Unit 43: Plant and Process Principles

Unit code: H/601 442 QCF level: 5 Credit value: 15

OUTCOME 3 - STATIC AND DYNAMIC FLUID SYSTEMS

TUTORIAL 4 - DYNAMIC FLUID SYSTEMS

3 **Understand static and dynamic fluid systems with reference to plant engineering**

Immersed surfaces: rectangular and circular surfaces, including retaining walls, tank sides, sluice gates, inspection covers, valve flanges; hydrostatic pressure and thrust on immersed surfaces

Centre of pressure: use of parallel axis theorem for immersed rectangular and circular surfaces

Viscosity shear stress; shear rate; dynamic viscosity: kinematic viscosity

Pipeline flow: head losses e.g. Bernoulli's equation and determination of head loss in pipes by D'Arcy's formula; Moody diagram; head loss due to sudden enlargement and contraction of pipe diameter; head loss at entrance to a pipe; head loss in valves; Reynolds' number; inertia and viscous resistance forces; laminar and turbulent flow; critical velocities

Impact of a jet: power of a jet normal thrust on a moving flat vane; thrust on a moving hemispherical cup; velocity diagrams to determine thrust on moving curved vanes; fluid friction losses; system efficiency

On completion of this outcome you should be able to do the following.

- Derive Bernoulli's equation for liquids.
- Define and explain laminar and turbulent flow.
- Find the pressure losses in piped systems due to fluid friction.
- Find the minor frictional losses in piped systems.

Let's start by revising basics. The flow of a fluid in a pipe depends upon two fundamental laws, the conservation of mass and energy.

1 PIPE FLOW

The solution of pipe flow problems requires the applications of two principles, the law of conservation of mass (continuity equation) and the law of conservation of energy (Bernoulli's equation)

1.1 CONSERVATION OF MASS

When a fluid flows at a constant rate in a pipe or duct, the mass flow rate must be the same at all points along the length. Consider a liquid being pumped into a tank as shown (fig. 1).

The mass flow rate at any section is $m = \rho A u_m$

ρ = density (kg/m^3) u_m = mean velocity (m/s) A = Cross Sectional Area (m^2)

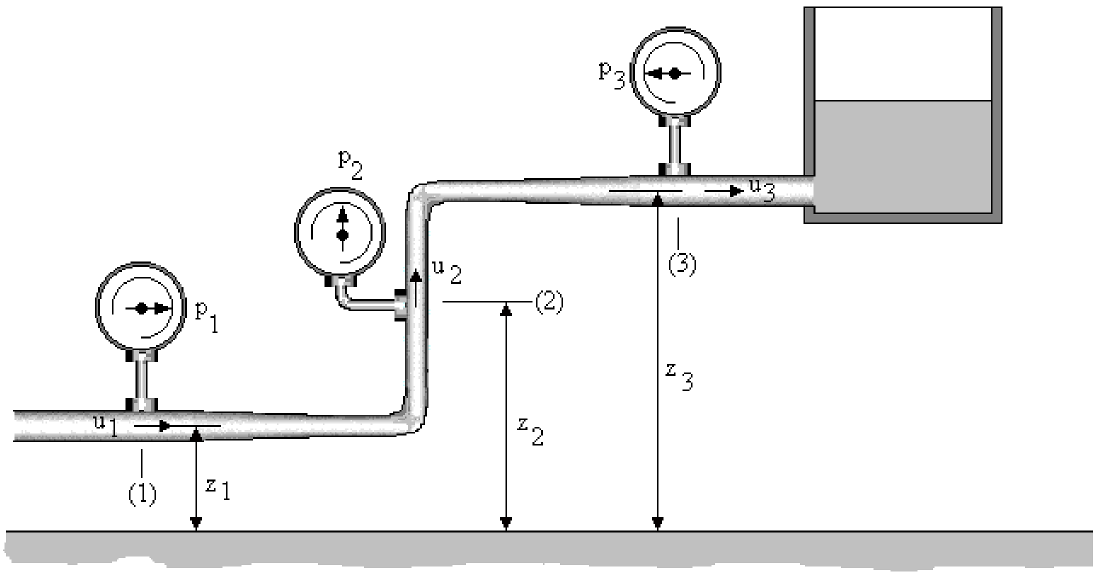

Fig. 1

For the system shown the mass flow rate at (1), (2) and (3) must be the same so

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 = \rho_3 A_3 u_3$$

In the case of liquids the density is equal and cancels so

$$A_1 u_1 = A_2 u_2 = A_3 u_3 = Q$$

1.2 CONSERVATION OF ENERGY

ENERGY FORMS

FLOW ENERGY
This is the energy a fluid possesses by virtue of its pressure.
The formula is ***F.E. = pQ Joules***

p is the pressure (Pascals)
Q is volume rate (m^3)

POTENTIAL OR GRAVITATIONAL ENERGY

This is the energy a fluid possesses by virtue of its altitude relative to a datum level.

The formula is ***P.E. = mgz Joules***

m is mass (kg)
z is altitude (m)

KINETIC ENERGY

This is the energy a fluid possesses by virtue of its velocity.

The formula is ***K.E.*** $= \frac{1}{2} m u_m^2$ ***Joules***

u_m is mean velocity (m/s)

INTERNAL ENERGY

This is the energy a fluid possesses by virtue of its temperature. It is usually expressed relative to 0^oC.

The formula is $U = mc\theta$

c is the specific heat capacity (J/kg ^oC)
θ is the temperature in ^oC

In the following work, internal energy is not considered in the energy balance.

SPECIFIC ENERGY

Specific energy is the energy per kg so the three energy forms as specific energy are as follows.

F.E./m $= pQ/m = p/\rho$ ***Joules/kg***
P.E/m. $= gz$ ***Joules/kg***
K.E./m $= \frac{1}{2} u^2$ ***Joules/kg***

ENERGY HEAD

If the energy terms are divided by the weight mg, the result is energy per Newton. Examining the units closely we have J/N = N m/N = metres.

It is normal to refer to the energy in this form as the energy head. The three energy terms expressed this way are as follows.

F.E./mg $= p/\rho g = h$
P.E./mg $= z$
K.E./mg $= u^2/2g$

The flow energy term is called the pressure head and this follows since earlier it was shown $p/\rho g = h$. This is the height that the liquid would rise to in a vertical pipe connected to the system.

The potential energy term is the actual altitude relative to a datum.

The term $u^2/2g$ is called the kinetic head and this is the pressure head that would result if the velocity is converted into pressure.

2. BERNOULLI'S EQUATION

Bernoulli's equation is based on the conservation of energy. If no energy is added to the system as work or heat then the total energy of the fluid is conserved. Remember that internal (thermal energy) has not been included.

