FREESTUDY

HEAT TRANSFER

TUTORIAL 3

ADVANCED STUDIES

This is the third tutorial in the series on heat transfer and covers some of the advanced theory of convection. The tutorials are designed to bring the student to a level where he or she can solve problems involving practical heat exchangers.

On completion of this tutorial the student should be able to do the following.

- Explain the logarithmic mean temperature difference.
- Describe the basic designs of heat exchangers.
- Parallel, Counter and Cross Flow Heat Exchangers.
- Explain the use of the overall heat transfer coefficient.
- Explain the factors involved in heat transfer.
- Apply dimensional analysis to heat transfer.
- Solve heat transfer problems for heat exchangers.

INTRODUCTION

Most heat exchanges make use of forced convection. Heat exchangers use a variety of surfaces a typical one being a tube with fins. The effectiveness of the heat exchanger depends on many things such as the following.

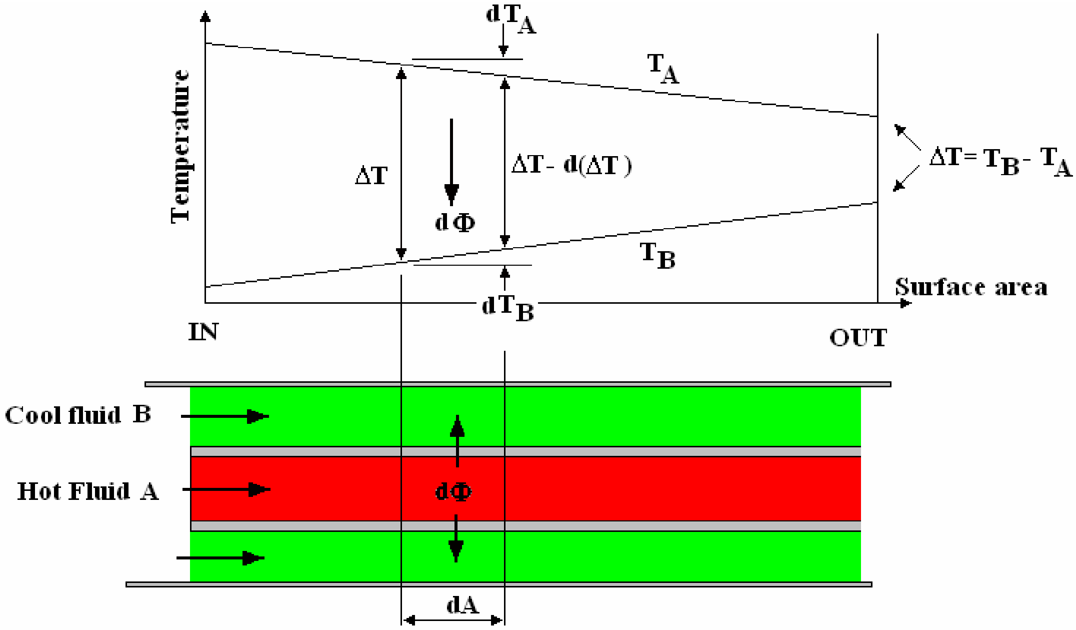
- The shape of the surface.
- The texture of the surface.
- The orientation to the direction of flow.
- The properties of the fluids.

Advanced studies involving thermodynamics and fluid mechanics have led to formulae for calculating the surface heat transfer coefficients for a variety of circumstances. This tutorial does not give the derivation in full of these and attempts to give the student an insight into the methods used. Students will find worked examples showing how to apply some of these formulae.

Many heat exchangers take the form of tubes with a fluid on both sides. The formulae used in tutorials 1 and 2 needs modifying to take account of the temperature change as the fluids travel through the heat exchangers.

HEAT TRANSFER THROUGH A LONG TUBE

Consider a hot fluid (A) flowing through a long tube exchanging heat with a cooler fluid (B) flowing in a parallel direction on the outside of the tube. As the fluids flow from inlet to outlet, fluid (A) cools and fluid (B) is heated. The net heat exchange is Φ. The temperature of the two fluids varies with the surface area or path length as shown.



Consider a short length with surface area dA. The temperature difference at any point is:-

$$\Delta T = T_B - T_A.$$

The heat exchange is $d\Phi$ over this small area.

The heat lost or gained by a fluid over the small length is given by $d\Phi = mc_p dT$

m is the mass flow rate and c_p the specific heat capacity and this is assumed to be constant in this work.

dT is the change in temperature over the path length.

Heat lost by fluid A $\quad d\Phi = m_A\, c_{pA}\, dT_A \qquad dT_A = \dfrac{d\Phi}{m_A c_{pA}}$

Heat gained by fluid B $\quad d\Phi = m_B\, c_{pB}\, dT_B \qquad dT_B = \dfrac{d\Phi}{m_B c_{pB}}$

$$dT_B + dT_A = d(\Delta T) = \frac{d\Phi}{m_B c_{pB}} + \frac{d\Phi}{m_A c_{pA}} \qquad d\Phi = \frac{d(\Delta T)}{\left(\dfrac{1}{m_B c_{pB}} + \dfrac{1}{m_A c_{pA}}\right)} \text{...........(i)}$$

Integrate between the outlet and inlet and

$$\Phi = \frac{\Delta T_o - \Delta T_i}{\dfrac{1}{m_B c_{pB}} + \dfrac{1}{m_A c_{pA}}} \text{................(ii)}$$

Now consider the change in the temperature difference over the short path length.

