Unit 4: Mechanical Principles

Unit code: **F/601/1450**

QCF level: **5**

OUTCOME 3 – POWER TRANSMISSION

TUTORIAL 3 –GEAR SYSTEMS

3 Power Transmission

Belt drives**:** flat and v-section belts; limiting coefficient friction; limiting slack and tight side tensions; initial tension requirements; maximum power transmitted

Friction clutches**:** flat single and multi-plate clutches; conical clutches; coefficient of friction; spring force requirements; maximum power transmitted by constant wear and constant pressure theories; validity of theories

Gear trains**:** simple, compound and epicycle gear trains; velocity ratios; torque, speed and power relationships; efficiency; fixing torques

On completion of this short tutorial you should be able to do the following.

- Describe the different types of gear systems.
- Describe a simple gear train.
- Describe a compound gear rain.
- Describe three types of epicyclic gear boxes
- Solve gear box ratios.
- Calculate the input and outputs speeds and torques of gear boxes.
- Calculate the holding torque on gear box cases

It is assumed that the student is already familiar with the following concepts.

- Angular motion.
- Power transmission by a shaft.

All these above may be found in the pre-requisite tutorials.

CONTENTS

1. <u>INTRODUCTION</u>

A gear box is a device for converting the speed of a shaft from one speed to another. In the process the torque T is also changed. This can be done with pulley and chain drives but gears have advantages over these system. A good example is that of winch in which a motor with a high speed and low torque is geared down to turn the drum at a low speed with a large torque. Similarly, a marine engine may use a reduction gear box to reduce the speed of the engine to that of the propeller. Other examples are motor vehicles, lathes, drills and many more. The diagram shows a typical winch that has a reduction gear box built inside the drum.

Figure 1

This tutorial is not about the design of gears but it should be mentioned that there are many types of gears, each with their own advantages. Here are some examples.

Figure 2

Gears are wheels which mesh with each other through interlocking teeth. Rotation of one wheel produces rotation of the other with no slip between them. The shape of the gear teeth is important in order to produce a smooth transfer of the motion. The most common shape is the INVOLUTE gear form but it is not our task to study this here.

The design of the gear teeth also affects the relative position of one gear to another. For example bevelled gears allow the axis of one gear to be inclined to the axis of another. Worm gears convert the motion through 90^{o} and so on. The design also affects the friction present in the transfer.

2. BASIC GEAR BOX THEORY

Consider a simple schematic of a gear box with an input and output shaft.

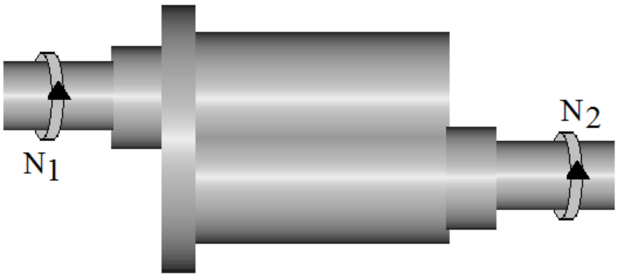


Figure 3

GEAR BOX RATIO

The ratio of the gear box is defined as $G.R. = \frac{\text{INPUT SPEED}}{\text{OUTPUT SPEED}} = \frac{N_1}{N_2}$

N is usually in rev/min but the ratio is the same whatever units of speed are used. If angular velocity is used then

$$G.R. = \frac{\text{INPUT SPEED}}{\text{OUTPUT SPEED}} = \frac{\omega_1}{\omega_2}$$

TORQUE AND EFFICIENCY

The power transmitted by a torque T Nm applied to a shaft rotating at N rev/min is given by $S.P. = \frac{2\pi NT}{60}$. In an ideal gear box, the input and output powers are the same so

$$\frac{2\pi N_1 T_1}{60} = \frac{2\pi N_2 T_2}{60} \qquad N_1 T_1 = N_2 T_2 \qquad \frac{T_2}{T_1} = \frac{N_1}{N_2} = G.R.$$

It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box, power is lost through friction and the power output is smaller than the power input. The efficiency is defined as:

$$\eta = \frac{\text{Power Out}}{\text{Power In}} = \frac{2\pi\ N_2 T_2 \text{ x } 60}{2\pi\ N_1 T_1 \text{ x } 60} = \frac{N_2 T_2}{N_1 T_1}$$

Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque T_3 must be applied to the body through the clamps.

