MECHANICAL PRINCIPLES

OUTCOME 3
CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

TUTORIAL 1 – CENTRIFUGAL FORCE

Centripetal acceleration and force: derivation of expressions for centripetal acceleration and centripetal force in terms of radius of rotation and angular or tangential velocity, calculation of centripetal force and centrifugal reaction in rotating systems (eg conical pendulum, governors, aircraft performing a vertical loop, tilting trains, fairground rides, satellites in geostationary orbit)

Centrifugal clutches: construction and operation, speed at which engagement commences, torque and power transmitted

***Stability of vehicles**:* maximum speed on unbanked level curves, wheel reactions, speed on a banked curve for no tendency to side-slip

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

On completion of this tutorial you should be able to

- Analyse problems involving vehicles skidding and overturning on bends.
- Analyse problems involving banked bends.

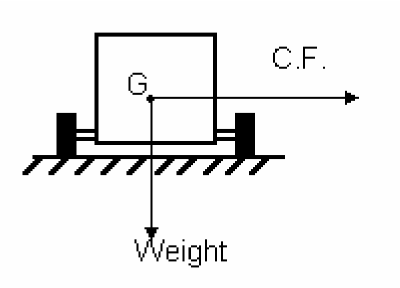
It is assumed that the student is already familiar with the following concepts.

- Newton's laws of Motion.
- Coulomb's laws of friction.
- Stiffness of a spring.
- The laws relating angular displacement, velocity and acceleration.
- The laws relating angular and linear motion.
- Basic vector theory.
- Basic stress and strain relationships.

All the above may be found in the pre-requisite tutorials.

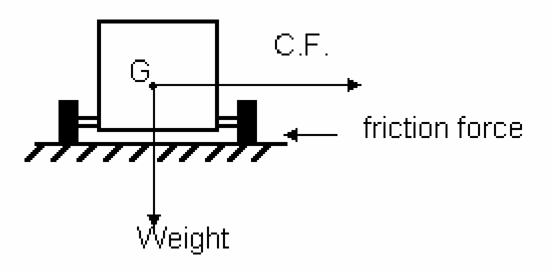
1. APPLICATION TO VEHICLES

1.1 HORIZONTAL SURFACE



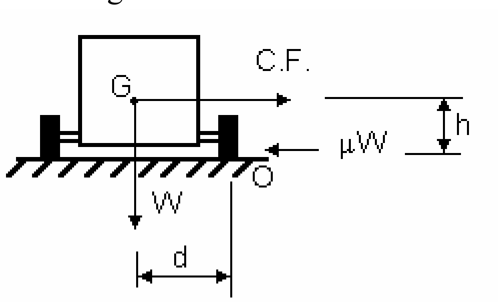
When a vehicle travels around a bend, it is subject to centrifugal force. This force always acts in a radial direction. If the bend is in a horizontal plane then the force always acts horizontally through the centre of gravity. The weight of the vehicle is a force that always acts vertically down through the centre of gravity. The diagram shows these two forces.

Figure 1



The centrifugal force tends to make the vehicle slide outwards. This is opposed by friction on the wheels. If the wheels were about to slide, the friction force would be μW where μ is the coefficient of friction between the wheel and the road.

Figure 2



The tendency would be for the vehicle to overturn. If we consider the turning moments involved, we may solve the velocity which makes it overturn. If the vehicle mass is M its weight is Mg. The radius of the bend is R.

Figure 3

Consider the turning moments about point O

The moment of force due to the weight is	$W \times d$ or $M g d$
The moment due to the centrifugal force is	$C.F. \times h$
Remember the formula for centrifugal force is	$C.F. = (Mv^2/R)$
The moment due to the centrifugal force is	$C.F. \times h = (Mv^2/R) \times h$

When the vehicle is about to overturn the moments are equal and opposite.