The total energy E_T at (1) and (2) on the diagram (fig.1) must be equal so :

$$E_T = p_1 Q_1 + mgz_1 + m\frac{u_1^2}{2} = p_2 Q_2 + mgz_2 + m\frac{u_2^2}{2}$$

Dividing by mass gives the specific energy form

$$\frac{E_T}{m} = \frac{p_1}{\rho_1} + gz_1 + \frac{u_1^2}{2} = \frac{p_2}{\rho_2} + gz_2 + \frac{u_2^2}{2}$$

Dividing by g gives the energy terms per unit weight

$$\frac{E_T}{mg} = \frac{p_1}{g\rho_1} + z_1 + \frac{u_1^2}{2g} = \frac{p_2}{g\rho_2} + z_2 + \frac{u_2^2}{2g}$$

Since $p/\rho g$ = pressure head h then the total head is given by the following.

$$h_T = h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g}$$

This is the head form of the equation in which each term is an energy head in metres. z is the potential or gravitational head and $u^2/2g$ is the kinetic or velocity head.

For liquids the density is the same at both points so multiplying by ρg gives the pressure form. The total pressure is as follows.

$$p_T = p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2}$$

In real systems there is friction in the pipe and elsewhere. This produces heat that is absorbed by the liquid causing a rise in the internal energy and hence the temperature. In fact the temperature rise will be very small except in extreme cases because it takes a lot of energy to raise the temperature. If the pipe is long, the energy might be lost as heat transfer to the surroundings. Since the equations did not include internal energy, the balance is lost and we need to add an extra term to the right side of the equation to maintain the balance. This term is either the head lost to friction h_L or the pressure loss p_L.

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

The pressure form of the equation is as follows.

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

The total energy of the fluid (excluding internal energy) is no longer constant.

Note that if one of the points is a ***free surface*** the pressure is normally atmospheric but if gauge pressures are used, the pressure and pressure head becomes zero. Also, if the surface area is large (say a large tank), the velocity of the surface is small and when squared becomes negligible so the kinetic energy term is neglected (made zero).

WORKED EXAMPLE No. 1

The diagram shows a pump delivering water through as pipe 30 mm bore to a tank. Find the pressure at point (1) when the flow rate is 1.4 dm^3/s. The density of water is 1000 kg/m^3. The loss of pressure due to friction is 50 kPa.

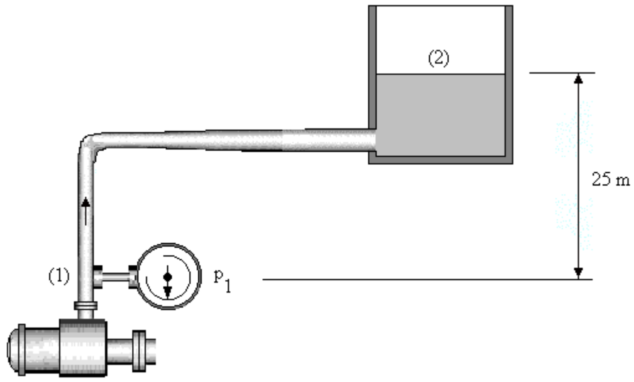

Fig. 2

SOLUTION

Area of bore A = π x $0.03^2/4 = 706.8 \times 10^{-6}$ m^2.
Flow rate Q = 1.4 dm^3/s = 0.0014 m^3/s
Mean velocity in pipe = Q/A = 1.98 m/s
Apply Bernoulli between point (1) and the surface of the tank.

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

Make the low level the datum level and $z_1 = 0$ and $z_2 = 25$.
The pressure on the surface is zero gauge pressure. P_L = 50 000 Pa
The velocity at (1) is 1.98 m/s and at the surface it is zero.

$$p_1 + 0 + \frac{1000 x 1.98^2}{2} = 0 + 1000 x 9.9125 + 0 + 50000 \qquad p_1 = 293.29 kPa \text{ gauge pressure}$$

WORKED EXAMPLE No.2

The diagram shows a tank that is drained by a horizontal pipe. Calculate the pressure head at point (2) when the valve is partly closed so that the flow rate is reduced to 20 dm^3/s. The pressure loss is equal to 2 m head.

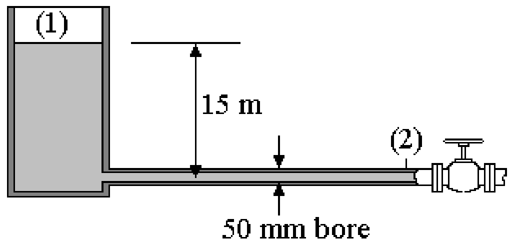

Fig. 3

SOLUTION

Since point (1) is a free surface, $h_1 = 0$ and u_1 is assumed negligible.
The datum level is point (2) so $z_1 = 15$ and $z_2 = 0$.
Q = 0.02 m3/s
$A_2 = \pi d^2/4 = \pi$ x $(0.05^2)/4 = 1.963 \times 10^{-3}$ m^2.
u_2 = Q/A = 0.02/1.963 x 10^{-3} = 10.18 m/s

Bernoulli's equation in head form is as follows.

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

$$0 + 15 + 0 = h_2 + 0 + \frac{10.18^2}{2 \text{ x } 9.81} + 2$$

$$h_2 = 7.72 m$$

WORKED EXAMPLE No.3

The diagram shows a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of 600 mm^2 and the exit has a bore area of 200 mm^2. Calculate the flow rate when the inlet pressure is 400 Pa. Assume there is no energy loss.

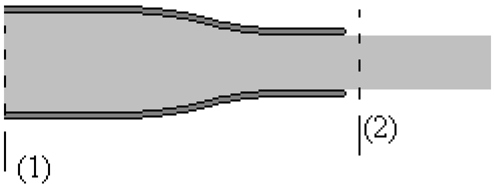

Fig. 4

SOLUTION

Apply Bernoulli between (1) and (2)

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

Using gauge pressure, p2 = 0 and being horizontal the potential terms cancel. The loss term is zero so the equation simplifies to the following.

$$p_1 + \frac{\rho u_1^2}{2} = \frac{\rho u_2^2}{2}$$

From the continuity equation we have

$$u_1 = \frac{Q}{A_1} = \frac{4Q}{\pi x 0.6^2} = 3.537Q$$

$$u_2 = \frac{Q}{A_2} = \frac{4Q}{\pi x 0.2^2} = 31.831Q$$

Putting this into Bernoulli's equation we have the following.

$$400 + 1000x\frac{(3.537Q)^2}{2} = 1000x\frac{(31.831Q)^2}{2}$$

$$400 + 6.255x10^3 Q^2 = 506.6Q^2$$

$$400 = 600.3x10^3 Q^2$$

$$Q^2 = \frac{400}{600.3x10^3} = 666x10^{-6}$$

$$Q = 0.0258\,m^3/s \text{ or } 25.8\,dm^3/s$$

3. HYDRAULIC GRADIENT

Consider a tank draining into another tank at a lower level as shown. There are small vertical tubes at points along the length to indicate the pressure head (h). Relative to a datum, the total energy head is

$h_T = h + z + u^2/2g$

This is shown as line A.