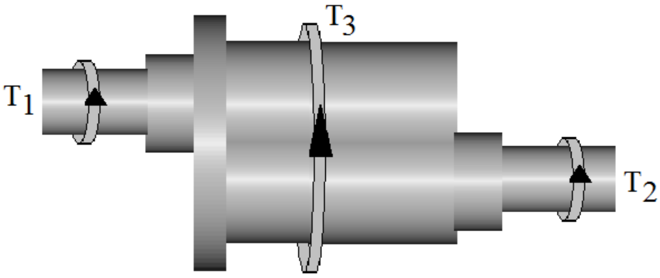


Figure 4

The total torque must add up to zero. $T_1 + T_2 + T_3 = 0$

If we use a convention that anti-clockwise is positive and clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.

WORKED EXAMPLE No.1

1. A gear box has an input speed of 1500 rev/min clockwise and an output speed of 300 rev/min anticlockwise. The input power is 20 kW and the efficiency is 70%. Determine the following.

i. The gear ratio
ii. The input torque.
iii. The output power.
iv. The output torque.
v. The holding torque.

SOLUTION

$$\text{G.R.} = \frac{\text{INPUT SPEED}}{\text{OUTPUT SPEED}} = \frac{N_1}{N_2} = \frac{1500}{300} = 5$$

$$\text{Power In} = \frac{2\pi N_1 T_1}{60} \qquad T_1 = \frac{60 \text{ x Power In}}{2\pi N_1}$$

$$T_1 = \frac{60 \text{ x } 20\,000}{2\pi \text{ x } 1500} = 127.3 \text{ Nm (Negative clockwise)}$$

$$\eta = 0.7 = \frac{\text{Power Out}}{\text{Power In}} \qquad \text{power Out} = 0.7 \text{ x Power In} = 0.7 \text{ x } 20 = 14 \text{ kW}$$

$$\text{Power out} = \frac{2\pi N_2 T_2}{60} \qquad T_2 = \frac{60 \text{ x Power Out}}{2\pi N_2}$$

$$T_2 = \frac{60 \text{ x } 14\,000}{2\pi \text{ x } 300} = 445.6 \text{ Nm (positive antic clockwise)}$$

$$T_1 + T_1 + T_3 = 0$$

$$-127.3 + 445.6 + T_3 = 0$$

$$T_3 = 127.3 - 445.6 = -318.3 \text{ Nm (Anticlockwise)}$$

SELF ASSESSMENT EXERCISE No.1

1. A gear box has an input speed of 2000 rev/min clockwise and an output speed of 500 rev/min anticlockwise. The input power is 50 kW and the efficiency is 60%. Determine the following.

i. The input torque. (238.7 Nm)
ii. The output power. (30 kW)
iii. The output torque. (573 Nm)
iv. The holding torque. (334.3 Nm clockwise)

2. A gear box must produce an output power and torque of 40 kW and 60 Nm when the input shaft rotates at 1000 rev/min. Determine the following.

    i. The gear ratio. (0.1571)
    ii. The input power assuming an efficiency of 70% (57.14 kW)

3. TYPES OF GEAR TRAINS

The meshing of two gears may be idealised as two smooth discs with their edges touching and no slip between them. This ideal diameter is called the Pitch Circle Diameter (PCD) of the gear.

3.1 SIMPLE GEAR TRAIN.

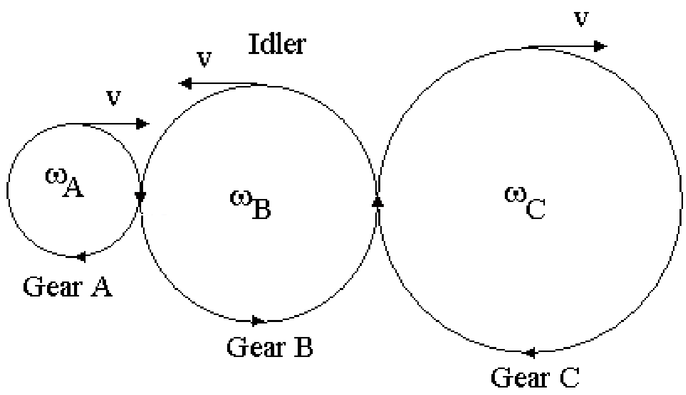


Figure 5

These are typically spur gears as shown in diagram 1. The direction of rotation is reversed from one gear to another. The only function of the idler gear is to change the direction of rotation. It has no affect on the gear ratio. The teeth on the gears must all be the same size so if gear A advances one tooth, so does B and C.

