EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

FURTHER MECHANICAL PRINCIPLES AND APPLICATIONS
UNIT 11 - NQF LEVEL 3

OUTCOME 3 - ROTATING SYSTEMS

TUTORIAL 1 - ANGULAR MOTION

CONTENT

Be able to determine the characteristics of rotating systems

***Rotating systems with uniform angular acceleration*:** systems e.g. simple (such as rotating rim, flywheel, motor armature, pump or turbine rotor), complex (such as systems where combined linear and angular acceleration is present, hoist and vehicle on an inclined track); kinetic parameters e.g. angular displacement, angular velocity, angular acceleration, equations for uniform angular motion $\omega_2 = \omega_1 + \alpha t$, $\theta = \omega_1 t + ½\alpha t^2$, $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $\theta = ½ (\omega_1 + \omega_2)t$; dynamic parameters e.g. radius of gyration, moment of inertia ($I = mk^2$), inertia torque ($T = I\alpha$), friction torque, application of D'Alembert's principle, mechanical work ($W = T\theta$), power (*Average Power* = *W/t*, *Instantaneous Power* = $T\omega$), rotational kinetic energy ($KE = ½I\omega^2$), application of principle of conservation of energy.

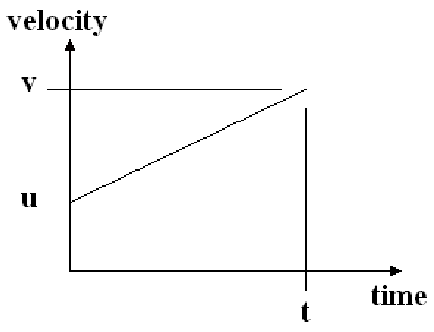
***Rotating systems with uniform centripetal acceleration*:** systems e.g. simple (such as concentrated mass rotating in a horizontal or vertical plane, vehicle on a hump-backed bridge, aircraft performing a loop), complex (such as centrifugal clutch, vehicle on a curved track); kinetic parameters e.g. expressions for centripetal acceleration ($a = \omega^2 r$, $a = v^2/r$); dynamic parameters e.g. expressions for centripetal force ($F_c = m\omega^2 r$, $F_c = mv^2/r$)

It is assumed that the student has studied Mechanical Principles and Applications Unit 6 and in particular the section on linear motion.

1. REVISION OF LINEAR MOTION

When a body moving at velocity u m/s accelerates uniformly to v m/s in time t the graph is as shown. From this we can deduce that:

$a = (v - u)/t$
$v = u + at$
$s = t/2\,(v + u)$
$s = ut + at^2/2$
$v^2 = u^2 + 2as$



t is time
s is the distance moved.
u is the initial velocity
v is the velocity at time t.
a is the uniform acceleration.

These equation apply equally to rotating bodies and this is the main area you need to study.

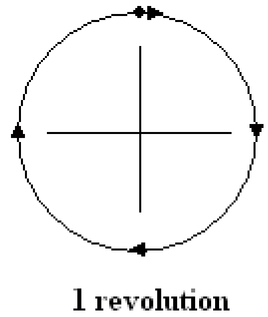
2. ANGULAR MOTION

This work applies to bodies revolving about a point at a fixed radius so the path taken by any point on the body is a circular path. This could apply to wheels of many forms, to vehicles (land, sea and air) going around a curve.

2.1. ANGLES

Angle has no units since it a ratio of arc length to radius. We use the names **REVOLUTION, DEGREE** and **RADIAN**.

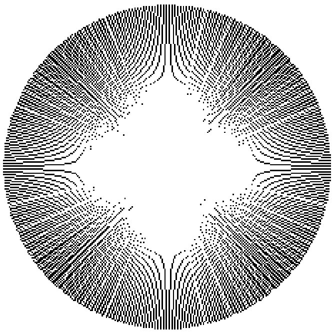
REVOLUTION



A point on a wheel that rotates one revolution traces out a circle. One revolution is the angle of rotation. This is a bit crude for use in calculations and we need smaller parts of the revolution.

