HEAT TRANSFER

TUTORIAL 1 – CONDUCTION

This is the first of a series of tutorials on basic heat transfer theory plus some elements of advanced theory. The tutorials are set at NQF Levels 3 and 4 and are designed to bring the student to a level where he or she can solve problems ranging from basic level to dealing with practical heat exchangers.

1. Introduction

Heat exchangers are used in so many fields of engineering from central heating systems to large industrial boilers, from sugar refining to oil distillation, from refrigerators to air conditioners and so on. The list is endless. The design of suitable units is based on theory and empirical data and this covers a vast range of work. The purpose of these tutorials is to provide the basic level of understanding and some advanced theory for anyone studying the topic from beginner's level. The tutorial only covers steady state heat transfer.

More advanced studies of conduction will enable you to study the following topics.

2. Non Steady Heat Transfer

This occurs when the temperature in a material is changing with time. Calculating the temperature at any point is beyond the scope of this tutorial.

3. Three Dimensional Heat Transfer

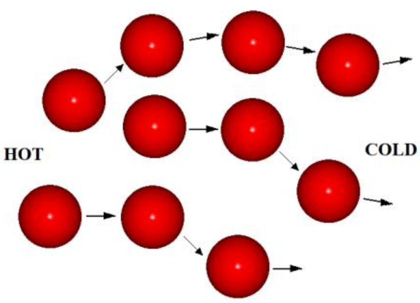
When a block of material is maintained at constant but different temperatures on all its faces, a steady flow of heat may be produced at each face but the temperature at any given point on the block is difficult to find and is also beyond the scope of this tutorial.

4. Conduction Theory

Heat transfer occurs from one body to another by three methods, conduction, convection and radiation. Most heat exchangers will use elements of all three. A net amount of heat is always transferred from the hotter body to the colder body.

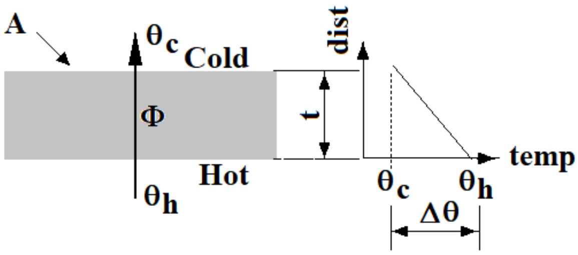
This tutorial covers conduction, the process by which heat is passed on through solids, liquids and gasses from one molecule to another.

Heat or more correctly, internal energy is basically the kinetic energy of the molecules vibrating or moving around in the material. The kinetic activity increases as the temperature is raised. If the material is all at one uniform temperature, the kinetic motion is uniform. When a hot material comes into contact with a colder material, the kinetic activity is passed on through contact and the transfer of momentum between the faster moving molecules and the slower moving molecules so the colder material becomes hotter. This is the basis of conduction.



5. Fourier's Conduction Law

Consider a flat solid material of thickness t and surface area A. The surface temperature on the hot side is θ_h on the cold side θ_c. Heat will flow from one side to the other at a rate of Φ Watts.



The heat transfer is directly proportional to the temperature difference Δθ and the area A. It is inversely proportional to the thickness t since the thicker it is, the less the heat transfer. It also depends on the material. Clearly copper conducts heat much better than concrete so we need a property called the Thermal Conductivity and this is usually given the symbol k.
It follows that

$$\Phi = -kA\frac{\Delta\theta}{t} = -kA\frac{(\theta_c - \theta_h)}{t} = kA\frac{(\theta_h - \theta_c)}{t}$$

The units of k are Watts/m K Here are some typical values of k for common material at ambient conditions.

Material	Aluminium	Copper	Steel	Concrete	Glass	Wood	Water	Air
k W/m K	201	385	63	0.1	1.0	0.15	0.59	0.024

WORKED EXAMPLE No. 1

Calculate the heat transfer through a flat copper plate 200 mm tall by 300 mm wide and 25 mm thick when the surface temperatures are 150^oC and 55^oC.