t = number of teeth on the gear.
D = Pitch circle diameter.
m = modem = D/t and this must be the same for all gears otherwise they would not mesh.
$m = D_A/t_A = D_B/t_B = D_C/t_C$
$D_A = m\ t_A \quad D_B = m\ t_B \quad D_C = m\ t_C$
ω = angular velocity.
v = linear velocity on the circle. $v = \omega D/2$

The velocity v of any point on the circle must be the same for all the gears, otherwise they would be slipping. It follows that

$$\frac{\omega_A D_A}{2} = \frac{\omega_B D_B}{2} = \frac{\omega_C D_C}{2}$$

$$\omega_A D_A = \omega_B D_B = \omega_C D_C$$

$$\omega_A m t_A = \omega_B m t_B = \omega_C m t_C$$

$$\omega_A t_A = \omega_B t_B = \omega_C t_C$$

or in terms of rev/min $N_A\ t_A = N_B\ t_B = N_C\ t_C$

The gear ratio is defined as GR = Input speed/Output speed

If gear A is the input and gear C the output, $GR = N_A / N_C = t_C / t_A$

WORKED EXAMPLE No.2

A simple train has 3 gears. Gear A is the input and has 50 teeth. Gear C is the output and has 150 teeth. Gear A rotates at 1500 rev/min anticlockwise. Calculate the gear ratio and the output speed.

The input torques on A is 12 Nm and the efficiency is 75%. Calculate the output power and the holding torque.

SOLUTION

$GR = N_A / N_C = t_C / t_A = 150/50 = 3$

$N_A / N_C = 3 \quad N_C = N_A /3 = 1500/3 = 500$ rev/min (anticlockwise)

$T_A = 12$ Nm $\quad$ P (input) $= 2\pi N_A T_A/60 = 2\pi \times 1500 \times 12/60 = 1885$ W

P (output) = P (Input) x $\eta = 1885 \times 0.75 = 1413.7$ W

$$T_C = \frac{60P(\text{output})}{2\pi N_C} = \frac{60 \times 1413.7}{2\pi \times 500} = 27 \text{ Nm}$$

$T_A + T_C + T_{hold} = 0$

$12 + 27 + T_{hold} = 0 \qquad T_{hold} = -39$ Nm (clockwise)

SELF ASSESSMENT EXERCISE No.2

A simple gear train has 2 spur gears. The input gear has 20 teeth and the output gear has 100 teeth. The input rotates at 2000 rev/min clockwise. Calculate the gear ratio and the output speed. (5 and 400 rev/min anticlockwise)

The input torque is 15 Nm and the efficiency is 65%. Calculate the output power and the holding torque. (2 042 W and 33.75 Nm clockwise)

3.2 COMPOUND GEARS

Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed.

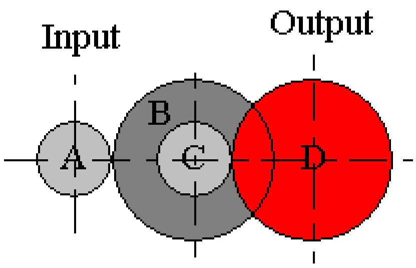


Figure 6

The velocity of each tooth on A and B are the same so $\omega_A t_A = \omega_B t_B$ as they are simple gears. Likewise for C and D, $\omega_C t_C = \omega_D t_D$.

$$\frac{\omega_A}{t_B} = \frac{\omega_B}{t_A} \quad \text{and} \quad \frac{\omega_C}{t_D} = \frac{\omega_D}{t_C}$$

$$\omega_A = \frac{t_B \ \omega_B}{t_A} \qquad \omega_C = \frac{t_D \ \omega_D}{t_C}$$

$$\omega_A \omega_C = \frac{t_B \ \omega_B}{t_A} \times \frac{t_D \ \omega_D}{t_C} = \frac{t_B t_D}{t_A t_C} \times \omega_B \omega_D$$

$$\frac{\omega_A \omega_C}{\omega_B \omega_D} = \frac{t_B t_D}{t_A t_C}$$

Since gears B and C are on the same shaft, $\omega_B = \omega_C$

$$\frac{\omega_A}{\omega_D} = \frac{t_B t_D}{t_A t_C} = \text{G.R.}$$

Since $\omega = 2\pi N$ then the gear ratio may be written as $\frac{N(IN)}{N(OUT)} = \frac{t_B t_D}{t_A t_C} = \text{G.R.}$

Gears B and D are the driven gears. Gears A and C are the driver gears. It follows that:

Gear ratio = product of driven teeth/product of driving teeth

This rule applies regardless of how many pairs of gears there are.