In terms of the overall heat transfer coefficient $\quad d\Phi = U\, dA\, \Delta T$............(iii)

Equate (i) and (iii)

$$d\Phi = U\, dA\, \Delta T = \frac{d(\Delta T)}{\left(\dfrac{1}{m_B c_{pB}} + \dfrac{1}{m_A c_{pA}}\right)} \qquad U\left(\frac{1}{m_B c_{pB}} + \frac{1}{m_A c_{pA}}\right) dA = \frac{d(\Delta T)}{\Delta T}$$

Integrating over the whole path $\quad U\left(\frac{1}{m_B c_{pB}}+\frac{1}{m_A c_{pA}}\right)A=\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)$

$$\left(\frac{1}{m_B c_{pB}}+\frac{1}{m_A c_{pA}}\right)=\frac{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}{U\,A}\text{ ....(iv)}$$

Substitute (iv) into (ii) $\quad \Phi=\frac{U\,A(\Delta T_o-\Delta T_i)}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$

For the whole system we may use $\quad \Phi = U\,A\,\Delta T_m$ where ΔT_m is a mean temperature difference.

Comparing it is apparent that $\quad \Delta T_m=\frac{(\Delta T_o-\Delta T_i)}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$

This is called the logarithmic mean temperature difference. If we did the same analysis for the fluids flowing in opposite directions, we would get the same result.
Next let's discuss the basic types of heat exchangers.

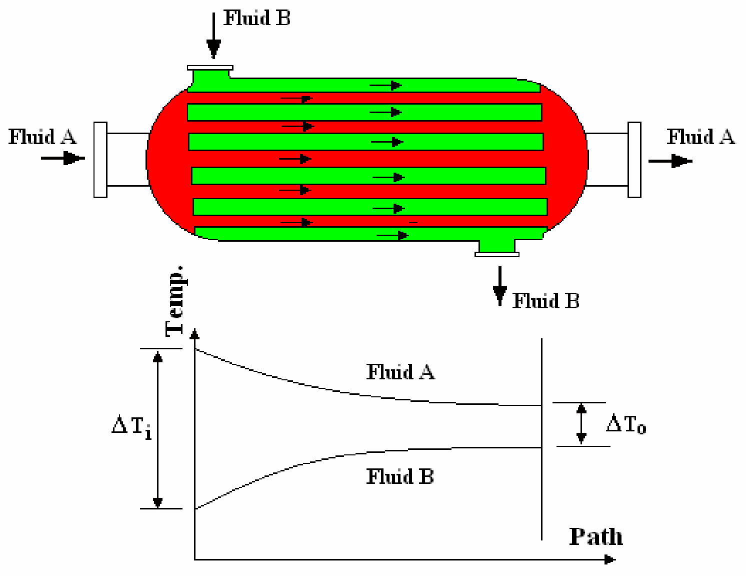
PARALLEL FLOW HEAT EXCHANGERS

The fluid being heated and the fluid being cooled flow in the same parallel direction as shown. The heat is convected to the wall of the tube, then conducted to the other side and then convected to the other fluid. If the tube wall is thin, the heat transfer rate is

$$\Phi = UA\frac{\Delta T_o-\Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

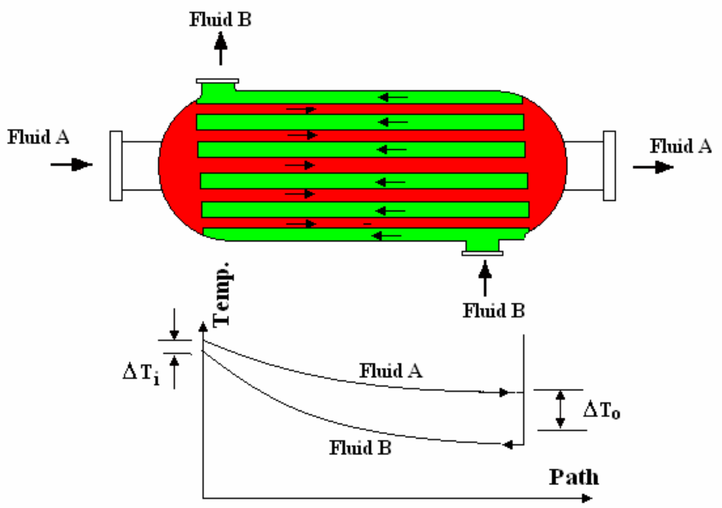
A is the surface are of the tubes.

The diagram shows typically how the temperature of the two fluids varies with the path length. Fluid 'B' gets hotter and fluid 'A' gets cooler so the temperature difference is greatest at inlet.