DEGREES

Traditionally we divide one revolution into 360 parts and call this a degree with symbol ^{o}. 1 revolution = 360^o



The picture shows a circle divided into 360 parts. They are so close they can hardly be seen individually. Even so, a single degree is not accurate enough for many applications so we divide a degree up into smaller parts called minutes.
1^o = 60 minutes or 60'
A minute can be divided up into even smaller bits called seconds and 1 minute = 60 seconds or 60".

In modern times we use decimals to express angles accurately so you are unlikely to use minutes and seconds.

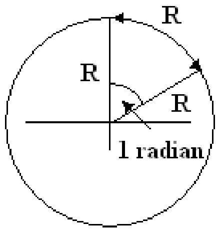
GRADS

In France, they divide the circle up into 400 parts and this is called a Grad. 1 revolution = 400 Grad.

This makes a quarter of a circle 100 Grads whereas in degrees it would be 90^o.

RADIAN

In Engineering and Science, we use another measurement of angle called the Radian. This is defined as the angle created by placing a line of length 1 radius around the edge of the circle as shown. In mathematical words it is the angle subtended by an arc of length one radius. This angle is called the RADIAN.



The circumference of a circle is 2πR. It follows that the number of radians that make a complete circle is

$$\frac{2\pi R}{R} \text{ or } 2\pi.$$

There are 2π radians in one revolution so $360^o = 2\pi$ radian. 1 radian = 360/2π = 57.296^o

In the following work we will be using degrees and radian so it is very important that you make sure your calculator is set to the units that you are going to use. You might find a button labelled DRG on your calculator. Press this repeatedly until the display shows either D (for degrees) or R (for Radian) or if you are French, G (for Grad). On other calculators you might have to do this by using the mode button so read your instruction book. Also note that since one revolution is 2π radian and also 360^o we convert degrees into radian as follows.

$$\theta \text{ (radian)} = \text{degrees x } 2\pi/360 = \text{degrees x } \pi/180$$

2.2 ANGULAR VELOCITY ω

The symbol for angular velocity is the lower case of the Greek letter Omega - ω.

If a body turns at constant speed, the angular velocity is the angle turned in 1 second. The angle turned in t seconds is then:

$\theta = \omega t$ and so $\omega = \frac{\theta}{t}$ rad/s

If the body is speeding up or slowing down we may express the instantaneous angular velocity as the rate of change of angle per second.

In calculus form we can write $\omega = \frac{d\theta}{dt}$

In practical cases, angular velocity or speed is usually given in revolutions/second or revolutions/minute. When solving problems we nearly always have to convert this into radians/s. Since a circle (or revolution) is 2π radian we convert rev/s into rad/s by $\omega = 2\pi N$.

EXAMPLE No. 1

A wheel rotates 200^o in 4 seconds. Calculate the following.

i. The angle turned in radians?

ii. The angular velocity in rad/s

SOLUTION

$\theta = (200/180)\pi = 3.49$ rad. $\omega = 3.49/4 = 0.873$ rad/s

SELF ASSESSMENT EXERCISE No. 1

1. A wheel rotates 5 revolutions in 8 seconds. Calculate the angular velocity in rev/s and rad/s. (Answers 0.625 rev/s and 3.927 rad/s)

2. A disc spins at 3000 rev/min. Calculate its angular velocity in rad/s. How many radians has it rotated after 2.5 seconds? (Answers 314.2 rad/s and 785.4 rad)

2.3 ANGULAR ACCELERATION α

Angular acceleration (symbol α - alpha) occurs when a wheel speeds up or slows down. It is defined as the rate of change of angular velocity. Since angular velocity is the rate of change of angle we can also say that acceleration is the rate of change of the rate of change of angle.