WORKED EXAMPLE No.3

Calculate the gear ratio for the compound chain shown below. If the input gear rotates clockwise, in which direction does the output rotate?

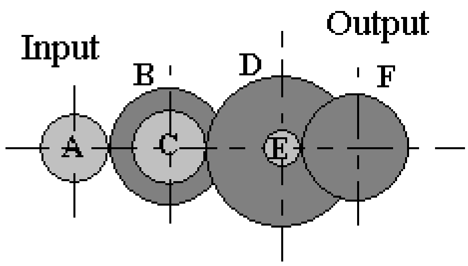


Figure 7

Gear A has 20 teeth
Gear B has 100 teeth
Gear C has 40 teeth
Gear D has 100 teeth
Gear E has 10 teeth
Gear F has 100 teeth

SOLUTION

The driving teeth are A, C and E.
The driven teeth are B, D and F
Gear ratio = product of driven teeth/product of driving teeth

Gear ratio = (100 x 100 x 100)/ (20 x 40 x 10) = 125

Alternatively we can say there are three simple gear trains and work ot the ratio for each.

First chain GR = 100/20 = 5
Second chain GR = 100/40 = 2.5
Third chain GR = 100/10 = 10
The overall ratio = 5 x 2.5 x 10 = 125

Each chain reverses the direction of rotation so if A is clockwise, B and C rotate anticlockwise so D and E rotate clockwise. The output gear F hence rotates anticlockwise.

SELF ASSESSMENT EXERCISE No.3

Gear A is the input and revolves at 1200 rev/min clockwise viewed from the left end. The input torque is 30 Nm and the efficiency is 70%.
Gear A has 50 teeth
Gear B has 150 teeth
Gear C has 30 teeth
Gear D has 60 teeth

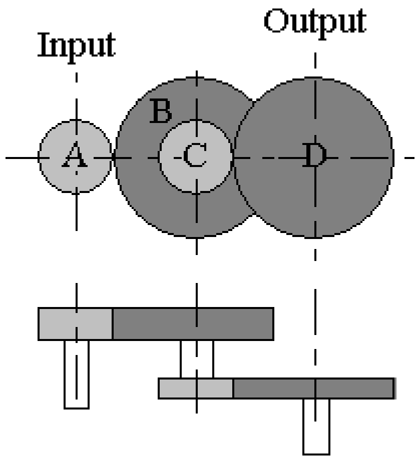


Figure 8

Calculate the following.

i. The output speed and its direction. (200 rev/min clockwise)
ii. The output power. (2639 W)
iii. The fixing torque. (156 Nm anticlockwise)

3.3 EPICYCLIC GEARS

Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. Here is a picture of a typical gear box.

Figure 9

This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gear boxes to electric screwdrivers.

BASIC THEORY

The diagram shows a gear B on the end of an arm A. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.

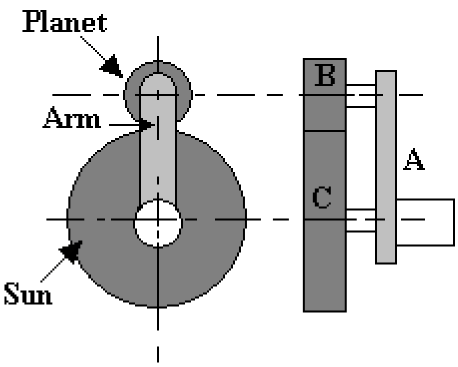


Figure 10

First consider what happens when the planet gear orbits the sun gear.

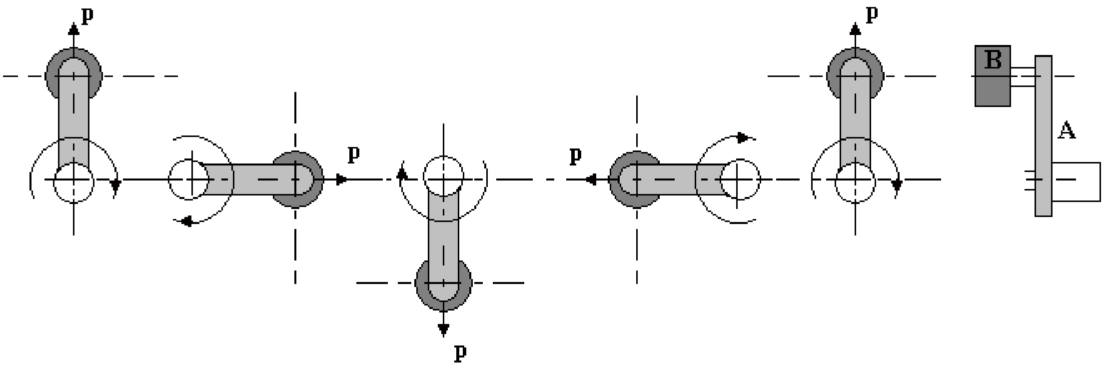


Figure 11

Observe point p and you will see that gear B also revolves once on its own axis. Any object orbiting around a centre must rotate once. Now consider that B is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and gear C is rotated once. B spins about its own centre and the number of revolutions it makes is the ratio t_C/t_B. B will rotate by this number for every complete revolution of C.