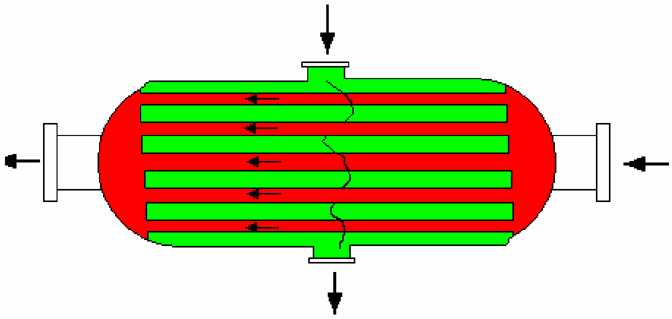
COUNTER FLOW

The fluids flow in opposite but parallel directions as shown. The temperature of the fluid being heated can be raised to near the inlet temperature of the fluid being cooled. This is important for exchangers that heat up air for combustion such as the exhaust gas heat exchangers on gas turbines. For a given heat transfer the surface area is less. The heat transfer formula is the same.



CROSS FLOW

The fluids flow at right angles to each other as shown. This design is often the result of the plant layout and is common in industrial boilers where the hot gasses flow in a path normal to the water and steam. (superheaters, recuperators and economisers).



The tubes are often finned to improve the efficiency. Sooty deposits on the outside are more likely to be dislodged in this configuration. Large steam condensers also use this style.

WORKED EXAMPLE No. 1

An exhaust pipe is 75 mm diameter and it is cooled by surrounding it with a water jacket. The exhaust gas enters at 350°C and the water enters at 10°C. The surface heat transfer coefficients for the gas and water are 300 and 1500 W/m^2 K respectively. The wall is thin so the temperature drop due to conduction is negligible. The gasses have a mean specific heat capacity c_p of 1130 J/kg K and they must be cooled to 100°C. The specific heat capacity of the water is 4190 J/kg K. The flow rate of the gas and water is 200 and 1400 kg/h respectively.

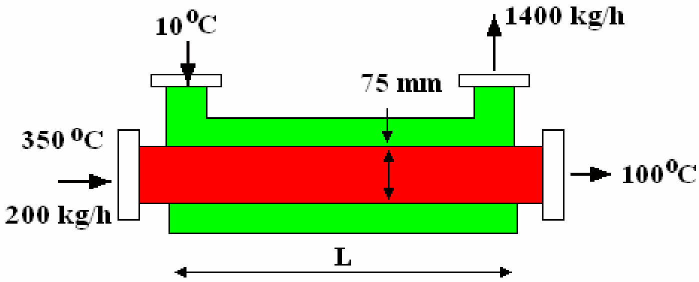
Calculate the required length of pipe for (a) parallel flow and (b) contra flow.

SOLUTION

Overall Heat Transfer Coefficient is U where $\frac{1}{U} = \frac{1}{h_g} + \frac{1}{h_w} + \frac{x}{k}$

If the wall is very thin so $x \approx 0$ $\quad \frac{1}{U} = \frac{1}{h_g} + \frac{1}{h_w} = \frac{1}{0.3} + \frac{1}{1.5} = 4 \quad U = 0.25$

PARALLEL FLOW



Heat lost by the gas is
$\Phi = mC_p\Delta T = 200 \text{ x } 1.13 (350 - 100)$
$\Phi = 56500$ kJ/h

Heat Gained by water
$\Phi = 56500 = mC_p\Delta T$
$56500 = 1400 \text{ x } 4.19 \text{ x } (\theta - 10)$
$\theta = 19.63°C$

$\Delta T_i = 340$ K $\quad \Delta T_o = 80.34$ K

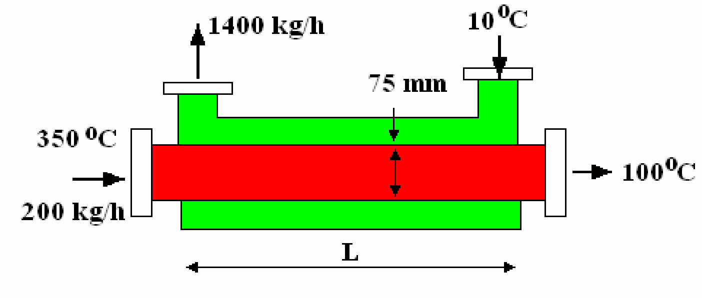
$$\Phi = UA\frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)} = \frac{56500}{3600}\text{kW}$$

$$15.694 = 0.25A\frac{80.34 - 340}{\ln\left(\frac{80.34}{340}\right)} = \frac{64.915A}{1.443} = 45A \quad A = 0.349 \text{ m}^2$$