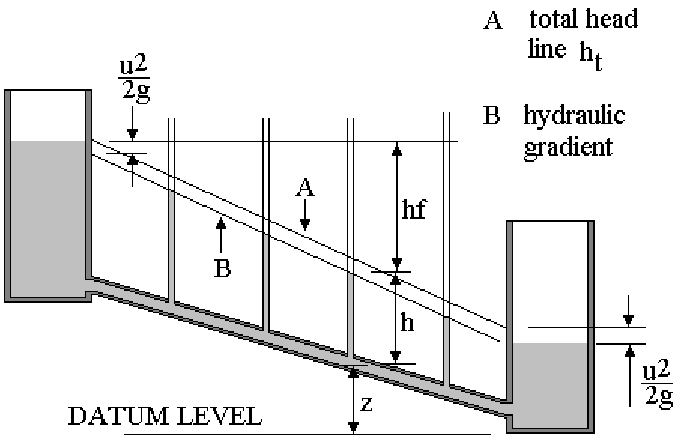

Fig. 5

The hydraulic grade line is the line joining the free surfaces in the tubes and represents the sum of h and z only. This is shown as line B and it is always below the line of h_T by the velocity head $u^2/2g$. Note that at exit from the pipe, the velocity head is not recovered but lost as friction as the emerging jet collides with the static liquid. The free surface of the tank does not rise.

The only reason why the hydraulic grade line is not horizontal is because there is a frictional loss h_f. The actual gradient of the line at any point is the rate of change with length $i = \delta h_f/\delta L$

SELF ASSESSMENT EXERCISE No.1

1. A pipe 100 mm bore diameter carries oil of density 900 kg/m^3 at a rate of 4 kg/s. The pipe reduces to 60 mm bore diameter and rises 120 m in altitude. The pressure at this point is atmospheric (zero gauge). Assuming no frictional losses, determine:

   i. The volume/s (4.44 dm^3/s)
   ii. The velocity at each section (0.566 m/s and 1.57 m/s)
   iii. The pressure at the lower end. (1.06 MPa)

2. A pipe 120 mm bore diameter carries water with a head of 3 m. The pipe descends 12 m in altitude and reduces to 80 mm bore diameter. The pressure head at this point is 13 m. The density is 1000 kg/m^3. Assuming no losses, determine

   i. The velocity in the small pipe (7 m/s)
   ii. The volume flow rate. (35 dm^3/s)

3. A horizontal nozzle reduces from 100 mm bore diameter at inlet to 50 mm at exit. It carries liquid of density 1000 kg/m^3 at a rate of 0.05 m^3/s. The pressure at the wide end is 500 kPa (gauge). Calculate the pressure at the narrow end neglecting friction. (196 kPa)

4. A pipe carries oil of density 800 kg/m^3. At a given point (1) the pipe has a bore area of 0.005 m^2 and the oil flows with a mean velocity of 4 m/s with a gauge pressure of 800 kPa. Point (2) is further along the pipe and there the bore area is 0.002 m^2 and the level is 50 m above point (1). Calculate the pressure at this point (2). Neglect friction. (374 kPa)

5. A horizontal nozzle has an inlet velocity u_1 and an outlet velocity u_2 and discharges into the atmosphere. Show that the velocity at exit is given by the following formulae.

   $$u_2 = \{2\Delta p/\rho + u_1^2\}^{½}$$
   and
   $$u_2 = \{2g\Delta h + u_1^2\}^{½}$$

4. LAMINAR and TURBULENT FLOW

The following work only applies to Newtonian fluids.

4.1 LAMINAR FLOW

A ***stream line*** is an imaginary line with no flow normal to it, only along it. When the flow is laminar, the streamlines are parallel and for flow between two parallel surfaces we may consider the flow as made up of parallel laminar layers. In a pipe these laminar layers are cylindrical and may be called ***stream tubes***. In laminar flow, no mixing occurs between adjacent layers and it occurs at low average velocities.

4.2 TURBULENT FLOW

The shearing process causes energy loss and heating of the fluid. This increases with mean velocity. When a certain critical velocity is exceeded, the streamlines break up and mixing of the fluid occurs. The diagram illustrates Reynolds coloured ribbon experiment. Coloured dye is injected into a horizontal flow. When the flow is laminar the dye passes along without mixing with the water. When the speed of the flow is increased turbulence sets in and the dye mixes with the surrounding water. One explanation of this transition is that it is necessary to change the pressure loss into other forms of energy such as angular kinetic energy as indicated by small eddies in the flow.

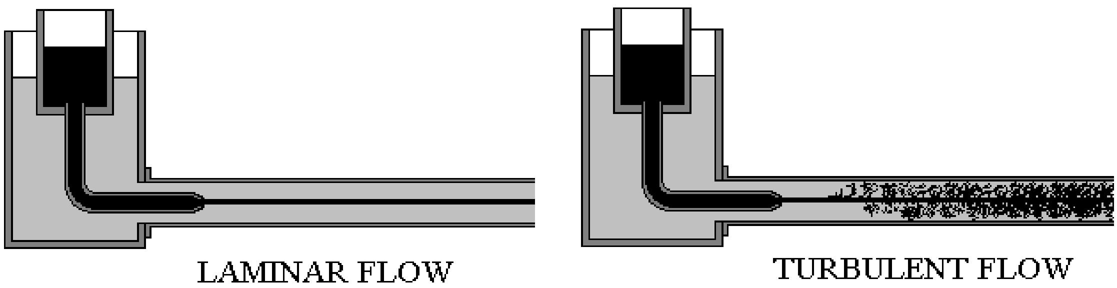

Fig. 6

4.3 LAMINAR AND TURBULENT BOUNDARY LAYERS

In chapter 2 it was explained that a ***boundary layer*** is the layer in which the velocity grows from zero at the wall (no slip surface) to 99% of the maximum and the thickness of the layer is denoted δ. When the flow within the boundary layer becomes turbulent, the shape of the boundary layers waivers and when diagrams are drawn of turbulent boundary layers, the mean shape is usually shown. Comparing a laminar and turbulent boundary layer reveals that the turbulent layer is thinner than the laminar layer.

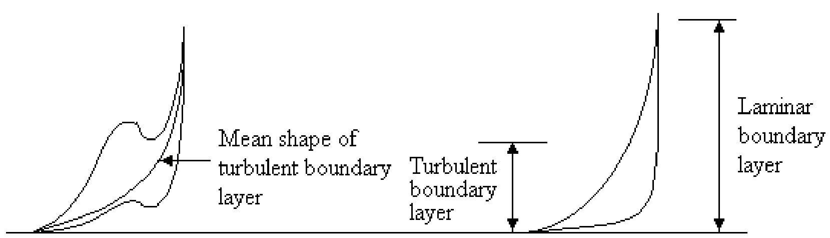

Fig. 7

5. CRITICAL VELOCITY - REYNOLDS NUMBER

When a fluid flows in a pipe at a volumetric flow rate Q m^3/s the average velocity is defined

$u_m = \frac{Q}{A}$ A is the cross sectional area.