Now consider that C is unable to rotate and the arm A is revolved once. Gear B will revolve $t_C/t_B + 1$ because of the orbit. It is this extra rotation that causes confusion. One way to get round this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

Suppose gear C is fixed and the arm A makes one revolution. Determine how many revolutions the planet gear B makes.

Step 1 is to revolve everything once about the centre.
Step 2 identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of B.
Step 3 is simply add them up and we find the total revs of C is zero and for the arm is 1.

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C by -1 rev	0	$+ t_C/t_B$	-1
3	Add	1	$1+ t_C/t_B$	0

The number of revolutions made by B is $(1 + t_C/t_B)$. Note that if C revolves -1, then the direction of B is opposite so $+ t_C/t_B$

WORKED EXAMPLE No.4

A simple epicyclic gear has a fixed sun gear with 100 teeth and a planet gear with 50 teeth. If the arm is revolved once, how many times does the planet gear revolve?

SOLUTION

Step	Action	A	B	C
1	Revolve all once	1	1	1
2	Revolve C -1	0	+100/50	-1
3	Add	1	3	0

Gear B makes 3 revolutions for every one of the arm.

The design so far considered has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done several ways.

METHOD 1

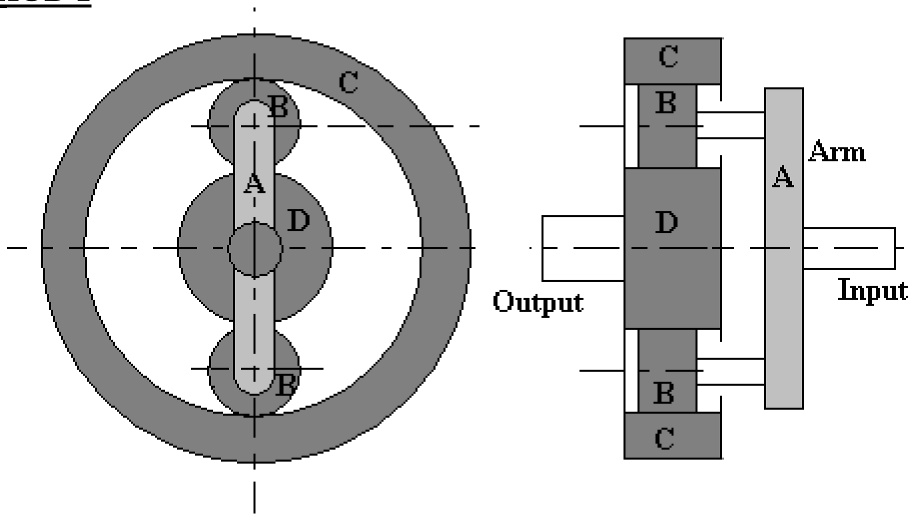


Figure 12

The arm is the input and gear D is the output. Gear C is a fixed internal gear and is normally part of the outer casing of the gear box. There are normally four planet gears and the arm takes the form of a cage carrying the shafts of the planet gears. Note that the planet gear and internal gear both rotate in the same direction.

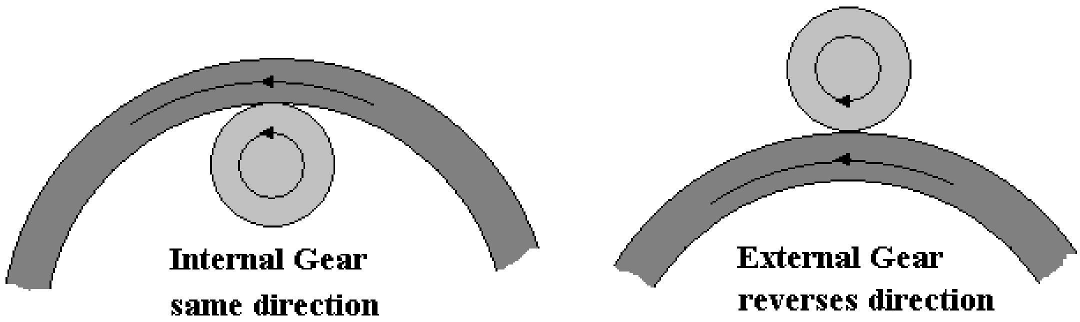


Figure 13

WORKED EXAMPLE No.5

An epicyclic gear box has a fixed outer gear C with 240 teeth. The planet gears have 20 teeth. The input is the arm/cage A and the output is the sun gear D.