$A = \pi DL$

$L = 0.349/(\pi \text{ x } 0.075) = 1.48$ m

CONTRA FLOW



$\Delta T_i = 330.4$ K $\Delta T_o = 90$ K

$$\Phi = 15.694 = UA\frac{\Delta T_o - \Delta T_i}{\ln\left(\dfrac{\Delta T_o}{\Delta T_i}\right)}$$

$$15.694 = 0.25A\frac{90 - 330.4}{\ln\left(\dfrac{90}{330.4}\right)}$$

A = 0.34 m^2

$$15.694 = \frac{60.1A}{1.3} = 46.23A$$

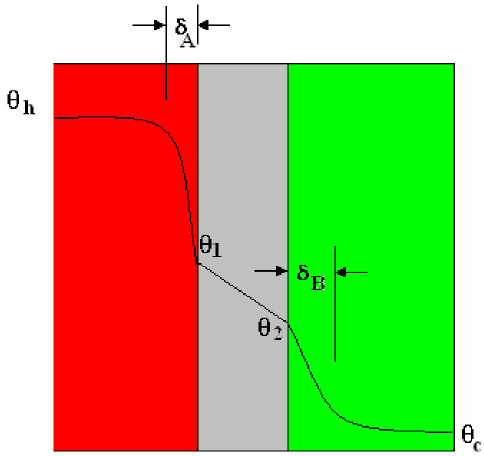
A = πDL L = 0.34/(π x 0.075) = 1.44 m

DERIVATION AND CALCULATION OF HEAT TRANSFER COEFFICIENTS

In the introduction it was stated that the surface heat transfer coefficients depend on many things such as shape and orientation of the surface, the velocity of the fluid and the properties of the fluid. The fluid properties also depend on the temperature. The velocity of the fluid is a major consideration and this varies with distance from the surface in the boundary layer and distance from the leading edge. All these factors are involved when deriving formula from basic principles. Perhaps the best place to start is with boundary layers.

BOUNDARY LAYER - LAMINAR AND TURBULENT

When a fluid flows over a surface, the velocity grows from zero at the surface to a maximum at distance δ. In theory, the value of δ is infinity but in practice it is taken as the height needed to obtain 99% of the mainstream velocity and this is very small.



The boundary layer is important for the following reason. When a fluid is convecting heat to or from a solid surface, the heat transfer close to the surface is by conduction through the boundary layer. Outside the boundary layer the temperature is the bulk temperature of the fluid but inside it decreases to the surface temperature.

The diagram shows how the temperature varies from the hot fluid on one side to cold fluid on the other. There is a boundary layer on both sides. Consider the hot side only.

The heat transfer using the conduction law from the bulk fluid at θ_h to the surface at θ_1 is

$$\Phi = (k/\delta)A(\theta_h - \theta_1)$$

The heat transfer using the convection law is $\Phi = hA(\theta_h - \theta_1)$

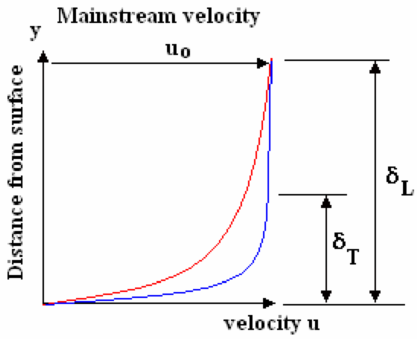
Equating to find h we have $\mathbf{h = k/\delta}$

So the surface heat transfer depends on the thickness of the boundary layer and the thermal conductivity of the fluid. The boundary layer thickness depends on several factors such as the velocity and distance from the leading edge making the calculation of h a lot less simple than implied above. The boundary layer shape is discussed next.

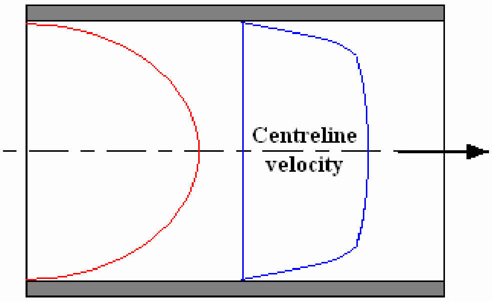
There are many theories and formulae that describe the shape of the boundary and this is covered in the tutorials on fluid mechanics. The following is a summary.

When the flow is laminar the boundary layer is a curve following precise law $u = y\left(\frac{\delta}{2\mu}\frac{dp}{dL} + \frac{u_o}{\delta}\right)$

When the flow is turbulent, the shape of the boundary layers waivers and the mean shape is usually shown. Comparing a laminar and turbulent boundary layer reveals that the turbulent layer is thinner than the laminar layer.



Flat Surface



Pipe

When laminar flow occurs in a round pipe the boundary layer grows in three dimensions from the wall and meets at the middle. The velocity u in a pipe of radius R at any radius r is given by

$$u = \frac{\Delta p}{4\mu L}\left(R^2 - r^2\right) \qquad \Delta p \text{ is the pressure drop.}$$