The Reynolds number is defined as $R_e = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$

If you check the units of R_e you will see that there are none and that it is a dimensionless number. You will learn more about such numbers in section ….?.

Reynolds discovered that it was possible to predict the velocity or flow rate at which the transition from laminar to turbulent flow occurred for any Newtonian fluid in any pipe. He also discovered that the critical velocity at which it changed back again was different. He found that when the flow was gradually increased, the change from laminar to turbulent always occurred at a Reynolds number of 2500 and when the flow was gradually reduced it changed back again at a Reynolds number of 2000. Normally, 2000 is taken as the critical value.

WORKED EXAMPLE No.4

Oil of density 860 kg/m^3 has a kinematic viscosity of 40 cSt. Calculate the critical velocity when it flows in a pipe 50 mm bore diameter.

SOLUTION

$$R_e = \frac{u_m D}{\nu}$$

$$u_m = \frac{R_e \nu}{D} = \frac{2000 x 40 x 10^{-6}}{0.05} = 1.6 \text{ m/s}$$

6. DERIVATION OF POISEUILLE'S EQUATION for LAMINAR FLOW

Poiseuille did the original derivation shown below which relates pressure loss in a pipe to the velocity and viscosity for LAMINAR FLOW. His equation is the basis for measurement of viscosity hence his name has been used for the unit of viscosity. Consider a pipe with laminar flow in it. Consider a stream tube of length ΔL at radius r and thickness dr.

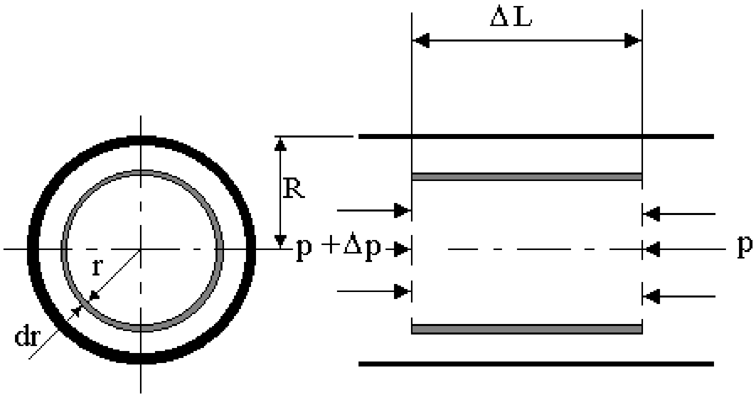

Fig.8

y is the distance from the pipe wall. $y = R - r$ $dy = -dr$ $\frac{du}{dy} = -\frac{du}{dr}$

The shear stress on the outside of the stream tube is τ. The force (F_s) acting from right to left is due to the shear stress and is found by multiplying τ by the surface area.

$Fs = \tau \times 2\pi r\,\Delta L$

For a Newtonian fluid , $\tau = \mu\frac{du}{dy} = -\mu\frac{du}{dr}$. Substituting for τ we get the following.

$$F_s = -2\pi r\Delta L\mu\frac{du}{dr}$$

The pressure difference between the left end and the right end of the section is Δp. The force due to this (F_p) is Δp x circular area of radius r.

$F_p = \Delta p \times \pi r^2$

Equating forces we have $-2\,\pi\,r\,\mu\,\Delta L\frac{du}{dr} = \Delta p\pi r^2$

$$du = -\frac{\Delta p}{2\,\mu\,\Delta L}rdr$$

In order to obtain the velocity of the streamline at any radius r we must integrate between the limits u = 0 when r = R and u = u when r = r.

$$\int_0^u du = -\frac{\Delta p}{2\,\mu\,\Delta L}\int_R^r rdr$$

$$u = -\frac{\Delta p}{4\,\mu\,\Delta L}\left(r^2 - R^2\right)$$

$$u = \frac{\Delta p}{4\,\mu\,L}\left(R^2 - r^2\right)$$

This is the equation of a Parabola so if the equation is plotted to show the boundary layer, it is seen to extend from zero at the edge to a maximum at the middle.

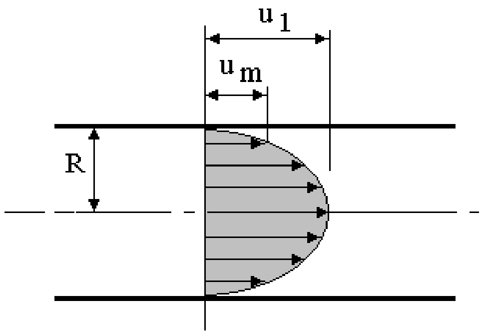

Fig.9

For maximum velocity put r = 0 and we get $u_1 = \frac{\Delta p R^2}{4\mu\mu\Delta}$

The average height of a parabola is half the maximum value so the average velocity is

$$u_m = \frac{\Delta p R^2}{8\mu\mu\Delta}$$

Often we wish to calculate the pressure drop in terms of diameter D. Substitute R=D/2 and rearrange.

$$\Delta p = \frac{32\mu 2\mu\Delta_m}{D^2}$$

The volume flow rate is average velocity x cross sectional area.

$$Q = \frac{\pi R^2 \Delta p R^2}{8\mu\mu\Delta} = \frac{\pi R^4 \Delta p}{8\mu\mu\Delta} = \frac{\pi D^4 \Delta p}{128\mu 28}$$

This is often changed to give the pressure drop as a friction head.

The friction head for a length L is found from $h_f = \Delta p/\rho g$

$$h_f = \frac{32\mu 2\mu_m}{\rho g D^2}$$

This is Poiseuille's equation that applies only to laminar flow.

WORKED EXAMPLE No. 5

A capillary tube is 30 mm long and 1 mm bore. The head required to produce a flow rate of 8 mm^3/s is 30 mm. The fluid density is 800 kg/m^3.

Calculate the dynamic and kinematic viscosity of the oil.

SOLUTION

Rearranging Poiseuille's equation we get

$$\mu = \frac{h_f \rho g D^2}{32 L u_m}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi \text{ x } 1^2}{4} = 0.785 \text{ mm}^2$$

$$u_m = \frac{Q}{A} = \frac{8}{0.785} = 10.18 \text{ mm/s}$$

$$\mu = \frac{0.03 \text{ x } 800 \text{ x } 9.81 \text{ x } 0.001^2}{32 \text{ x } 0.03 \text{ x } 0.01018} = 0.0241 \text{ N s/m or } 24.1 \text{ cP}$$

$$\nu = \frac{\mu}{\rho} = \frac{0.0241}{800} = 30.11 \text{ x } 10^{-6} \text{ m}^2/\text{s or } 30.11 \text{ cSt}$$

WORKED EXAMPLE No.6

Oil flows in a pipe 100 mm bore with a Reynolds number of 250. The dynamic viscosity is 0.018 Ns/m^2. The density is 900 kg/m^3.

