125 mm pump.

Efficiency	0	48	66	66	45	%
Q_A	0	0.0148	0.0295	0.0441	0.059	
H_B	0	26	104			

Plotting H_B we get the result shown. We require the speed to produce operating point B for the same size (125 mm).

From the Flow Coefficient between points A and B.

$$\frac{Q_B}{N_B D_B^{\,3}} = \frac{Q_A}{N_A D_A^{\,3}} \qquad Q_A = 0.025\,\text{m}^3/\text{s and } H_A = 70 \text{ m}$$

$$\frac{0.016}{N_B} = \frac{0.025}{150}$$

$N_B = 96$ rad/s

Check by repeating the process with the head coefficient.

$$\frac{g\,H_B}{N_B^{\,2} D_B^{\,2}} = \frac{g\,H_A}{N_A^{\,2} D_A^{\,2}} \qquad\qquad N_B = N_A \sqrt{\frac{H_B}{H_A}} = 150\sqrt{\frac{30.5}{70}} = 99$$

The efficiency at this point is 62%

Water Power = mgH = 16 x 9.81 x 30.5 = 4787 W

Power Input = WP/η = 4787/0.62 = 7720 W