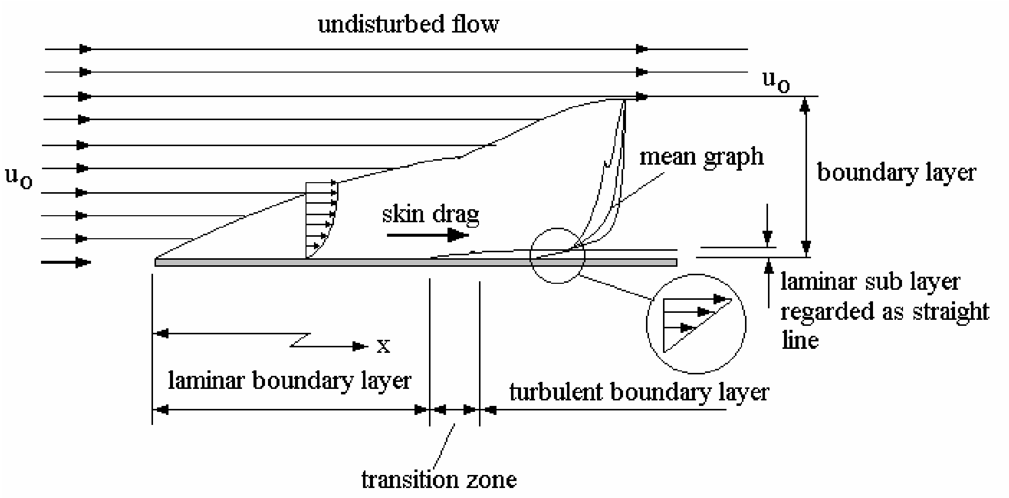
There are many formulae and theories for the shape of the boundary layer such as that given by Prandtl.

$$u = u_1(y/\delta)^{1/7}$$

This law fits the turbulent case well for Reynolds' numbers below 10^7:

FORMATION OF BOUNDARY LAYER

When a fluid first encounters a solid surface, the boundary layer grows with distance until it becomes fully formed. Eventually it changes from laminar to turbulent. This also affects the heat transfer.



DIMENSIONLESS GROUPS

One of the best ways to analyse how all the factors come together is the use of dimensional analysis and the dimensionless groups that they form. Those required to study this topic should refer to the section on fluid dynamics.

A dimensionless group is a combination of variables that have no overall dimensions when grouped in a certain pattern and when evaluated yields a number with no units.

LIST OF VARIABLES THAT AFFECT HEAT TRANSFER

Dynamic viscosity μ
Kinematic viscosity $\nu = \mu / \rho$
Density ρ
Thermal conductivity k
Specific heat capacity c_p
Temperature θ
Mean velocity u_o
Characteristic length l or diameter D or distance from leading edge x.
Coefficient of cubical expansion β

There are many dimensionless groups for heat transfer and a summary is given next.

Here is a summary of the groups that occur.

REYNOLDS NUMBER $\quad Red = \dfrac{\rho u_o D}{\mu} = \dfrac{u_o D}{\nu}$ or $Rex = \dfrac{\rho u_o x}{\mu} = \dfrac{u_o x}{\nu}$

This group occurs in studies of fluid friction and is widely used in determining whether the flow is laminar or turbulent. It may be based on pipe diameter D or distance from the edge of a surface x.

NUSSELT NUMBER $\quad Nux = \dfrac{hx}{k}$

This group is very important for convection. The solution of h often depends on evaluating Nux first.

PRANDTL NUMBER $\quad Pr = \dfrac{c_p \mu}{k}$

This is a group that relates some of the thermal properties.

GRASHOF NUMBER $\quad Grx = \dfrac{\beta g \rho^2 x^3 \theta}{\mu^2}$

This is a group is important in convection because it determines how the buoyancy of the fluid is affected by temperature.

STANTON NUMBER $\quad St = \dfrac{h}{\rho\, u\, c_p} = \dfrac{Nu}{Re\, Pr}$

The following demonstrates that $Nu = \phi Pr\ Re$

DIMENSIONAL ANALYSIS

Tutorials on dimensional analysis may be found in the section on fluid mechanics. The following is the application of this method to heat transfer. There are many dimensionless groups for heat transfer. A dimensionless group is a combination of variables that have no overall dimension when grouped in a certain pattern and when evaluated yields a number with no units.

The surface heat transfer coefficient 'h' is a function of the following variables.
Dynamic viscosity μ, density ρ, thermal conductivity k, specific heat capacity c, temperature θ, mean velocity u and Characteristic length l

The solution to this problem is much easier if we use energy E as a base unit.

The dimensions used here are:
Length $\rightarrow$ L Mass $\rightarrow$ M Time $\rightarrow$ T Temperature $\rightarrow \theta$ Energy $\rightarrow$ E

First convert the units into dimensions.
h Watts/m^2 K $\rightarrow$ Energy/s m^2 K $\rightarrow$ E/(T L^2 θ)
μ kg/m s $\rightarrow$ M/(LT)
ρ kg/m^3 $\rightarrow$ M/L^3
k Watts/m K $\rightarrow$ Energy/s m K $\rightarrow$ E/(T L θ)
c J/kg K $\rightarrow$ E/(M θ)
θ K $\rightarrow \theta$
u m/s $\rightarrow$ L/T
l m $\rightarrow$ L

The general relationship is $h = \phi(\mu,\rho,k,c,\theta,u,l)$

This may be represented as a power series $h = \text{constant}\ \mu^a\ \rho^b\ k^c\ c^d\ \theta^e\ u^f\ l^g$

There are 8 quantities and 5 dimensions so there will be 8 – 5 = 3 dimensionless groups. The solution is easier if told in advance that we solve a, b, c ,e and g in terms of d and f

Put in the dimensions. Any not present are implied index 0

$ET^{-1}\ L^{-2}\ \theta^{-1}\ M^0 = \text{constant}\ (ML^{-1}T^{-1})^a\ (ML^{-3})^b\ (ET^{-1}\ L^{-1}\ \theta^{-1})^c\ (EM^{-1}\ \theta^{-1})^d\ \theta^e\ (LT^{-1})^f\ L^g$

Equate dimensions on left and right of equality.

