EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

PRINCIPLES AND APPLICATIONS of FLUID MECHANICS UNIT 13

NQF LEVEL 3

OUTCOME 1 - PHYSICAL PROPERTIES AND CHARACTERISTIC BEHAVIOUR OF FLUIDS

TUTORIAL 2 - VISCOSITY

CONTENT

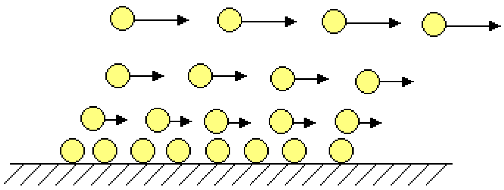
Know about the physical properties and characteristic behaviour of fluids

Surface tension**:** surface tension coefficient; capillary action

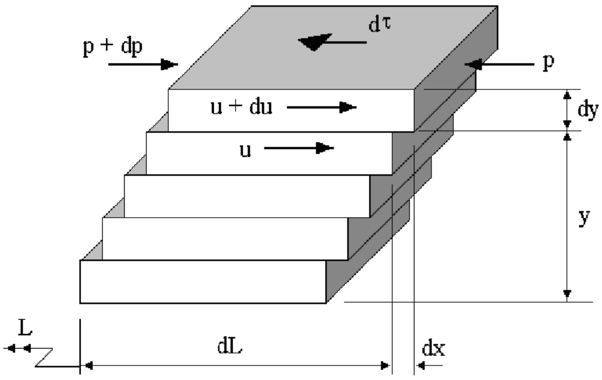
Viscosity**:** viscous behaviour e.g. dynamic viscosity, kinematic viscosity, effect of shearing in Newtonian fluids (water, lubricating oils) and non-Newtonian fluids (pseudoplastic, Bingham plastic, Casson plastic, dilatent); bearings e.g. plain journal, plain thrust; system parameters e.g. bearing dimensions, speed, viscosity of lubricant, viscous resistance, power loss

1. VISCOSITY

Tutorial 1 gave a general explanation and definition for viscosity. In the following we will only consider the case where a liquid wets the surface. We will derive equations for the forces needed to make a fluid flow and provide a definition for viscosity.



The diagram illustrates molecules flowing over a surface. The molecules in contact with the surface stick to it and do not move. The molecules above are pulled back by the forces of attraction but we find that the further you move from the surface, the faster the molecules move.



The next diagram illustrates how the fluid moves in layers, one above the other. The layer next to the surface is stuck to it (it wets the surface). The layers of fluid above the surface are moving so there must be shearing taking place between the layers of the fluid.

Let us suppose that the fluid is flowing over a flat surface in laminated layers from left to right as shown.

Consider two layers at a distance y from the surface. In a small time dt the upper layer moves a distance dx relative to the one below it. The thickness of the layer is dy. Denote velocity with the letter 'u'

The increase in velocity between the two layers is $du = \frac{dx}{dt}$

Because the layers are slipping over each other, there is shear stress 'τ' acting on the slipping surface and the layer must be subjected to a corresponding shear strain 'γ'. We define this as:

$$\gamma = \frac{\text{sideways deformation}}{\text{height of the layer being deformed}} = \frac{dx}{dy}$$

The rate of shear strain is defined as follows.

$$\dot{\gamma} = \frac{\text{shear strain}}{\text{time taken}} = \frac{\gamma}{dt} = \frac{dx}{dt\,dy} = \frac{du}{dy}$$

This is also called the 'velocity gradient' since it measures the rate of change of velocity with distance y.

NEWTONIAN FLUIDS.

It is found that fluids such as water, oil and air, behave in such a manner that the shear stress between layers is directly proportional to the rate of shear strain. $\tau = \text{constant} \times \dot{\gamma}$

Fluids that obey this law are called ***NEWTONIAN FLUIDS***. It is the constant in this formula that we know as the dynamic viscosity of the fluid.