Determine the pressure drop per metre length, the average velocity and the radius at which it occurs.

SOLUTION

$R_e = \rho\, u_m\, D/\mu$.

Hence

$u_m = Re\, \mu/ \rho D$

u_m = (250 x 0.018)/(900 x 0.1) = 0.05 m/s

$\Delta p = 32 \mu L\, u_m /D^2$

Δp = 32 x 0.018 x 1 x 0.05/0.1^2

Δp= 2.88 Pascals.

$u = \{\Delta p/4L\mu\}(R^2 - r^2)$ which is made equal to the average velocity 0.05 m/s

0.05 = (2.88/4 x 1 x 0.018)($0.05^2 - r^2$)

r = 0.035 m or 35.3 mm.

SELF ASSESSMENT EXERCISE No. 2

1. Oil flows in a pipe 80 mm bore diameter with a mean velocity of 0.4 m/s. The density is 890 kg/m^3 and the viscosity is 0.075 Ns/m^2.

   Show that the flow is laminar and hence deduce the pressure loss per metre length. (150 Pa)

2 Calculate the maximum velocity of water that can flow in laminar form in a pipe 20 m long and 60 mm bore. Determine the pressure loss in this condition. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 N s/m^2. (0.0333 m/s and 5.92 Pa)

3. Oil flow in a pipe 100 mm bore diameter with a Reynolds Number of 500. The density is 800 kg/m^3. The dynamic viscosity μ = 0.08 Ns/m^2.

   Calculate the velocity of a streamline at a radius of 40 mm. (0.36 m/s)

$$-\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \quad 4a$$

When a viscous fluid is subjected to an applied pressure it flows through a narrow horizontal passage as shown below. By considering the forces acting on the fluid element and assuming steady fully developed laminar flow, show that the velocity distribution is given by

b. Using the above equation show that for flow between two flat parallel horizontal surfaces distance t apart the velocity at any point is given by the following formula.

$$\mathbf{u = (1/2\mu)(dp/dx)(y^2 - ty)}$$

c. Carry on the derivation and show that the volume flow rate through a gap of height ‘t’ and width ‘B’ is given by $Q = -B\frac{dp}{dx}\frac{t^3}{12\mu}$.

d. Show that the mean velocity ‘u_m’ through the gap is given by $u_m = -\frac{1}{12\mu}\frac{dp}{dx}t^2$

5 The volumetric flow rate of glycerine between two flat parallel horizontal surfaces 1 mm apart and 10 cm wide is 2 cm^3/s. Determine the following.

   i. the applied pressure gradient dp/dx. (240 kPa per metre)
   ii. the maximum velocity. (0.06 m/s)

   For glycerine assume that μ= 1.0 Ns/m^2 and the density is 1260 kg/m^3.

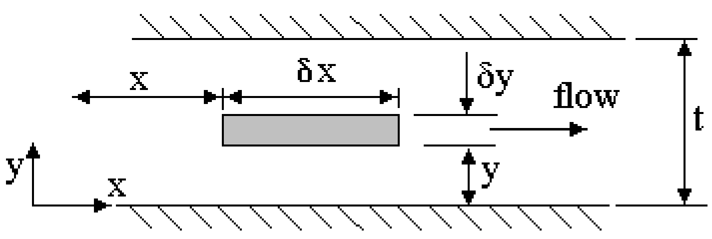

Fig.10

7. FRICTION COEFFICIENT

The friction coefficient is a convenient idea that can be used to calculate the pressure drop in a pipe. It is defined as follows.

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}}$$

7.1 DYNAMIC PRESSURE

Consider a fluid flowing with mean velocity u_m. If the kinetic energy of the fluid is converted into flow or fluid energy, the pressure would increase. The pressure rise due to this conversion is called the dynamic pressure.

KE = $\frac{1}{2} m u_m^2$

Flow Energy = p Q   Q is the volume flow rate and $\rho = m/Q$

Equating $\frac{1}{2} m u_m^2 = p\,Q$ $p = mu^2/2Q = \frac{1}{2}\rho u_m^2$

7.2 WALL SHEAR STRESS τ_o

The wall shear stress is the shear stress in the layer of fluid next to the wall of the pipe.

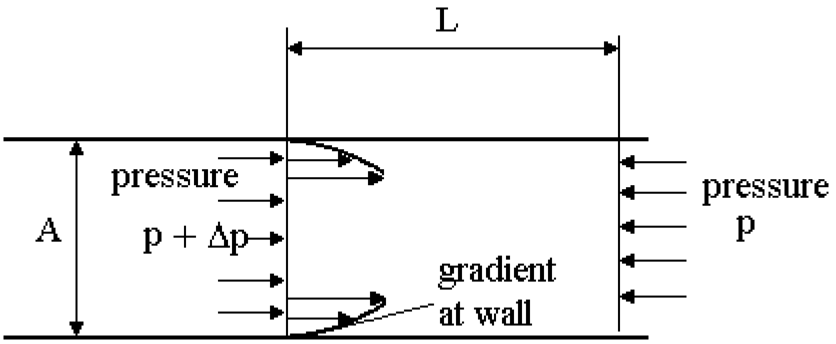

Fig.11

The shear stress in the layer next to the wall is $\tau_o = \mu\left(\frac{du}{dy}\right)_{wall}$

The shear force resisting flow is $F_s = \tau_o \pi L D$

The resulting pressure drop produces a force of $F_p = \frac{\Delta p \pi D^2}{4}$

Equating forces gives $\tau_o = \frac{D\,\Delta p}{4L}$

7.3 FRICTION COEFFICIENT for LAMINAR FLOW

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$$

From Poiseuille's equation $\Delta p = \frac{32\mu L u_m}{D^2}$ Hence $C_f = \left(\frac{2D}{4L\rho u_m^2}\right)\left(\frac{32\mu L u}{D^2}\right) = \frac{16\mu}{\rho u_m^2 D} = \frac{16}{R_e}$

8. DARCY FORMULA

This formula is mainly used for calculating the pressure loss in a pipe due to turbulent flow but it can be used for laminar flow also.

Turbulent flow in pipes occurs when the Reynolds Number exceeds 2500 but this is not a clear point so 3000 is used to be sure. In order to calculate the frictional losses we use the concept of friction coefficient symbol C_f. This was defined as follows.

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$$

Rearranging equation to make Δp the subject

$$\Delta p = \frac{4C_f L\rho u_m^2}{2D}$$

This is often expressed as a friction head h_f

$$h_f = \frac{\Delta p}{\rho g} = \frac{4C_f L u_m^2}{2gD}$$

This is the Darcy formula. In the case of laminar flow, Darcy's and Poiseuille's equations must give the same result so equating them gives

$$\frac{4C_f L u_m^2}{2gD} = \frac{32\mu L u_m}{\rho g D^2}$$

$$C_f = \frac{16\mu}{\rho u_m D} = \frac{16}{R_e}$$

This is the same result as before for laminar flow.