Energy $1 = c + d$ $c = 1 - d$

Mass $0 = a + b - d$ Hence $b = d - a$ or $a = d - b$

Length $-2 = -a - 3b - c + f + g$ substitute to get rid of b and c

$-2 = -a - 3(d - a) - (1 - d) + f + g$

$-2 = -a - 3d + 3a - 1 + d + f + g$

$-1 = 2a - 2d + f + g$

Temperature $-1 = -c - d + e$ substitute for c

$-1 = -(1 - d) - d + e$ Hence $e = 0$

Time $-1 = -a - c - f$ substitute for c and a

$-1 = -(d - b) - (1 - d) - f$ hence $b = f$

Now finish with $a = d - b = d - f$

From $-1 = 2a - 2d + f + g$ substitute for unknowns

$-1 = 2(d - f) - 2d + f + g$

$-1 = 2d - 2f - 2d + f + g$

$-1 = -f + g$ $g = f - 1$

Next form the groups by substituting the indices into $h = \text{constant}\ \mu^a\ \rho^b\ k^c\ c^d\ \theta^e\ u^f\ l^g$

$h = \text{constant}\ \mu^{d-f}\ \rho^f\ k^{1-d}\ c^d\ \theta^0\ u^f\ l^{f-1}$

$$h = \text{const}\left(\frac{k}{l}\right)\left(\frac{\mu\, c}{k}\right)^d\left(\frac{\rho\, u\, l}{\mu}\right)^f \qquad \frac{h\, l}{k} = \text{const}\left(\frac{\mu\, c}{k}\right)^d\left(\frac{\rho\, u\, l}{\mu}\right)^f$$

$Nu = h\,l/k$ $Pr = c\mu/k$ $Re = \rho\, u\, l/\mu = u\, l/\nu$

The Prandtl number is a function of fluid properties only and so may be looked up in tables at the appropriate temperature.

SOME STANDARD CASES

Studies of fluids and boundary layers have led to formulae for the surface heat transfer coefficient under specified conditions. **In the following you need to be conversant with tables of fluid properties in order to look them up. Here are some of the better known formulae.**

NATURAL CONVECTION WITH A VERTICAL SURFACE

$$Nux = 0.509(Pr)^{\frac{1}{3}}(Pr + 0.952))^{-\frac{1}{4}}(Grx)^{\frac{1}{4}}$$

This has an approximate solution of $h = 1.42\ (\Delta T/D)^{1/4}$ when Gr is in the range 10^4 to 10^9 and
$h = 0.93\ (\Delta T/D)^{1/4}$ when Gr is in the range 10^9 to 10^{12}

FORCED CONVECTION OVER A HORIZONTAL SURFACE

$$Nu = 0.332\ Pr^{1/3}\ Re^{1/2}\ (Ta/Ts)^{0.117}$$

TURBULENT FLOW THROUGH A PIPE (Dittus-Boelter Equation)

$$Nud = 0.023\ Re^{0.8}\ Pr^{0.4}$$

NATURAL CONVECTION FROM A HORIZONTAL CYLINDER

$$Nu = 0.527 Pr^{\frac{1}{2}}(Pr + 0.952)^{-\frac{1}{4}} Gr^{\frac{1}{4}}$$

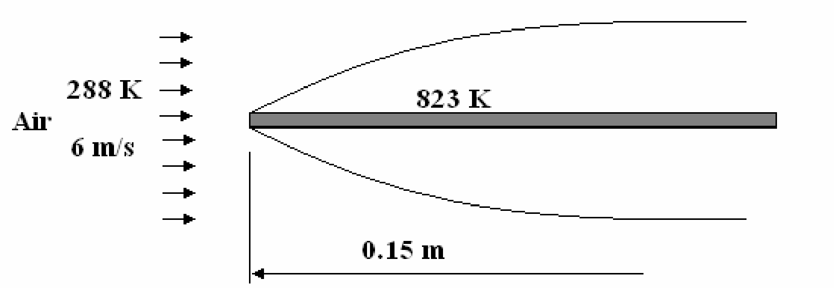
This has an approximate solution of $h = 1.32(\Delta T/D)^{1/4}$

WORKED EXAMPLE No.2

Air at 288 K and bulk velocity 6 m/s flows over a flat horizontal plate with a temperature of 823 K at all points on its surface. Given that $Nu = 0.332\ Pr^{1/3}\ Re^{1/2}\ (Ta/Ts)^{0.117}$
Calculate the heat transfer rate from both sides over the first 150 mm.

Ta is the bulk air temperature and Ts is the surface temperature. The fluid properties should be obtained at the mean temperature.