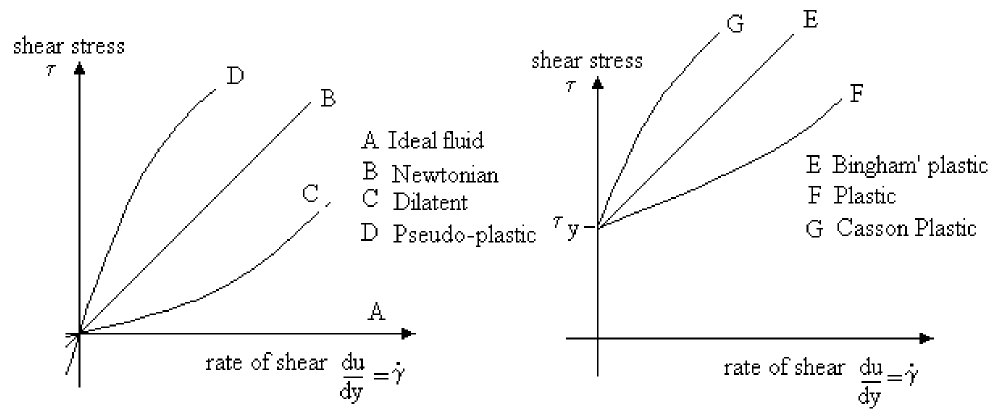
$$\textbf{\textit{DYNAMIC VISCOSITY}}\ \mu = \frac{\text{shear stress}}{\text{rate of shear}} = \frac{\tau}{\dot{\gamma}} = \tau\frac{dy}{du}\ \text{N s/m}^2.\ \textbf{\textit{(1cP = 0.001 N s/m}}^2\textbf{\textit{)}}$$

Note that $\frac{du}{dy}$ is called the velocity gradient and the meaning of this becomes more clear when you study the boundary layer formation. Newtonian fluids will be covered in more detail later in this tutorial.

2. NON-NEWTONIAN FLUIDS

We defined a Newtonian fluid as one that obeyed the law $\tau = \mu \frac{du}{dy}$. There is a range of other liquid or semi-liquid materials that do not obey this law and produce strange flow characteristics. Such materials include various foodstuffs, paints, cements and so on. Many of these are in fact solid particles suspended in a liquid with various concentrations.

All Non-Newtonian fluids have a law relating shear stress τ and the rate of shear strain $\dot{\gamma} = \frac{du}{dy}$. The graphs below show the way various fluids behave.



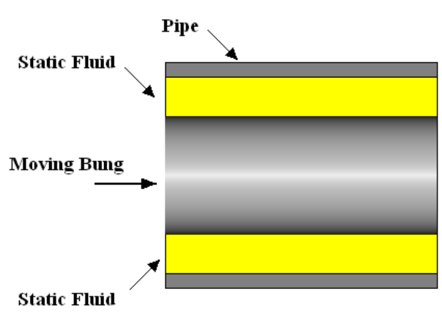
Graph A shows **an ideal fluid that has no viscosity** and hence has no shear stress at any point. This is often used in theoretical models of fluid flow.

Graph B shows a Newtonian Fluid. The graph is a straight line and the gradient is the dynamic viscosity μ.

Graph C shows the relationship for a ***Dilatent fluid***. The gradient and hence viscosity increases with γ and such fluids are also called ***shear-thickening.*** This phenomenon occurs with some solutions of sugar and starches. Did you know that if you run on a lake of custard it will be like running on a solid surface but if you stand still you will sink into it.

Graph D shows the relationship for a ***Pseudo-plastic***. The gradient and hence viscosity reduces with γ and they are called ***shear-thinning.*** Most foodstuffs are like this as well as clay and liquid cement.

The graphs E, F and G show fluids that do not shear at all until a minimum force and shear stress 'τ_y' is reached. This is plastic behaviour but unlike plastics, there may be no elasticity prior to shearing. Note that fluids with a shear yield stress will flow in a pipe as a plug. Within a certain radius, the shear stress will be insufficient to produce shearing so inside that radius the fluid flows as a solid plug.



Graph E shows the relationship for a ***Bingham plastic.*** This is the special case where the behaviour is the same as a Newtonian fluid except for the existence of the yield stress. Foodstuffs containing high level of fats approximate to this model (butter, margarine, chocolate and Mayonnaise).

Graph F shows the relationship for a ***plastic*** fluid that exhibits shear thickening characteristics.

Graph G shows the relationship for a ***Casson fluid***. This is a plastic fluid that exhibits shear-thinning characteristics. This model was developed for fluids containing rod like solids and is often applied to molten chocolate and blood.

MATHEMATICAL MODELS

It is doubtful that you are expected to use mathematical models at this level but it is included for those who want to do more advanced studies. The graphs that relate shear stress τ and rate of shear strain γ are based on models or equations. Most are mathematical equations created to represent empirical data (i.e. found by doing experiments). Oddly, the maths for this is easier than for a Newtonian fluid.