Calculate the number of teeth on the sun gear and the ratio of the gear box.

SOLUTION

The PCD of the outer gear must the sum of PCD of the sun plus twice the PCD of the planets so it follows that the number of teeth are related as follows.

$t_C = t_D + 2\ t_B$
$240 = t_D + 2 \times 20$
$t_D = 240 - 40 = 200$

Identify that gear C is fixed and the arm must do one revolution so it must be rotated back one revolution holding the input stationary.

Step	Action	A	B	C	D
1	Revolve all once	1	1	1	1
2	Revolve C -1	0	-240/20	-1	240/200
3	Add	1	-11	0	2.2

The ratio A/D is then 1: 2.2 and this is the gear ratio.

METHOD 2

In this case the sun gear D is fixed and the internal gear C is made into the output.

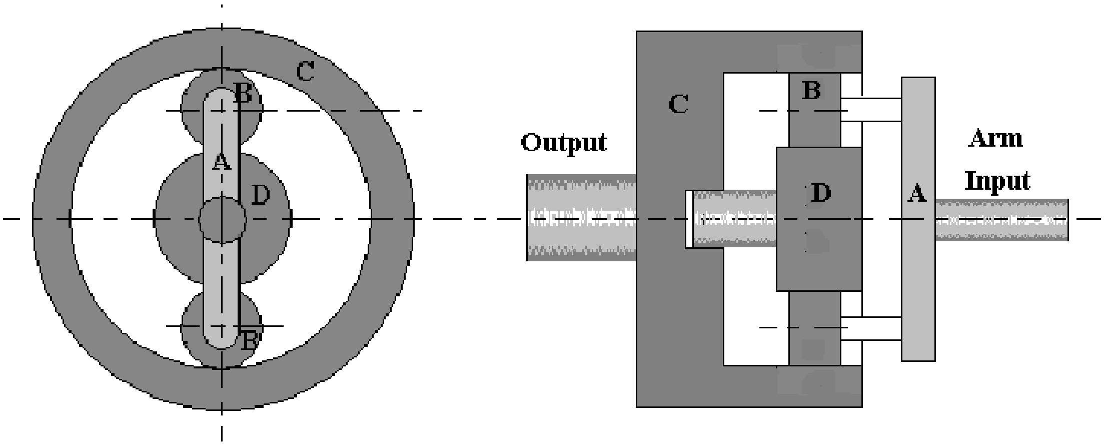


Figure 14

WORKED EXAMPLE No.6

An epicyclic gear box has a fixed sun gear D and the internal gear C is the output with 300 teeth. The planet gears B have 30 teeth. The input is the arm/cage A.

Calculate the number of teeth on the sun gear and the ratio of the gear box.

SOLUTION

$t_C = t_D + 2\ t_B$
$300 = t_D + 2 \times 30$
$t_D = 300 - 60 = 240$

Identify that gear D is fixed and the arm must do one revolution so it must be D that is rotated back one revolution holding the arm stationary.

Step	Action	A	B	C	D
1	Revolve all once	1	1	1	1
2	Revolve D -1	0	240/30	240/300	-1
3	Add	1	9	1.8	0

The ratio A/C is then 1: 1.8 and this is the gear ratio. Note that the solution would be the same if the input and output are reversed but the ratio would be 1.8.

METHOD 3

In this design a compound gear C and D is introduced. Gear B is fixed and gears C rotate upon it and around it. Gears C are rigidly attached to gears D and they all rotate at the same speed. Gears D mesh with the output gear E.