SOLUTION



$Nu = 0.332\ Pr^{1/3}\ Re^{1/2}\ (Tw/Ts)^{0.117}$
Mean temperature is (288 + 823)/2 = 555.5 K
From fluids tables Pr = 0.68 $\upsilon = 4.515 \times 10^{-5}\ m^2/s$ $k = 4.39 \times 10^{-2}$ W/m K
$Re = v\ l/\upsilon = 6 \times 0.15/\ 4.515 \times 10^{-5} = 19.93 \times 10^3$

$$Nu = 0.332(0.68)^{\frac{1}{3}}(10.93 \times 10^3)^{\frac{1}{2}}\left(\frac{823}{288}\right)^{0.117}$$

$$Nu = 46.604$$

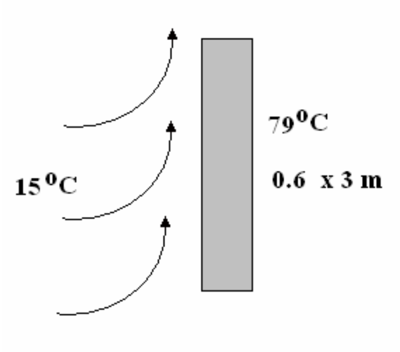
$$Nu = 46.604 = \frac{h\,l}{k}$$

$$h = 46.604\frac{k}{l} = 46.604\frac{4.39 \times 10^{-2}}{0.15} = 13.64\ W/m^2K$$

Heat Transfer from one side = Φ = h A (Tw-Ts) = 13.64 x (1 x 0.15) x (823-288) = 1094.6 W
For the two sides double up.

WORKED EXAMPLE No.3

Calculate the heat transfer to air at 15°C by natural convection with a vertical surface 0.6 m tall and 3 m wide maintained at 79°C at all points.



The Nusselt Number is given by

$$Nux = 0.509(Pr)^{\frac{1}{3}}(Pr + 0.952))^{-\frac{1}{4}}(Grx)^{\frac{1}{4}}$$

β = coefficient of cubical expansion given by β = 1/T

SOLUTION

β = 1/T = 1/(273 + 15) = 1/288

Use the fluid tables for dry air at (15°C) 288 K and atmospheric pressure
Taking atmospheric pressure as 1.013 bar
Gas constant R = 287 J/kg K
$\rho = p/RT = 1.013 \times 10^5/(287 \times 288) = 1.226$ kg/m^{3}This can be also be found in the tables.
μ is the dynamic viscosity $\mu = 1.788 \times 10^{-5}$ kg/m s
Pr = 0.69
Thermal conductivity $k = 2.53 \times 10^{-2}$ W/ m K
Taking x as the height of the wall

$$\text{Grashof Number } Grx = \frac{\beta g \rho x^3 \theta}{\mu^2} = \frac{\left(\frac{1}{288}\right) 9.81 \text{ x } 1.226\,(0.6)^3 \text{ x } 288}{\left(1.788 \text{ x } 10^5\right)^2} = 9.959 \text{ x } 10^9$$

$$Nux = 0.509(Pr)^{\frac{1}{3}}(Pr + 0.952))^{-\frac{1}{4}}(Grx)^{\frac{1}{4}}$$

$$Nux = 0.509(0.69)^{\frac{1}{3}}(1.642))^{-\frac{1}{4}}\left(9.959 \text{ x } 10^9\right)^{\frac{1}{4}} = 125.531$$

$$Nu = 125.531 = \frac{h\,x}{k}$$

$$h = 125.531\frac{k}{x} = 125.531\frac{2.53 \text{ x} 10^{-2}}{0.6} = 5.293 \text{ W/m}^2\text{K}$$

This the value at height 0.6 m. We need the mean over the height and this is 4/3 of the value
h = (4/3)5.293 = 7.057 W/m^2 K

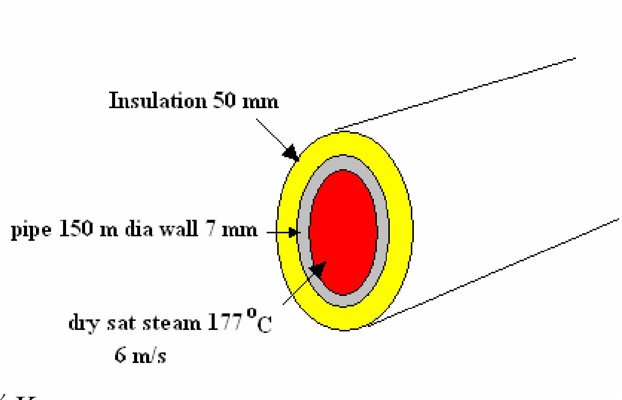
Heat Transfer from one side = Φ = h A (79-15) = 7.057 x (0.6 x 3) x (79-15) = 813 W

WORKED EXAMPLE No.4

Dry saturated steam at 177°C flows in a pipe with a bore of 150 mm with a mean velocity of 6 m/s. The pipe has a wall 7 mm thick and is covered with a layer of insulation 50 mm thick. The surrounding atmospheric air is at 17 °C. Calculate the heat transfer rate for 1 m length. The Nusselt number is given by $Nu = 0.23\, Re^{0.8}\, Pr^{0.4}$
k for pipe = 50 W/m K
k for insulation = 0.06 W/m K
The surface heat transfer coefficient for a long horizontal cylinder is $h = 1.32(\Delta T/D)^{1/4}$