Hirschel and Bulkeley developed the power law for non-Newtonian equations. This is as follows.

$\tau = \tau_y + K\dot{\gamma}^n$ K is called the consistency coefficient and n is a power.

In the case of a Newtonian fluid n = 1 and $\tau_y = 0$ and K = μ (the dynamic viscosity) $\tau = \mu\dot{\gamma}$

For a Bingham plastic, n = 1 and K is also called the plastic viscosity μ_p. The relationship reduces to $\tau = \tau_y + \mu_p\dot{\gamma}$

For a ***dilatent fluid,*** $\tau_y = 0$ and n>1

For a ***pseudo-plastic,*** $\tau_y = 0$ and n<1

The model for both is $\tau = K\dot{\gamma}^n$

The ***Herchel-Bulkeley*** model is as follows. $\tau = \tau_y + K\dot{\gamma}^n$

This may be developed as follows.

$\tau = \tau_y + K\dot{\gamma}^n$

$\tau - \tau_y = K\dot{\gamma}^n$ sometimes writtend as $\tau - \tau_y = \mu_p\dot{\gamma}^n$ where μ_p is called the plastic viscosity.

dividing by $\dot{\gamma}$

$$\frac{\tau}{\dot{\gamma}} - \frac{\tau_y}{\dot{\gamma}} = K\frac{\dot{\gamma}^n}{\dot{\gamma}} = K\dot{\gamma}^{n-1}$$

$\frac{\tau}{\dot{\gamma}} = \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1}$ The ratio is called the apparent viscosity μ_{app}

$$\mu_{app} = \frac{\tau}{\dot{\gamma}} = \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1}$$

For a Bingham plastic n = 1 so $\mu_{app} = \frac{\tau_y}{\dot{\gamma}} + K$

For a Fluid with no yield shear value $\tau_y = 0$ so $\mu_{app} = K\dot{\gamma}^{n-1}$

The *Casson* fluid model is quite different in form from the others and is as follows.

$$\tau^{\frac{1}{2}} = \tau_y^{\frac{1}{2}} + K\dot{\gamma}^{\frac{1}{2}}$$

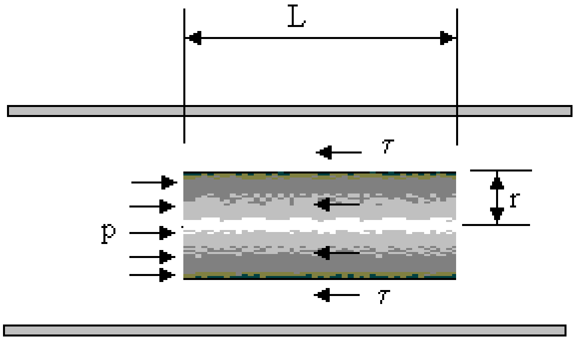
WORKED EXAMPLE No. 1

The Herchel-Bulkeley model for a non-Newtonian fluid is as follows. $\tau = \tau_y + K\dot{\gamma}^n$.

Derive an equation for the minimum pressure required drop per metre length in a straight horizontal pipe that will produce flow.

Given that the pressure drop per metre length in the pipe is 60 Pa/m and the yield shear stress is 0.2 Pa, calculate the radius of the slug sliding through the middle.

SOLUTION



The pressure difference p acting on the cross sectional area must produce sufficient force to overcome the shear stress τ acting on the surface area of the cylindrical slug. For the slug to move, the shear stress must be at least equal to the yield value τy. Balancing the forces gives the following.

$p \times \pi r^2 = \tau_y \times 2\pi r L$

$p/L = 2\tau_y / r$

$60 = 2 \times 0.2/r$ $\quad$ $r = 0.4/60 = 0.0066$ m or 6.6 mm

WORKED EXAMPLE No. 2

A Bingham plastic flows in a pipe and it is observed that the central plug is 30 mm diameter when the pressure drop is 100 Pa/m.

Calculate the yield shear stress.

Given that at a larger radius the rate of shear strain is 20 s^{-1} and the consistency coefficient is 0.6 Pa s, calculate the shear stress.

SOLUTION

For a Bingham plastic, the same theory as in the last example applies.

$p/L = 2\tau_y / r$ $\quad 100 = 2\tau_y/0.015$ $\quad$ $\tau_y = 100 \times 0.015/2 = 0.75$ Pa

A mathematical model for a Bingham plastic is

$\tau = \tau_y + K\dot{\gamma} = 0.75 + 0.6 \times 20 = 12.75$ Pa

WORKED EXAMPLE No. 3

Research has shown that tomato ketchup has the following viscous properties at 25°C.