SOLUTION

$$\Phi = kA\frac{(\theta_h - \theta_c)}{t} = 385 \times (0.2 \times 0.3)\frac{(150 - 55)}{t} = 87\ 780\ \mathrm{W}$$

6. Electrical Analogy and Sign Convention

If we make the analogy that Φ is equivalent to electric current I and Δθ is equivalent to voltage ΔV, we may make an analogy with Ohm's law

$$\text{Electrical Resistance}\quad R = \frac{\Delta V}{I} \qquad \text{Thermal Resistance}\quad R = \frac{\Delta\theta}{\Phi} = \frac{t}{kA}$$

We should note that Δθ and ΔV are both strictly negative quantities because Δ means the last figure minus the first figure so $\Delta\theta = (\theta_c - \theta_h)$. If we stick to this, we must put a minus sign in front of the previous equations. If we simply let Δθ represent the temperature difference $\Delta\theta = (\theta_h - \theta_c)$ and remember that heat flows from hot to cold, we will have no difficulty solving basic problems.

7. Calculus Form

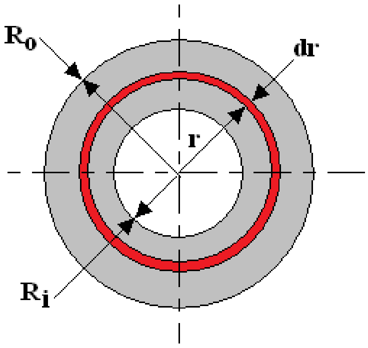
In reality, the temperature drops in the direction of heat flow so if we measure the distance from the hot side as x, the heat flow through a very thin layer thickness dx will be:-

$$\Phi = -kA\frac{d\theta}{dx}$$

This is important when analysing non-steady flow conduction and in the following work.

8. Conduction through a Tube

Consider heat flowing from the inside of a tube to the outside at a steady Φ. This is the same at all radii.



The heat flowing through an elementary layer of radius r and length L is

$$\Phi = -kA\frac{d\theta}{dx} = -k \times 2\pi Lr\frac{d\theta}{dr}$$

Rearrange to get

$$\frac{dr}{r} = -\frac{2\pi kLd\theta}{\Phi}$$

Now integrate between limits.

$$\int_{R_i}^{R_o}\frac{dr}{r} = -\frac{2\pi kL}{\Phi}\int_{\theta_i}^{\theta_o}d\theta$$

$$[\ln(r)]_{R_i}^{R_o} = -\frac{2\pi kL}{\Phi}[\theta]_{\theta_i}^{\theta_o}$$

$$\ln\left(\frac{R_o}{R_i}\right) = -\frac{2\pi kL}{\Phi}(\theta_o - \theta_i)$$

$$\Phi = -\frac{2\pi kL}{\ln\left(\frac{R_o}{R_i}\right)}(\theta_o - \theta_i) = \frac{2\pi kL}{\ln\left(\frac{R_o}{R_i}\right)}(\theta_i - \theta_o)$$

WORKED EXAMPLE No. 2

Calculate the heat transfer through a copper tube 5 m long with inner diameter 80 mm and outer diameter 100 mm. The inside temperature is 200 ^oC and the outside temperature is 70^oC.

SOLUTION

$$\Phi = \frac{2\,\pi\,k\,L}{Ln\left(\frac{R_o}{R_i}\right)}(\theta_i - \theta_o) = \frac{2\,\pi\text{ x }385\text{ x }5}{Ln\left(\frac{50}{40}\right)}(200-70) = 7\ 046\ 438\text{ W}$$

9. Conduction through a Hollow Sphere

The derivation is similar to that for a tube. The surface area of a sphere is $4\pi r^2$

$$\Phi = -kA\frac{d\theta}{dx} = -k \times 4\pi r^2\frac{d\theta}{dr}$$

Rearrange to get

$$\frac{dr}{r^2} = -\frac{4\pi k d\theta}{\Phi}$$

Now integrate between limits.