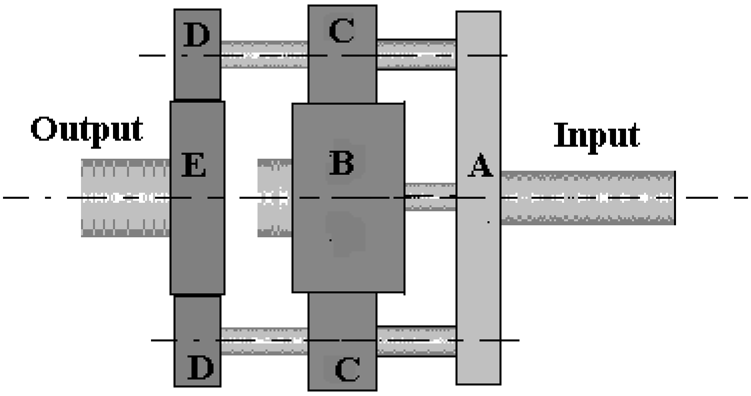


Figure 15

WORKED EXAMPLE No.7

An epicyclic gear box is as shown above. Gear C has 100 teeth, B has 50, D has 50 and E has 100.

Calculate the ratio of the gear box.

SOLUTION

Identify that gear B is fixed and that A must do one revolution so it must be B that is rotated back one revolution holding A stationary.

Step	Action	A	B	C/D	E
1	Revolve all once	1	1	1	1
2	Revolve B -1	0	-1	½	-¼
3	Add	1	0	1 ½	¾

The ratio A/E is then ¾ :1 or 3:4

Note that the input and output may be reversed but the solution would be the same with a ratio of 4:3 instead of 3:4

SELF ASSESSMENT EXERCISE No.4

1. An epicyclic gear box is designed as shown. The input D rotates at 200 rev/min clockwise viewed from the left with a torque of 40 Nm. The efficiency is 75%.

   Calculate the following.

   i. The gear box ratio. (16:1)
   ii. The output speed and its direction. (12.5 rev/min clockwise)
   iii. The power output. (628.3 W)
   iv. The holding torque. (520 Nm anticlockwise)

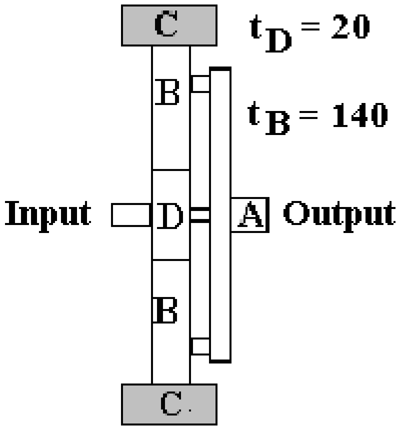


Figure 16

2. An epicyclic gear box is designed as shown. The input A rotates at 100 rev/min clockwise viewed from the right with a torque of 20 Nm. The efficiency is 65%.

   Calculate the following.

   i. The gear box ratio. (1:5)
   ii. The output speed and its direction. (500 rev/min clockwise)
   iii. The power output. (136.1 W)
   iv. The holding torque. (22.6 Nm anticlockwise)

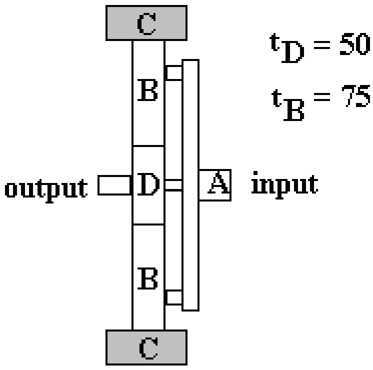


Figure 17

3. An epicyclic gear box as shown has a fixed sun gear D and the internal gear C is the output with 400 teeth. The planet gears B have 150 teeth. The input is the arm/cage A. The output must deliver 5 kW of power at 900 rev/min. The input power is 7 kW.

   Calculate the following.

   i. The gear box ratio. (4:5)
   ii. The input speed and its direction. (720 rev/min same direction as input)
   iii. The efficiency. (71.4%)
   iv. The holding torque. (145.9 Nm)

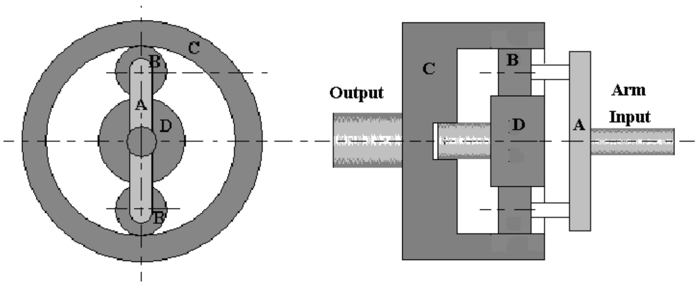


Figure 18

4. An epicyclic gear box as shown has a fixed sun gear B with 150 teeth. Gear C has 30 teeth and it is compounded with D which has 130 teeth. The input shaft delivers 200 W at 2400 rev/min. The gear box efficiency is 55%.