If the wheel changes its velocity by $\Delta\omega$ in t seconds, the acceleration is $\alpha = \Delta\omega / t$ rad/s^2

In calculus form we can express this as $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

WORKED EXAMPLE No. 2

A disc is spinning at 2 rad/s and it is uniformly accelerated to 6 rad/s in 3 seconds. Calculate the angular acceleration.

SOLUTION

$\alpha = \Delta\omega/t = (\omega_2 - \omega_1)/t = (6 - 2)/3 = 1.33$ rad/s^2

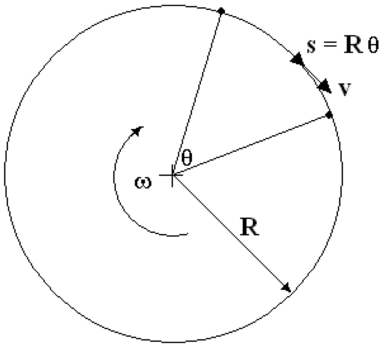
SELF ASSESSMENT EXERCISE No. 2

1. A wheel at rest accelerates to 8 rad/s in 2 seconds. Calculate the acceleration. (Answer 4 rad/s^2)

2. A flywheel spins at 5000 rev/min and is decelerated uniformly to 2000 rev/min in 12 seconds. Calculate the acceleration in rad/s^2. (Answer -26.2 rad/s^2)

2.4 LINK BETWEEN ANGULAR AND LINEAR MOTION

Consider a point moving on a circular path as shown.

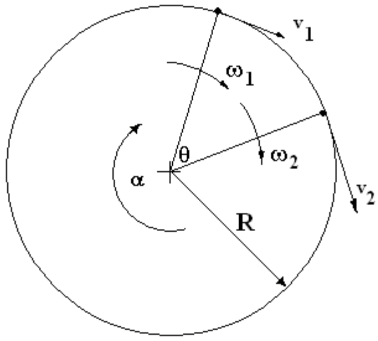


In time t it rotates about the centre by angle θ and travels along the arc. The distance travelled on the circular path is the length of the arc 's' and:

s = Rθ

The velocity along the circular path is **v = s/t = Rθ/t**

v = R ω



Next suppose that the point accelerates from angular velocity ω_1 to ω_2. The velocity along the curve also changes from v_1 to v_2.

Angular acceleration = $\alpha = (\omega_2 - \omega_1)/t$

Substituting $\omega = v/R$

$\alpha = (v_2/R - v_1/R)/t = a/R$ hence:

a = R α

It is apparent that to change an angular quantity into a linear quantity all we have to do is multiply it by the radius.

WORKED EXAMPLE No. 3

A car travels around a circular track of radius 40 m at a velocity of 8 m/s. Calculate its angular velocity.

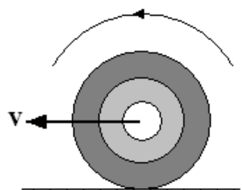
SOLUTION

$v = \omega R \quad \omega = v/R = 8/40 = 0.2$ rad/s

WORKED EXAMPLE No. 4

A car has wheels 1 m diameter. Calculate the angular speed of the wheels in rev/minute when the vehicle travels at 100 km/h.

SOLUTION



If the vehicle moves forwards at v m/s then the surface of the wheel must be moving at v m/s otherwise it would be sliding on the surface.

v = 100 000 m/h or 100 000/3600 m/s

v = 27.777m/s

$\omega R = \omega \times 0.5 \qquad \omega = 27.777/0.5 = 555.555$ rad/s

$N = \omega/2\pi = 555.555/2\pi = 8.842$ rev/s or 530.5 rev/minute

SELF ASSESSMENT EXERCISE No. 3

1. The vanes on a steam turbine rotate at 3000 rev/min at a mean radius of 0.3 m. What is the linear velocity of the vanes?

   (Answer 94.2 m/s)

2. A winch rotates at 30 rev/min. The mean radius to the centre of the rope being drawn in is 0.2 m. Calculate the speed of the rope in m/s.

