INSTRUMENTATION AND CONTROL

TUTORIAL 1 – BASIC ENGINEERING SCIENCE

This tutorial provides minimal engineering science necessary to complete the rest of the tutorials. Greater depth of the individual topics can be found on the web site. It is useful to anyone studying measurement systems and instrumentation but it is provided mainly in support of the EC module D227 – Control System Engineering. This tutorial is very basic.

On completion of this tutorial, you should be able to explain and use the following basic relationships used in mechanical and electrical engineering science.

- Basic units.

- Displacement, velocity and acceleration.

- Newton's Second Law of Motion.

- Springs.

- Pressure and flow.

- Resistance, current, capacitance and inductance.

- Stress and strain.

- Resistivity and temperature coefficient of resistance.

1. UNITS

There are only 5 basic units used in engineering. These are

mass	kilogramme (kg)
length	metre (m)
time	seconds (s)
current	Ampere (A)
temperature	Kelvin (K)

Also used in physics is luminous intensity   Candela (Cd). All other quantities used are multiples of these units.

2. FUNDAMENTAL MECHANICAL LAWS

2.1. VELOCITY

Linear
Velocity = rate of change of distance (m/s)
v = distance moved/time taken for constant velocity or x/t.
$v = dx/dt$ for instantaneous velocity.

Angular
Angular velocity = rate of change of angle (rad/s)
ω = angle turned/time taken for constant velocity or θ/t.
$\omega = d\theta/dt$ for varying conditions.

2.2 ACCELERATION

Linear
Acceleration = rate of change of velocity (m/s^2).
$a = dv/dt = d^2x/dt^2$

Angular
Angular acceleration = rate of change of velocity (rad/s^2)
$\alpha = d\omega/dt = d^2\theta/dt^2$

2.3. NEWTON'S 2nd LAW OF MOTION

This concerns the force required to overcome the inertia of a body. Inertia (or mass) is that property of a body which opposes changes to its motion.

Linear   Angular

Force = Mass x acceleration   Torque = Moment of inertia x angular acceleration
$F = M\, dv/dt$ $T = I\, d\omega/dt$
$F = M\, d^2x/dt^2$ $T = I\, d^2\theta/dt^2$

2.4. SPRING

Linear
Force = constant x change in length (N)
$F = kx$

Angular
Torque = Constant x angle (N m)
$T = k\,\theta$

2.5. PRESSURE

Pressure = Force per unit area (N/m^2)

$p = F/A$

1 Pascal = 1 N/m^2

1 bar = 10^5 Pa

2.6. FLOW RATE IN PIPES

Flow rate = Cross sectional area x mean velocity

$Q = A \, dx/dt$

3.   FUNDAMENTAL ELECTRICAL LAWS

3.1   CURRENT $i = dq/dt$ (Coulomb/s or Amperes)

3.2   RESISTANCE $V=IR$ (Ohm's Law)

3.3   INDUCTANCE

This law is equivalent to the 2nd. law of motion. Inductance (L Henries) is that property of a component that resists changes to the flow of current. The emf (E) produced is

$$E = L \, di/dt$$
$$E = L \, d^2q/dt^2$$

3.4. CAPACITANCE

This law is equivalent to the spring law. Capacitance (C Farads) is the property of a component which enables it to store electric charge.

$$Q = C \, V \quad \text{(Coulombs)}$$

SELF ASSESSMENT EXERCISE No.1

1.   A spring is compressed 30 mm by a force of 50 N. Calculate the spring constant. (1.67 N/mm)

2.   A mass of 30 kg is accelerated at a rate of 4 m/s^2. Calculate the force used to do it. (120 N)

3.   A flywheel is accelerated at a rate of 4 rad/s^2 by a torque of 10 Nm. Calculate the moment of inertia. (2.5 kg m^2)

4.   A force of 5000 N acts on an area of 400 mm^2. Calculate the pressure in Pascals. (12.5 MPa)

5.   A pipe 100 mm diameter carries oil at a mean velocity of 2 m/s. Calculate the flow rate in m^3/s. (0.0157 m^3/s)

6.   The current in an inductive coil changes at 30 A/s and the back emf is 3 V. Calculate the inductance in H. (0.1 H)

7.   A capacitor has a charge of 2 Coulombs at 24 V stored on it. Calculate the capacitance in μF. (83.3 μF)

8.   A flywheel turns 120 radians in 3 seconds at a constant rate. Calculate the angular velocity. (40 rad/s)

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4.   STRESS AND STRAIN

4.1.   DIRECT STRESS **s**

When a force F acts directly on an area A as shown in figure 1, the resulting direct stress is the force per unit area and is given as

$$\sigma = F/A.$$

where

F is the force normal to the area in Newtons

A is the area in m^2

and

σ (sigma) is the direct stress in N/m^2 or Pascals.

Since 1 Pa is a small unit kPa , MPa and GPa are commonly used also.

If the force pulls on the area so that the material is stretched then it is a tensile force and stress and this is positive.
If the force pushes on the surface so that the material is compressed, then the force and stress is compressive and negative.

4.2.   DIRECT STRAIN **e**
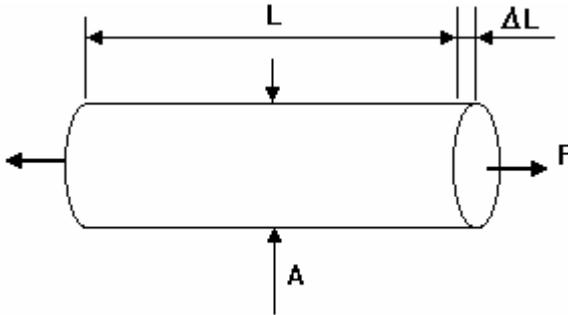
Consider a piece of material of length L as shown in figure 1. The direct stress produces a change in length ΔL. The direct strain produced is ε (epsilon) defined as $\varepsilon = \Delta L/L$

The units of change in length and original length must be the same and the strain has no units.