   Calculate the following.

   i. The number of teeth on E. (50)
   ii. The gear box ratio. (12:1)
   iii. The output speed and its direction. (200 rev/min in reverse direction)
   iv. The holding torque. (4.46 Nm)

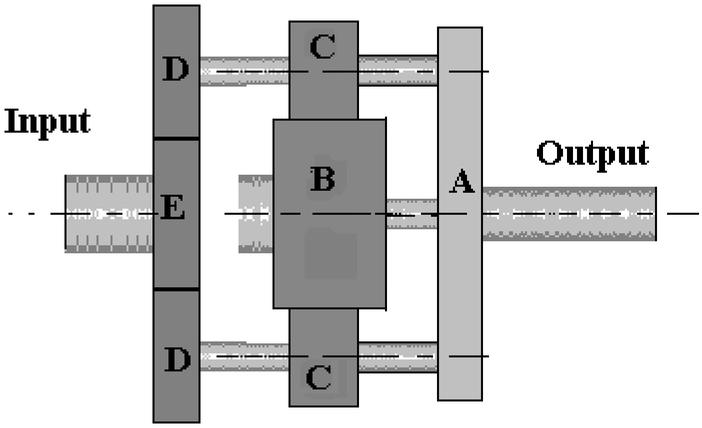


Figure 19

3.4 MORE ADVANCED EPICYCLIC BOXES

Sometimes the case of the epicyclic gear box is allowed to rotate. In this case the solution should be along these lines.

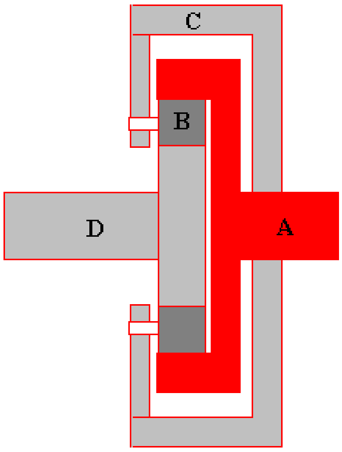


Figure 20

	A	B	C	D
Keep C stationary give A 1 rev	1	t_A/t_B	0	$-t_A/t_D$
Multiply by x (revs of A)	x	xt_A/t_B	0	$-xt_A/t_D$
Lock the gears and rotate all y times	x + y		y	$-xt_A/t_D + y$

x is the revolutions of A and y the revolutions of C

Given the speed of any two gears, the speed of the other may be deduced.

WORKED EXAMPLE No.8

An epicyclic gear box is as shown above. Gear B has 28 teeth and D has 64. Shaft D rotates at 200 rev/min and the case C is allowed to rotate at 100 rev/min in the same direction. Calculate the speed of shaft A.

Calculate the ratio of the gear box.

SOLUTION

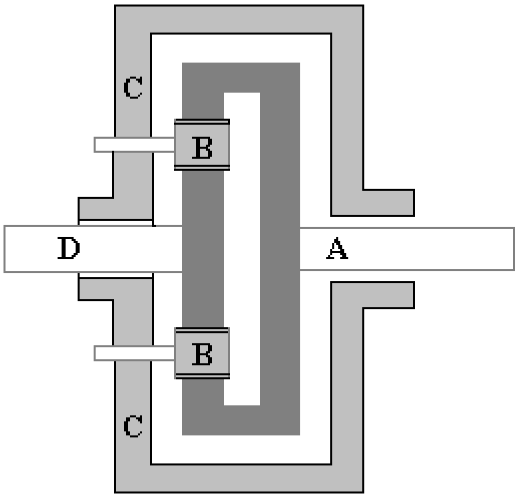
The teeth on A must be 64 + 28 + 28 = 120

	A	B	C	D
Keep C stationary give A 1 rev	1	120/28	0	-120/64
Multiply by x (revs of A)	x	120x/28	0	-120x/64
Lock the gears and rotate all y times	x + y	120x/28+y	y	-120x/64+ y

The speed of C is 100 rev/min so y = 100
The speed of D is 200 rev/min so -120x/64+ y = 200
-120x/64 + 100 = 200 x = -53.3 rev/min (opposite direction)

SELF ASSESSMENT EXERCISE No.5

1. An epicyclic gear box is designed as shown. The number of teeth on D is 80 and on B is 40.



The input D rotates at 300 rev/min clockwise viewed from the left and the case C is allowed to rotate at 150 rev/min in the opposite direction. Calculate the speed of shaft A. (-75 rev/min)