9. FLUID RESISTANCE

The above equations may be expressed in terms of flow rate Q by substituting u = Q/A

$$h_f = \frac{4C_f L u_m^2}{2gD} = \frac{4C_f L Q^2}{2gDA^2}$$

Substituting $A = \pi D^2/4$ we get the following.

$$h_f = \frac{32C_f L Q^2}{g\pi^2 D^5} = RQ^2$$

R is the fluid resistance or restriction. $R = \frac{32C_f L}{g\pi^2 D^5}$

If we want pressure loss instead of head loss the equations are as follows.

$$p_f = \rho g h_f = \frac{32\rho C_f L Q^2}{\pi^2 D^5} = RQ^2$$

R is the fluid resistance or restriction. $R = \frac{32\rho C_f L}{\pi^2 D^5}$

It should be noted that R contains the friction coefficient and this is a variable with velocity and surface roughness so R should be used with care.

10 MOODY DIAGRAM AND RELATIVE SURFACE ROUGHNESS

In general the friction head is some function of u_m such that $h_f = \phi u_m^n$. Clearly for laminar flow, n =1 but for turbulent flow n is between 1 and 2 and its precise value depends upon the roughness of the pipe surface. Surface roughness promotes turbulence and the effect is shown in the following work.

Relative surface roughness is defined as $\varepsilon = k/D$ where k is the mean surface roughness and D the bore diameter.

An American Engineer called Moody conducted exhaustive experiments and came up with the Moody Chart. The chart is a plot of C_f vertically against R_e horizontally for various values of ε. In order to use this chart you must know two of the three co-ordinates in order to pick out the point on the chart and hence pick out the unknown third co-ordinate. For smooth pipes, (the bottom curve on the diagram), various formulae have been derived such as those by Blasius and Lee.

BLASIUS $C_f = 0.0791\,R_e^{0.25}$

LEE $C_f = 0.0018 + 0.152\,R_e^{0.35}$.

The Moody diagram shows that the friction coefficient reduces with Reynolds number but at a certain point, it becomes constant. When this point is reached, the flow is said to be fully developed turbulent flow. This point occurs at lower Reynolds numbers for rough pipes.

A formula that gives an approximate answer for any surface roughness is that given by Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$

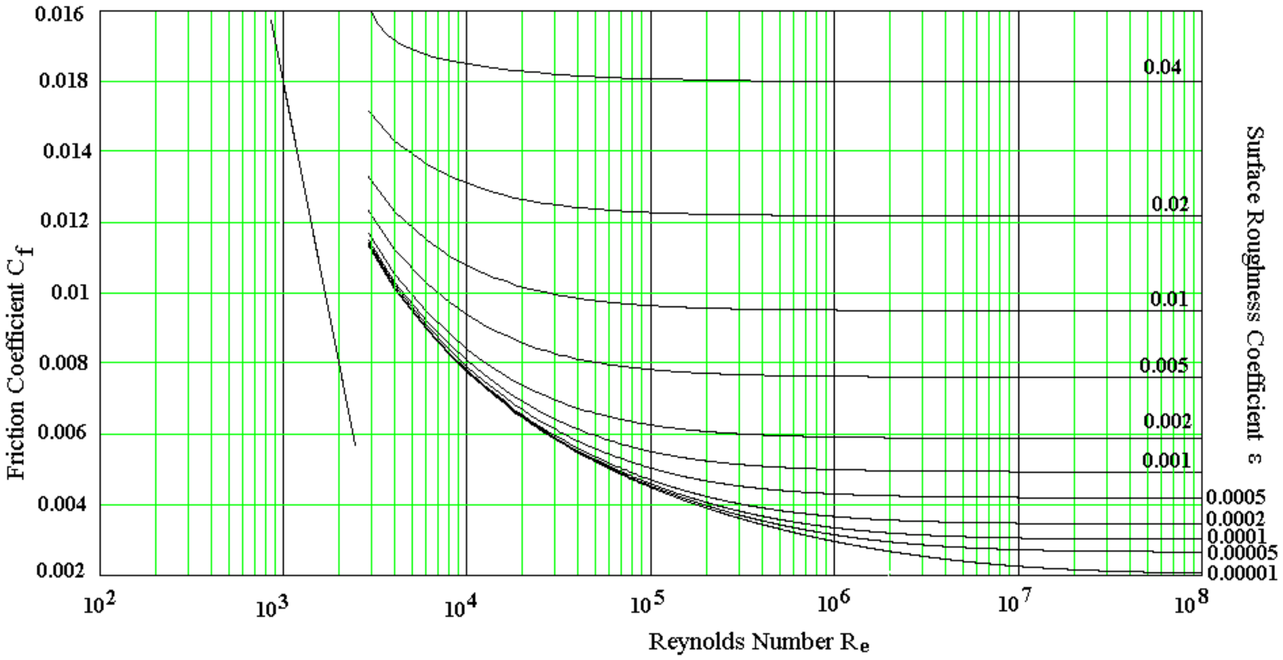

Fig.12 CHART

WORKED EXAMPLE No. 7

Determine the friction coefficient for a pipe 100 mm bore with a mean surface roughness of 0.06 mm when a fluid flows through it with a Reynolds number of 20 000.

SOLUTION

The mean surface roughness ε = k/d = 0.06/100 = 0.0006
Locate the line for ε = k/d = 0.0006.
Trace the line until it meets the vertical line at Re = 20 000. Read of the value of C_f horizontally on the left. Answer C_f = 0.0067

Check using the formula from Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{20000} + \left(\frac{0.0006}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{20000} + \left(\frac{0.0006}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = 12.206$$

$$C_f = 0.0067$$

WORKED EXAMPLE No. 8

Oil flows in a pipe 80 mm bore with a mean velocity of 4 m/s. The mean surface roughness is 0.02 mm and the length is 60 m. The dynamic viscosity is 0.005 N s/m^2 and the density is 900 kg/m^3. Determine the pressure loss.

SOLUTION

Re = ρud/μ = (900 x 4 x 0.08)/0.005 = 57600

ε= k/d = 0.02/80 = 0.00025

From the chart C_f = 0.0052

$h_f = 4C_fLu2/2dg$ = $(4 \times 0.0052 \times 60 \times 4^2)/(2 \times 9.81 \times 0.08)$ = 12.72 m

$\Delta p = \rho g h_f$ = 900 x 9.81 x 12.72 = 112.32 kPa.

SELF ASSESSMENT EXERCISE No. 3

1. A pipe is 25 km long and 80 mm bore diameter. The mean surface roughness is 0.03 mm. It caries oil of density 825 kg/m^3 at a rate of 10 kg/s. The dynamic viscosity is 0.025 N s/m^2.

   Determine the friction coefficient using the Moody Chart and calculate the friction head. (Ans. 3075 m.)

2. Water flows in a pipe at 0.015 m^3/s. The pipe is 50 mm bore diameter. The pressure drop is 13 420 Pa per metre length. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 N s/m^2.