Consistency coefficient K = 18.7 Pa s^n
Power n = 0.27
Shear yield stress = 32 Pa

Calculate the apparent viscosity when the rate of shear is 1, 10, 100 and 1000 s^{-1} and conclude on the effect of the shear rate on the apparent viscosity.

SOLUTION

This fluid should obey the Herchel-Bulkeley equation so

$$\mu_{app} = \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1}$$

$$\mu_{app} = \frac{32}{\dot{\gamma}} + 18.7\dot{\gamma}^{0.27-1}$$

Evaluating at the various strain rates we get.

$\gamma = 1$ $\mu_{app} = 18.8$
$\gamma = 10$ $\mu_{app} = 3.482$
$\gamma = 100$ $\mu_{app} = 0.648$
$\gamma = 1000$ $\mu_{app} = 0.12$

The apparent viscosity reduces as the shear rate increases.

SELF ASSESSMENT EXERCISE No. 1

Find examples of the following non- Newtonian fluids by searching the web.

Pseudo Plastic

Bingham's Plastic

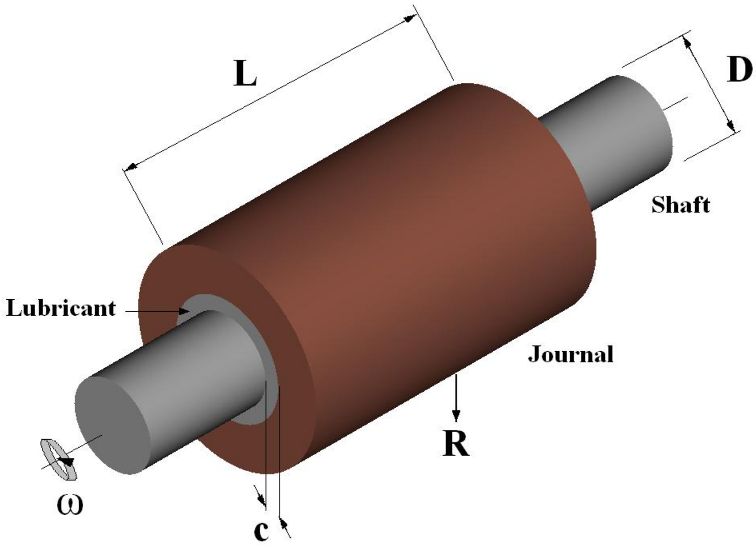
Casson Plastic

Dilatent Fluid

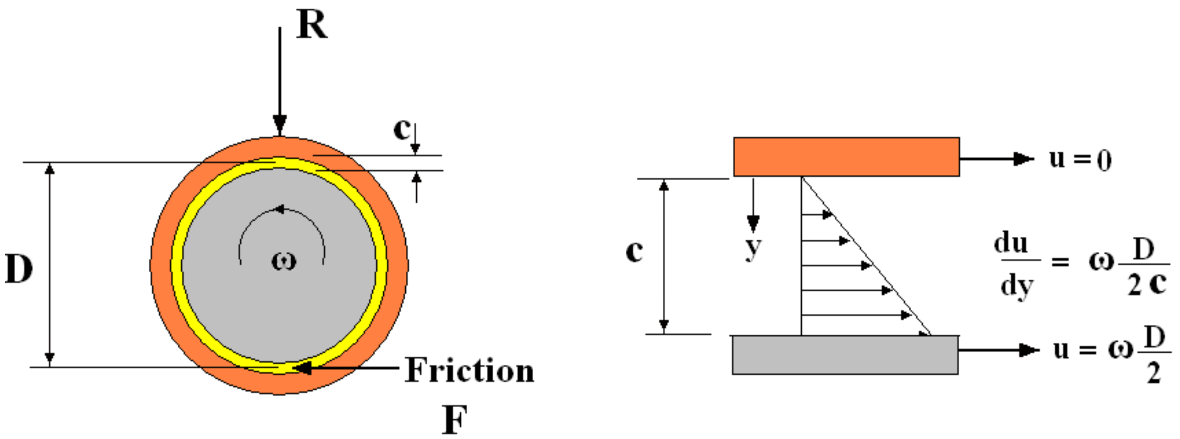
The study of friction in bearings is called TRIBOLOGY and this is a very complicated area of study. A Journal bearing is a good example to introduce you to the subject.