SOLUTION

From the fluids table for d.s.s. at 177°C we find Pr = 1.141 and
$v_g = 0.2 \text{ m}^3/\text{kg}$
$\mu = 14.88 \times 10^{-6}$ kg m/s
$k = 33.65\ 10^{-3}$ W/m K
D = 0.15 m
u = 6 m/s
$Re = u\,D/\mu\, v_g = 30242$
k for pipe = 50 W/m K
k for insulation = 0.06 W/m K



$Nu = 0.023\ Re^{0.8}\ Pr^{0.4} = 93.139$
$Nu = h\ D/k = 587.7$
$h = 93.139 \times 33.65\ 10^{-3}/0.15 = 20.89 \text{ W/m}^2 \text{ K}$

Heat transfer = Φ per metre length

From steam to pipe $\Phi = h_1\ A\ \Delta T = h_1\pi D\ \Delta T = 20.89\ \pi \times 0.15\ \Delta T = 9.846\ \Delta T$

Through the pipe wall $\Phi = 2\pi k\Delta T/\ln(D_2/D_1) = 2\pi \times 50\ \Delta T/\ln(164/150) = 3521\ \Delta T$

Through the insulation $\Phi = 2\pi k\Delta T/\ln(D_2/D_1) = 2\pi \times 0.06\ \Delta T/\ln(264/164) = 0.792\ \Delta T$

From the outer surface $h_2 = 1.32\ (\Delta T/D)^{1/4} = 1.32\ (\Delta T/0.264)^{1/4} = 1.842(\Delta T)^{1/4}$

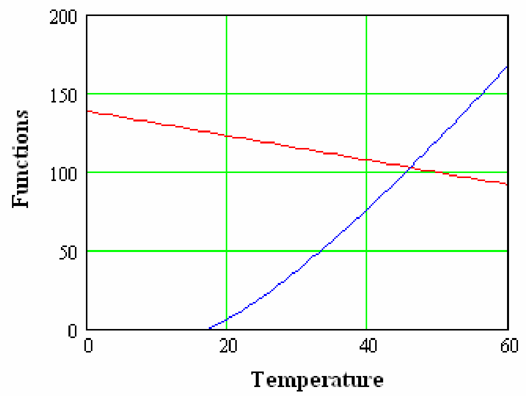
$\Phi = h\ A\ \Delta T = 1.8421(\Delta T)^{1/4}\ \pi \times 0.264\ \Delta T = 1.528(\Delta T)^{5/4}$

Because the surface temperature is unknown we need the heat transfer between the steam and the surface and the surface and the air.

Steam to Surface $\Phi = U\ (177 - T_s) = U\ (177 - T_s)$

U is found from $\frac{1}{U} = \frac{1}{9.846} + \frac{1}{3521} + \frac{1}{0.792}$ $\quad \frac{1}{U} = 1.364 \quad U = 0.733$

$\Phi = 0.782(177 - T_s)$
Surface to Air $\Phi = 1.527(T_s - 17)^{5/4}$
Equate $\Phi = 0.733(177 - T_s) = 1.527(T_s - 17)^{5/4}$
Possibly the best way to solve this is by plotting $0.733(177 - T_s)$ and $1.527(T_s - 17)^{5/4}$ against temperature to find the temperature that makes them the same. This appears to be 45°C



$\Phi = 0.733(177 - T_s) = 0.733(177 - 45) = 102.4$ W
$\Phi = 1.527(T_s - 17)^{5/4} = 1.527(45 - 17)^{5/4} = 102.8$ W Say 102.6 W

SELF ASSESSMENT EXERCISE No.1

1. Air at 300 K and bulk velocity 8 m/s flows over a flat horizontal plate with a temperature of 900 K at all points on its surface. Given that $Nu = 0.332\ Pr^{1/3}\ Re^{1/2}\ (Ta/Ts)^{0.117}$
   Calculate the heat transfer rate from one side over the first 100 mm and the first 200 mm..

   Ta is the bulk air temperature and Ts is the surface temperature. The fluid properties should be obtained at the mean temperature.
   (1.16 kW and 1.64 kW)

2. Calculate the heat transfer to air at 2°C by natural convection with a vertical surface 1 m tall and 1 m wide maintained at 110°C at all points.

   The Nusselt Number is given by $Nux = 0.509(Pr)^{\frac{1}{3}}(Pr + 0.952)^{-\frac{1}{4}}(Grx)^{\frac{1}{4}}$
   β = coefficient of cubical expansion given by $\beta = 1/T$
   (676 W)

3. Dry saturated steam at 5 bar flows in a pipe with a bore of 200 mm with a mean velocity of 4 m/s. The pipe has a wall 4 mm thick and is covered with a layer of insulation 60 mm thick. The surrounding atmosphere is at 0°C. Calculate surface temperature of the lagging and the heat transfer rate for 1 m length. The Nusselt number is given by $Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$

   k for pipe = 55 W/m K
   k for insulation = 0.08W/m K
   The surface heat transfer coefficient for a long horizontal cylinder is $h = 1.32(\Delta T/D)^{1/4}$

   (29.3°C and 129 W)