   Determine the following.
   i. The wall shear stress (167.75 Pa)
   ii. The dynamic pressures (29180 Pa).
   iii. The friction coefficient (0.00575)
   iv. The mean surface roughness (0.0875 mm)

3. Explain briefly what is meant by fully developed laminar flow. The velocity u at any radius r in fully developed laminar flow through a straight horizontal pipe of internal radius r_o is given by

$$\mathbf{u = (1/4\mu)(r_o^2 - r^2)dp/dx}$$

   dp/dx is the pressure gradient in the direction of flow and μ is the dynamic viscosity. The wall skin friction coefficient is defined as $C_f = 2\tau_o/(\rho u_m^2)$.

   Show that $C_f = 16/R_e$ where $R_e = \rho u_m D/\mu$ an ρ is the density, u_m is the mean velocity and τ_o is the wall shear stress.

4. Oil with viscosity 2 x 10^{-2} Ns/m^2 and density 850 kg/m^3 is pumped along a straight horizontal pipe with a flow rate of 5 dm^3/s. The static pressure difference between two tapping points 10 m apart is 80 N/m^2. Assuming laminar flow determine the following.

   i. The pipe diameter.
   ii. The Reynolds number.

   Comment on the validity of the assumption that the flow is laminar.

11 MINOR LOSSES

Minor losses occur in the following circumstances.

i. Exit from a pipe into a tank.
ii. Entry to a pipe from a tank.
iii. Sudden enlargement in a pipe.
iv. Sudden contraction in a pipe.
v. Bends in a pipe.
vi. Any other source of restriction such as pipe fittings and valves.

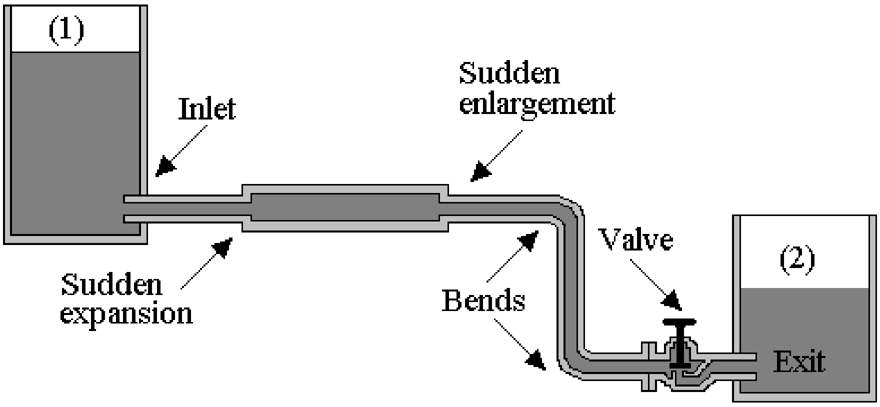

Fig.13

In general, minor losses are neglected when the pipe friction is large in comparison but for short pipe systems with bends, fittings and changes in section, the minor losses is the dominant factor.

In general, the minor losses are expressed as a fraction of the kinetic head or dynamic pressure in the smaller pipe.

Minor head loss = $k\,u^2/2g$ Minor pressure loss = $\frac{1}{2}k\rho u^2$

Values of k can be derived for standard cases but for items like elbows and valves in a pipeline, it is determined by experimental methods.

Minor losses can also be expressed in terms of fluid resistance R as follows.

$$h_L = k\frac{u^2}{2} = k\frac{Q^2}{2A^2} = k\frac{8Q^2}{\pi^2 D^4} = RQ^2 \quad \text{Hence } R = \frac{8k}{\pi^2 D^4}$$

$$p_L = k\frac{8\rho g Q^2}{\pi^2 D^4} = RQ^2 \quad \text{hence } R = \frac{8k\rho g}{\pi^2 D^4}$$

Before you go on to look at the derivations, you must first learn about the coefficients of contraction and velocity.

10.1 COEFFICIENT OF CONTRACTION Cc

The fluid approaches the entrance from all directions and the radial velocity causes the jet to contract just inside the pipe. The jet then spreads out to fill the pipe. The point where the jet is smallest is called the *VENA CONTRACTA*.

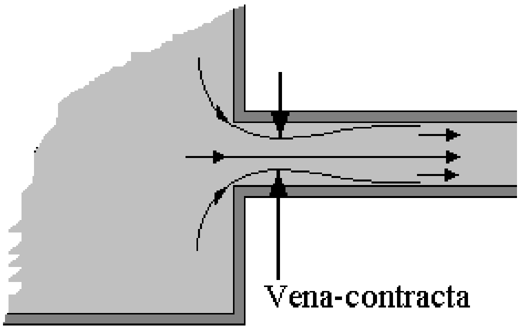

Fig.14

The coefficient of contraction C_c is defined as $C_c = A_j/A_o$

A_j is the cross sectional area of the jet and A_o is the c.s.a. of the pipe. For a round pipe this becomes $C_c = d_j^2/d_o^2$.

10.2 COEFFICIENT OF VELOCITY C_v

The coefficient of velocity is defined as **C_v = actual velocity/theoretical velocity**

In this instance it refers to the velocity at the vena-contracta but as you will see later on, it applies to other situations also.

10.3 EXIT FROM A PIPE INTO A TANK.

The liquid emerges from the pipe and collides with stationary liquid causing it to swirl about before finally coming to rest. All the kinetic energy is dissipated by friction. It follows that all the kinetic head is lost so k = 1.0

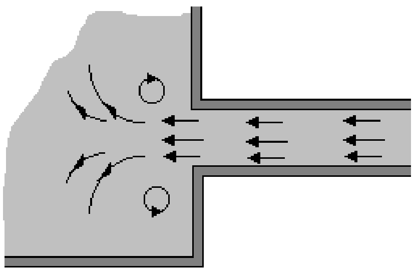

Fig.15

10.4 ENTRY TO A PIPE FROM A TANK

The value of k varies from 0.78 to 0.04 depending on the shape of the inlet. A good rounded inlet has a low value but the case shown is the worst.

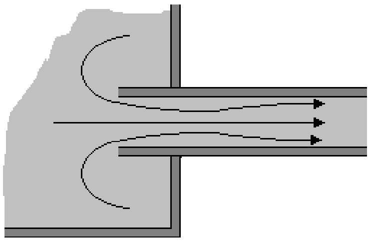

Fig.16

10.5 SUDDEN ENLARGEMENT

This is similar to a pipe discharging into a tank but this time it does not collide with static fluid but with slower moving fluid in the large pipe. The resulting loss coefficient is given by the following expression.

$$k = \left\{1 - \left(\frac{d_1}{d_2}\right)^2\right\}^2$$

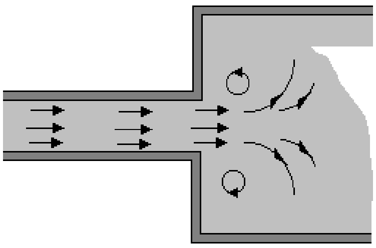

Fig.17

10.6 SUDDEN CONTRACTION

This is similar to the entry to a pipe from a tank. The best case gives k = 0 and the worse case is for a sharp corner which gives k = 0.5.