Equating the moments about point O we get	$M = (Mv^2/R) \times h = Mg\,d$
Rearrange to make v the subject.	$\mathbf{v = (g\,d\,R/h)^{1/2}}$

If the vehicle was about to slide sideways without overturning then the centrifugal force would be equal and opposite to the friction force. In this case

$$\mu W = Mv^2/R$$

$$\mu Mg = Mv^2/R$$

$$\mathbf{v = (\mu g R)^{1/2}}$$

Comparing the two equations it is apparent that it skids if $\mu < d/h$ and overturn if $\mu > d/h$

WORKED EXAMPLE No.1

A wheeled vehicle travels around a circular track of radius 50 m. The wheels are 2 m apart (sideways) and the centre of gravity is 0.8 m above the ground. The coefficient of friction is 0.4. Determine whether it overturns or slides sideways and determine the velocity at which it occurs

SOLUTION

Overturning

$v = (g\ d\ R/h)^{1/2} = (9.81 \times 1 \times 50/0.8)^{1/2} = 24.76$ m/s

Sliding sideways

$v = (\mu g R)^{1/2} = (0.4 \times 9.81 \times 50)^{1/2} = 14$ m/s

It follows that it will slide sideways when the velocity reaches 14 m/s.

1.2 BANKED SURFACE

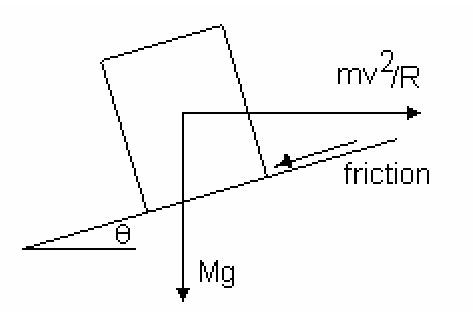


Figure 4

The road surface is flat and inclined at θ degrees to the horizontal but the bend is still in the horizontal plane. The analysis of the problem is helped if we consider the vehicle simply as block on an inclined plane as shown.

The C.F. still acts horizontally and the weight vertically. The essential distances required for moments about point O are the vertical and horizontal distances from O and these are h' and d' as shown.

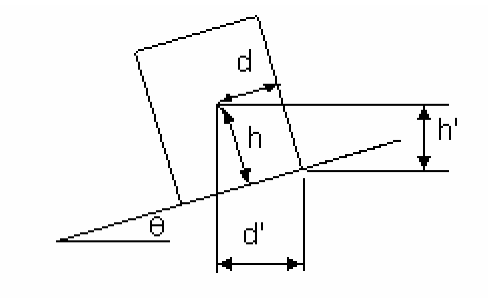


Figure 5

When the vehicle is just on the point of overturning the moments about the corner are equal and opposite as before.

By applying trigonometry to the problem you should be able to show that

$h' = (h - d\tan\theta)\cos\theta$

$d' = (h\tan\theta + d)\cos\theta$

Equating moments about the corner we have $Mv^2 h'/R = Mg\ d'$

$v^2 h' = (h - d\tan\theta)\cos\theta / R = g(h\tan\theta + d)\cos\theta$

Make v the subject. $$v^2 = gR\left\{\frac{\frac{d}{h} + \tan\theta}{1 - \frac{d}{h}\tan\theta}\right\}$$

This is the velocity at which the vehicle overturns. If the vehicle slides without overturning, then the total force **parallel** to the road surface must be equal to the friction force. Remember the friction force is μN where N is the total force acting normal to the road surface. Resolve all forces parallel and perpendicular to the road. The parallel forces are F $\cos\theta$ and W $\sin\theta$ as shown. The normal forces are F $\sin\theta$ and W $\cos\theta$ as shown. The friction force opposing sliding is μN