3. JOURNAL BEARINGS

The cylinder is the bearing and would be part of some machine. The shaft is often called the journal. Either the shaft rotates inside the bearing or the bearing rotates around the shaft. The purpose of the bearing is to support the shaft when a side load 'R' is applied. Clearly with no lubrication the two would rub and friction and wear would result. The introduction of lubrication into the clearance gap between the two will reduce both. The lubricant must remain in the bearing so often it is pumped in under pressure and flows out of the ends.



The diagram shows how friction due to viscous shearing in the lubricant 'F' acts to oppose rotation. This produces an opposing torque 'T' and the power used to overcome this is power lost as heat.



The simplest theory assumes that the journal runs concentrically so that there is a uniform clearance gap 'c'. The lubricant in the gap sticks to the surface of the bearing and to the surface of the shaft. If the bearing is stationary and the shaft revolving then the velocity of the lubricant must change from zero to the velocity of the surface of the shaft.

The shaft diameter is D. 'ω' is the angular velocity of the shaft and the radius of the surface is D/2 so $u = \omega D/2$ on the shaft surface. Assume that the velocity in the lubricant varies from 0 to $\omega D/2$ in a linear manner as shown.

The velocity gradient is then the same at all points:

$$\frac{du}{dy} = \frac{\omega D}{2c}$$

Lubricants are normally Newtonian so the shear stress between any of these layers is then

$$\tau = \eta \frac{du}{dy} = \eta \frac{\omega D}{2c}$$

Note that the alternative symbol η is used for dynamic viscosity to avoid confusion with the coefficient of friction later.

The shear force between any layer is then F = τ x area

The area could be the inside area of the bearing or the outside area of the shaft and there will be a slight difference so let us take D as the diameter to the middle of the oil film. If D>>c then this will be a very small error. A = πDL where L is the length of the bearing.

$$F = \tau \, x \, \text{Area} = \tau \, \pi \, D \, L = \eta \frac{\omega D}{2c} \pi \, D \, L$$

Since shaft speeds are normally given in rev/s we may substitute ω = 2πN

$$F = \frac{\eta D^2 \pi^2 N \; L}{c}$$

This is the friction force opposing rotation. The resulting friction torque is T = FD/2

$$T = \frac{\eta \, \pi \;\; L \, \omega D^3}{4c}$$

$$T = \frac{\eta \, \pi^2 \;\; L \, N \, D^3}{2c}$$

The power lost to friction is P = ωT = 2πNT

We often use the concept of friction coefficient defined as:

$$\mu = \frac{F}{R} \text{ (This is why η was chosen for viscosity).}$$

Hence $$\mu = \frac{\eta \, \pi \;\; L \, \omega \, D^2}{2cR} \text{ or } \frac{\eta \, \pi^2 \;\; LND^2}{cR}$$

WORKED EXAMPLE No. 4

A journal runs in a bearing 60 mm diameter and 60 mm long at 20 rev/s. The clearance gap is 1 mm. The lubricant in the clearance gap has a dynamic viscosity of 50 cP. The bearing must carry a side load of 800 N.

Calculate the following:
- The Friction force.
- The friction torque.
- The power loss.
- The coefficient of friction.

SOLUTION

$$F = \frac{\eta D^2 \pi^2 N \; L}{c} = \frac{0.05 \, x \, 0.06^2 \, x \, \pi^2 \; x \, 20 \, x \, 0.06}{0.001} = 2.132 \text{ N}$$

T = F D/2 = 2.132 x 0.03 = 0.064 Nm

P = 2πNT = 2π x 20 x 0.064 = 8.04 W

μ = F/R = 2.132/800 = 0.00267

SELF ASSESSMENT EXERCISE No.2

1. A journal runs in a bearing 30 mm diameter and 50 mm long at 30 rev/s. The clearance gap is 0.8 mm. The lubricant in the clearance gap has a dynamic viscosity of 120 cP. The bearing must carry a side load of 600 N.