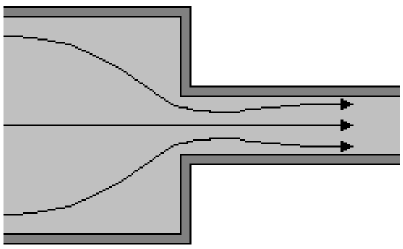

Fig.18

10.7 BENDS AND FITTINGS

The k value for bends depends upon the radius of the bend and the diameter of the pipe. The k value for bends and the other cases is on various data sheets. For fittings, the manufacturer usually gives the k value. Often instead of a k value, the loss is expressed as an equivalent length of straight pipe that is to be added to L in the Darcy formula.

WORKED EXAMPLE No.9

A tank of water empties by gravity through a horizontal pipe into another tank. There is a sudden enlargement in the pipe as shown. At a certain time, the difference in levels is 3 m. Each pipe is 2 m long and has a friction coefficient $C_f = 0.005$. The inlet loss constant is K = 0.3.

Calculate the volume flow rate at this point.

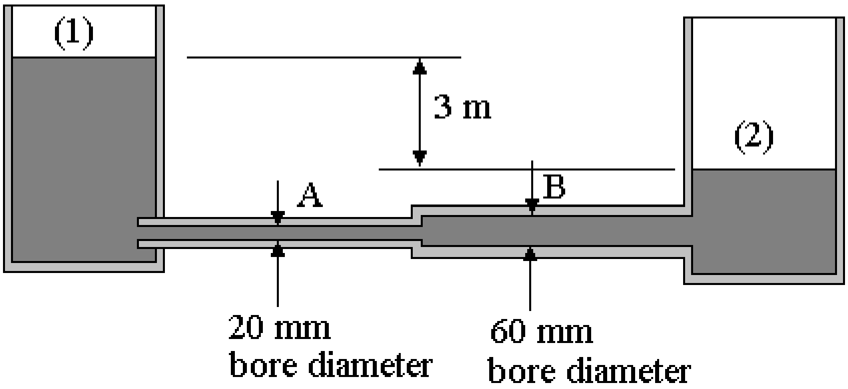

Fig.19

SOLUTION

There are five different sources of pressure loss in the system and these may be expressed in terms of the fluid resistance as follows.

The head loss is made up of five different parts. It is usual to express each as a fraction of the kinetic head as follows.

Resistance pipe A $R_1 = \frac{32C_f L}{gD_A{}^5\pi^2} = \frac{32 \text{ x } 0.005 \text{ x } 2}{g \text{ x } 0.025^5\pi^2} = 1.0328 \text{ x } 10^6 \text{ s}^2\text{m}^{-5}$

Resistance in pipe B $R_2 = \frac{32C_f L}{gD_B^5\pi^2} = \frac{32 \text{ x } 0.005 \text{ x } 2}{g \text{ x } 0.06^5\pi^2} = 4.250\text{x}10^3 \text{ s}^2\text{m}^{-5}$

Loss at entry K=0.3 $R_3 = \frac{8K}{g\pi^2 D_A{}^4} = \frac{8 \text{ x } 0.3}{g\,\pi^2 \text{ x } 0.02^4} = 158 \text{ s}^2\text{m}^{-5}$

Loss at sudden enlargement. $k = \left\{1 - \left(\frac{d_A}{d_B}\right)^2\right\}^2 = \left\{1 - \left(\frac{20}{60}\right)^2\right\}^2 = 0.79$

$$R_4 = \frac{8K}{g\pi^2 D_A{}^4} = \frac{8\text{x}0.79}{g\pi^2 \text{ x } 0.02^4} = 407.7 \text{ s}^2\text{m}^{-5}$$

Loss at exit K=1 $R_5 = \frac{8K}{g\pi^2 D_B{}^4} = \frac{8\text{x}1}{g\pi^2 \text{ x } 0.06^4} = 63710 \text{ s}^2\text{m}^{-5}$

Total losses.

$$h_L = R_1Q^2 + R_2Q^2 + R_3Q^2 + R_4Q^2 + R_5Q^2$$

$$h_L = (R_1 + R_2 + R_2 + R_4 + R_5)Q^2 = 1.101 \text{ x } 10^6 Q^2$$

BERNOULLI'S EQUATION

Apply Bernoulli between the free surfaces (1) and (2)

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

On the free surface the velocities are small and about equal and the pressures are both atmospheric so the equation reduces to the following.

$z_1 - z_2 = h_L = 3$ $3 = 1.101 \text{ x } 10^6 Q^2$

$Q^2 = 2.724 \text{ x } 10^{-6}$ $Q = 1.65 \text{ x } 10^{-3} \text{ m}^3/\text{s}$

SELF ASSESSMENT EXERCISE No.4

1. A pipe carries oil at a mean velocity of 6 m/s. The pipe is 5 km long and 1.5 m diameter. The surface roughness is 0.8 mm. The density is 890 kg/m^3 and the dynamic viscosity is 0.014 N s/m^2. Determine the friction coefficient from the Moody chart and go on to calculate the friction head h_f. (Ans. C_f = 0.0045 h_f = 110.1 m)

2. The diagram shows a tank draining into another lower tank through a pipe. Note the velocity and pressure is both zero on the surface on a large tank. Calculate the flow rate using the data given on the diagram. (Ans. 7.16 dm^3/s)

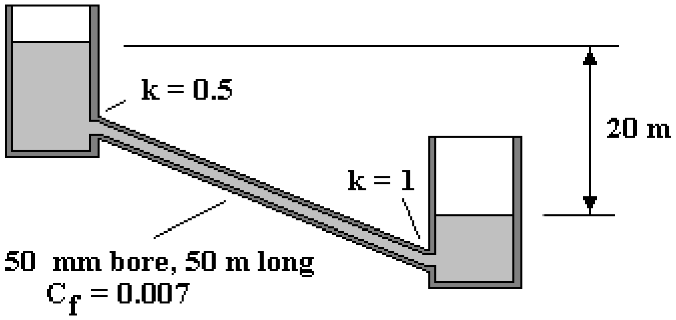

Fig.20

3. A domestic water supply consists of a large tank with a loss free-inlet to a 10 mm diameter pipe of length 20 m, that contains 9 right angles bends. The pipe discharges to atmosphere 8.0 m below the free surface level of the water in the tank.

   Evaluate the flow rate of water assuming that there is a loss of 0.75 velocity heads in each bend and that friction in the pipe is given by the Blasius equation $C_f = 0.079(Re)^{-0.25}$ (0.118 dm^3/s).

   The dynamic viscosity is 0.89 x 10^{-3} and the density is 997 kg/m^3.