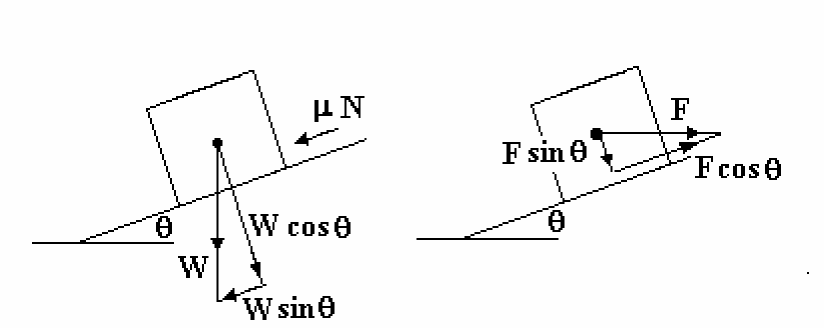


Figure 6

The forces acting parallel to the surface must be equal and opposite when sliding is about to occur.

Balancing all three forces we have $\mu N + W\sin\theta = F\cos\theta$

The total normal force N is the sum of the two normal forces. $N = W\cos\theta + F\sin\theta$

Substituting

$$\mu\{W\cos\theta + F\sin\theta\} + W\sin\theta = F\cos\theta$$

$$\mu W\cos\theta + \mu F\sin\theta + W\sin\theta = F\cos\theta$$

$$\mu Mg\cos\theta + \mu M\frac{v^2}{R}\sin\theta + Mg\sin\theta = M\frac{v^2}{R}\cos\theta$$

$$\mu g\cos\theta + \mu\frac{v^2}{R}\sin\theta + g\sin\theta = \frac{v^2}{R}\cos\theta$$

$$\mu + \mu\frac{v^2}{Rg}\tan\theta + \tan\theta = \frac{v^2}{Rg}$$

$$\frac{v^2}{Rg}\{\mu\tan\theta - 1\} = -\tan\theta - \mu$$

$$\frac{v^2}{Rg}\{1 - \mu\tan\theta\} = \tan\theta + \mu$$

$$v^2 = Rg\frac{\{\mu + \tan\theta\}}{\{1 - \mu\tan\theta\}}$$

This gives the velocity at which the vehicle slides.

WORKED EXAMPLE No.2

A motor vehicle travels around a banked circular track of radius 80 m. The track is banked at 15° to the horizontal. The coefficient of friction between the wheels and the road is 0.5. The wheel base is 2.4 m wide and the centre of gravity is 0.5 m from the surface measured normal to it. Determine the speed at which it overturns or skids.

SOLUTION

1. OVERTURNING

$$v^2 = gR\left\{\frac{\frac{d}{h} + \tan\theta}{1 - \frac{d}{h}\tan\theta}\right\} = 9.81X80\left\{\frac{\frac{1.2}{0.5} + \tan(15)}{1 - \frac{1.2}{0.5}\tan(15)}\right\}$$

Hence v = 76.6 m/s

2. SKIDDING

$$v^2 = Rg\frac{\{\mu + \tan\theta\}}{\{1 - \mu\tan\theta\}} = 80x9.81x\frac{\{0.5 + \tan 15\}}{\{1 - 0.5\tan 15\}}$$

Hence v = 26.3 m/s

The vehicle will skid before it overturns.

SELF ASSESSMENT EXERCISE No.1

1. A motor vehicle travels around a banked circular track of radius 110 m. The track is banked at 8° to the horizontal. The coefficient of friction between the wheels and the road is 0.4. The wheel base is 3 m wide and the centre of gravity is 0.6 m from the surface measured normal to it. Determine the speed at which it overturns or skids.
(66.3 m and 24.9 m)

2. A vehicle has a wheel base of 2.1 m and the centre of gravity is 1.1 m from the bottom. Calculate the radius of the smallest bend it can negotiate at 120 km/h. The coefficient of friction between the wheels and the track is 0.3.
(For skidding R = 377.5 m and for overturning R = 118.7 m hence the answer is 377.5 m)

2. Repeat question 3 given that the bend is banked at 10°.

(For skidding R = 225.2 m and for overturning R = 83.3 m hence the answer is 225.2 m)