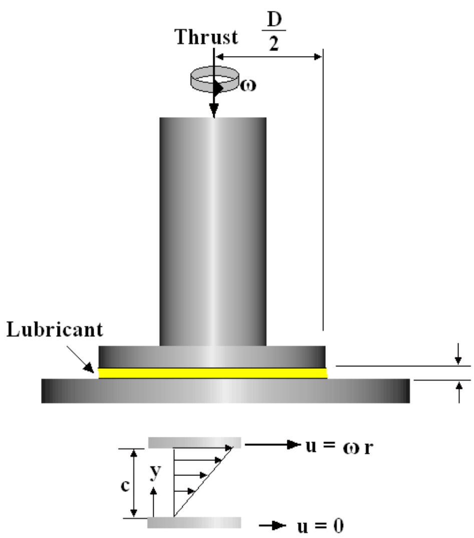
   Calculate the following:
   The Friction force. (2 N)
   The friction torque. (0.03 Nm)
   The power loss. (5.63 W)
   The coefficient of friction. (0.00333)

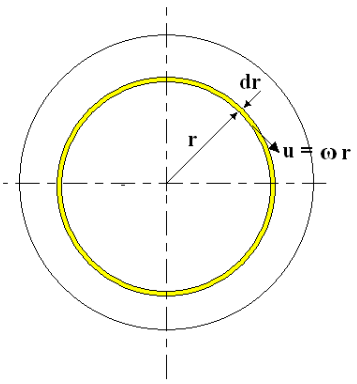
2. For the same bearing as in Q1, plot a graph of the coefficient of friction against speed N from N = 0 to N = 100 rev/s. What is the relationship between them?

3. A journal runs in a bearing 25 mm diameter and 10 mm long at 15 rev/s. The lubricant in the clearance gap has a dynamic viscosity of 80 cP. The bearing must carry a side load of 20 N. The coefficient of friction must be 0.006. Calculate the clearance gap. (0.616 mm)

4. THRUST BEARINGS

These are a little more complicated than journal bearings. They are bearings designed to support an axial force acting along the length of a shaft. For example a vertical rotor will need to rest on its end so some kind of thrust bearing is needed. The simplest form is a flat disc on the end of the shaft rotating on top of a flat surface as shown. Consider a round flat disc diameter 'D' and radius D/2 rotating at angular velocity ω rad/s on top of a flat surface and separated from it by an oil film of thickness c. The lubricant is normally pumped in under pressure and leaks away around the edges.





The complication is that the surface velocity varies with radius otherwise the derivation is the same as for the journal. At a given radius r we assume the velocity varies from zero on the stationary surface to u = ωr at the moving surface. It follows that shearing is greatest at the outer edge.

Assume the velocity gradient is linear in which case $\frac{du}{dy} = \frac{\omega r}{c}$

The shear stress on the ring is $\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{c}$

Now consider the shear force acting on a small elementary ring radius r and width dr as shown in the diagram.

This will be a small part of the total shear force so dF = τ x area of the ring = τ x 2πr dr

The shear force is $dF = \frac{\mu \omega r}{c} x\, 2\pi r\, dr = 2\pi r^2 dr\, \mu \frac{\omega}{c}$

The torque produced by this force is a small part of the total torque dT

$dT = r\, dF = 2\pi r^3\, dr\, \mu \frac{\omega}{c}$

The total torque is found by integrating between the middle and the outside.

$T = \int_0^{D/2} 2\pi r^3 dr\, \mu \frac{\omega}{c} = \frac{\mu \pi \omega D^4}{32\, c}$ Note D is the diameter of the disc.

In terms of revolution ω = 2πN hence :

$T = \frac{\mu \pi^2 N D^4}{16\, c}$ Note the gap c depends on the weight being supported and the pressure of the oil and advanced studies are needed to work out the weight supported by a given gap.

WORKED EXAMPLE No. 5

A shaft rests on a thrust bearing is 120 mm and revolves at 50 rev/s. The clearance gap is 1 mm. The lubricant in the clearance gap has a dynamic viscosity of 80 cP. Calculate the friction torque and power loss..

SOLUTION

$T = \frac{\mu \pi^2 N D^4}{16\, c} = \frac{0.08\, x\, \pi^2\, x\, 50\, x\, 0.12^4}{16\, x\, 0.001}$

T = 0.512 Nm

P = 2πNT = 2π x 50 x 0.512 =160.7 W

SELF ASSESSMENT EXERCISE No. 3

1. A vertical rotor has a shaft 30 mm diameter that rests in a recess and the weight is supported by a film of oil 0.5 mm thick and dynamic viscosity 110 cP. Calculate the friction torque and power loss at 1420 rev/min.

   (0.0026 Nm and 0.387 W)

2. A vertical rotor has a shaft 100 mm diameter that rests in a recess and the weight is supported by a film of oil 0.75 mm thick and dynamic viscosity 60 cP. Calculate the friction torque and power loss at 30 rev/s.

   (0.148 Nm and 27.9 W)