Figure 1

Strains are normally very small so often to indicate a strain of 10^{-6} we use the name micro strain and write it as $\mu\varepsilon$.
For example we would write a strain of 7×10^{-6} as $7\mu\varepsilon$.

Tensile strain is positive and compressive strain is negative.

4.3.   MODULUS OF ELASTICITY E

Many materials are elastic up to a point. This means that if they are deformed in any way, they will spring back to their original shape and size when the force is released. It has been established that so long as the material remains elastic, the stress and strain are related by the simple formula

$$E = \sigma /\varepsilon$$

and E is called the MODULUS OF ELASTICITY. The units are the same as those of stress.

ELASTIC LIMIT

A material is only elastic up to a certain point. If the elastic limit is exceeded, the material becomes permanently stretched. The stress-strain graph for some metals are shown below. The modulus of elasticity does not apply above the elastic limit. Strain gauges should not be stretched beyond the elastic limit of the strain gauge material which is approximately 3000με.
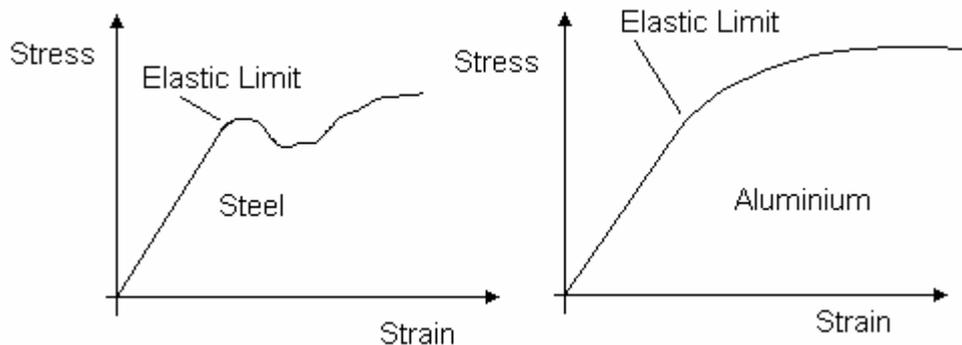

Figure 2

A typical value for the elastic limit of strain gauges is 3000 με.

WORKED EXAMPLE No.1

A metal bar which is part of a frame is 50 mm diameter and 300 mm long. It has a tensile force acting on it of 40 kN which tends to stretch it. The modulus of elasticity is 205 GPa. Calculate the stress and strain in the bar and the amount it stretches.

SOLUTION
$F = 40 \times 10^3$ N.

$A = \pi D^2/4 = \pi \times 50^2/4 = 1963$ mm^2

$\sigma = F/A = (40 \times 10^3)/(1963 \times 10^{-6}) = 20.37 \times 10^6$ N/m^2 = 20.37 MPa

$E = \sigma /\varepsilon = 205 \times 10^9$ N/m^2

$\varepsilon = \sigma/E = (20.37 \times 10^6)/(205 \times 10^9) = 99.4 \times 10^{-6}$ or 99.4 με

$\varepsilon = \Delta L/L$

$\Delta L = \varepsilon \times L = 99.4 \times 10^{-6} \times 300$ mm = 0.0298 mm

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4.4. POISSONS' RATIO

Consider a piece of material in 2 dimensions as shown in figure 2. The stress in the y direction is σ_y and there is no stress in the x direction. When it is stretched in the y direction, it causes the material to get thinner in all the other directions at right angles to it. This means that a negative strain is produced in the x direction. For elastic materials it is found that the applied strain (ε_y) is always directly proportional to the induced strain (ε_x) such that

$$(e_x)/(e_y) = - n$$

where ν (Nu)is an elastic constant called Poissons' ratio.
The strain produced in the x direction is $e_x = - n e_y$

Figure 3

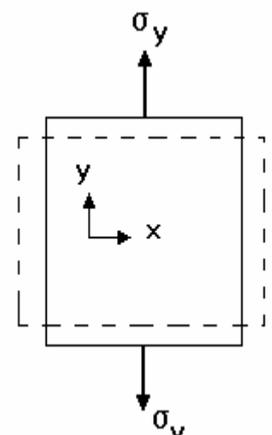## 5.   RESISTIVITY

The resistance of a conductor increases with length L and decreases with cross sectional area A so we may say R is directly proportional to L and inversely proportional to A.

R = Constant x L/A
The constant is the resistivity of the material ρ hence R = ρL/A Ohms

6.   TEMPERATURE COEFFICIENT OF RESISTANCE

The resistance of conductors changes with temperature. This is a problem when strain gauge devices are used. Usually the resistance increases with temperature. The amount by which the resistance changes per degree per ohm of the original resistance is called the temperature coefficient of resistance and is denoted α. The units are Ohms per Ohm per degree.

Let the resistance of a conductor be R_O at 0°C.
Let the resistance be R_1 at θ_1 ^{O}C. The change in resistance = $\alpha \theta_1 R_O$
The new resistance is $R_1 = R_O + \alpha \theta_1 R_O$
Let the resistance be R_2 at θ_2 ^{O}C. The change in resistance = $\alpha \theta_2 R_O$
The new resistance is $R_2 = R_O + \alpha \theta_2 R_O$
If the temperature changes from θ_1 to θ_2 the resistance changes by
$\Delta R = R_2 - R_1 = (R_O + \alpha \theta_2 R_O) - (R_O + \alpha \theta_1 R_O)$ **DR = R_0 a Dq**