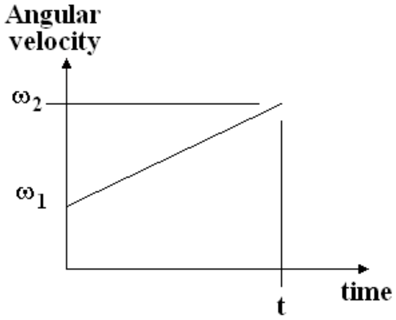
   (Answer 0.628 m/s)

3. The wheels on a train are 1.5 m diameter and they revolve at 25 rev/min without slipping. What is the speed of the train in km/h?

   (Answer 7.0686 km/h)

2.5 EQUATIONS OF MOTIONS

If a point moving at angular velocity ω_1 accelerates uniformly to ω_2 at a rate of α rad/s^2 in time t seconds, the graph is as shown.



By the same reasoning as used for linear motion:

Angle rotated = θ

θ = area under the graph = $\omega_1 t + (\omega_1 - \omega_2)t/2$

$\theta = \omega_1 t + (\omega_2 - \omega_1)t/2$ ....(1)

$\theta = \omega_1 t + \omega_2 t/2 - \omega_1 t/2 = \omega_2 t/2 + \omega_1 t/2$

$$\boldsymbol{\theta = \frac{t}{2}(\omega_2 - \omega_1)}$$

The acceleration = $\alpha = (\omega_2 - \omega_1)/t$ from which $(\omega_2 - \omega_1) = \alpha t$

Substituting this into equation (1) gives:

$$\boldsymbol{\theta = \omega_1 t + \alpha t^2/2}$$

Since $\omega_2 = \omega_1$ + the increase in velocity

$\Omega_2 = \omega_1 + \alpha t$ and squaring we get $\omega_2^2 = \omega_1^2 + 2\alpha[\alpha t^2/2 + \omega_1 t]$

$$\boldsymbol{\omega_2^2 = \omega_1^2 + 2\alpha\theta}$$

If we compare these with the equations for linear motion we see they are the same.

LINEAR	ANGULAR
$a = (v - u)/t$	$a = (\omega_1 - \omega_2)/t$
$s = ut + at^2/2$	$\theta = \omega_1 t + \alpha t^2/2$
$s = (u + v)t/2$	$\theta = (\omega_1 + \omega_2)t/2$
$v^2 = u^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

SELF ASSESSMENT EXERCISE No. 5

1. A wheel accelerates from rest to 3 rad/s in 5 seconds. Sketch the velocity - time graph and determine the angle rotated.

   (Answer 7.5 radian).

2. A wheel accelerates from rest to 4 rad/s in 4 seconds. It then rotates at a constant speed for 3 seconds and then decelerates uniformly to rest in 5 seconds. Sketch the velocity time graph and determine

   i. The angle rotated. (30 radian)
   ii. The initial angular acceleration. (1 rad/s^2)
   iii. The average angular velocity. (2.5 rad/s)

3. A roller 0.4 m diameter rolls down a slope starting from rest. It takes 10 seconds make 6 complete rotations along the sloping surface accelerating uniformly as it moves. Calculate the following:

   i. The angular velocity at the end. (7.54 rad/s)
   ii. The linear velocity at the end. (1.508 m/s)
   iii. The angular acceleration. (0.754 rad/s^2)
   iv. The linear acceleration. (0.151 m/s^2)
   v. The distance travelled. (7.54 m)

4. A large centrifugal air compressor rotates at 360 rev/min. The power is turned off and it decelerates to rest in 5 seconds. What is the angular acceleration?

   (-7.54 rad/s^2)

5. A large steam turbine rotating at 3000 rev/min has the steam supply cut off and it slows down at a uniform rate to 500 rev/min in 50 seconds. What is the angular acceleration?

   (-5.235 rad/s^2)

6. Assuming the turbine in Q5 continues to decelerate at the same rate, how long does it take to come to rest from the time the steam was cut off?

   (10 s)