$$\int_{R_i}^{R_o}\frac{dr}{r^2} = -\frac{4\pi k}{\Phi}\int_{\theta_i}^{\theta_o} d\theta$$

$$[-r^{-1}]_{R_i}^{R_o} = -\frac{4\pi k}{\Phi}[\theta]_{\theta_i}^{\theta_o}$$

$$(-R_o^{-1} + R_i^{-1}) = -(R_o^{-1} - R_i^{-1}) = -\frac{4\pi k}{\Phi}(\theta_o - \theta_i)$$

$$\Phi = \frac{4\pi k}{(R_o^{-1} - R_i^{-1})}(\theta_o - \theta_i) = \frac{4\pi k}{(R_i^{-1} - R_o^{-1})}(\theta_i - \theta_o)$$

WORKED EXAMPLE No. 3

A spherical steel reaction vessel has an outer radius of 1.5 m and is covered in lagging 200 mm thick. The thermal conductivity of the lagging is 0.1 W/m K. The temperature at the surface of the steel is 340°C and the surface temperature of the lagging is 45°C. Calculate the heat loss.

SOLUTION

$$\Phi = \frac{4\pi k}{(R_i^{-1} - R_o^{-1})}(\theta_i - \theta_o) = \frac{4\pi \times 0.1}{(1.5^{-1} - 1.7^{-1})}(340 - 45) = 4.7 \text{ kW}$$

SELF ASSESSMENT EXERCISE No. 1

1. Calculate the heat loss through flat sheet of glass 2 m x 1 m and 5 mm thick when the surface temperatures are 20°C and 5 °C.
   (Answer 6 kW)

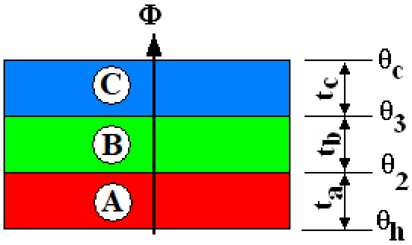
2. A steam pipe has an external diameter of 75 mm and it is covered with lagging 25 mm thick with a thermal conductivity of 0.09 W/m K. The surface temperature of the lagging is 300°C on the inside and 80°C on the outside. Calculate the heat loss per metre length.
   (Answer 432.5 W/m)

3. A steel pipe has an inner diameter of 50 mm and outer diameter of 100 mm. The outside temperature is 400 °C and the inside temperature is 120 °C. The thermal conductivity 60 W/m K. Calculate the heat flow from the outside to the inside for 1 m length.
   (Answer -152 kW)

4. A spherical vessel 2 m diameter is lagged to a depth of 300 mm. The thermal conductivity of the lagging is 0.1 W/m K. The temperature of the inside and outside of the lagging is 180°C and 40°C respectively. Calculate the heat loss.
   (Answer 762.4 W)

10. Compound Layers

If heat is conducted through a compound layer, the problem is analogous to electricity flowing through a series of resistors. Consider a flat wall made up of three layers of different materials A, B and C as shown. The heat transfer rate is the same through each layer. Using the thermal resistance we have:



$$R_A = \frac{t_A}{k_A A} \quad R_B = \frac{t_B}{k_B A} \quad R_C = \frac{t_C}{k_C A}$$

$$(\theta_h - \theta_2) = \Phi R_A \quad \theta_2 = \theta_h - \Phi R_A$$

$$(\theta_2 - \theta_3) = \Phi\, R_B \qquad \theta_3 = \theta_2 - \Phi R_B = \theta_h - \Phi R_A - \Phi R_B$$

$$(\theta_3 - \theta_C) = \Phi R_C \quad \theta_C = \theta_3 - \Phi R_C = \theta_h - \Phi R_A - \Phi R_B - \Phi R_C$$

$$\theta_C = \theta_h - \Phi(R_A + R_B + R_C\,)$$

$$\Phi = \frac{\theta_h - \theta_C}{R_A + R_B + R_C} = \frac{\theta_h - \theta_C}{R}$$

All we need to do is calculate the resistance of each part and add them up.

For Cylindrical layers

$$R = \frac{\Delta\theta}{\Phi} = \frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi k L}$$

For Spherical layers

$$R = \frac{\Delta\theta}{\Phi} = \frac{[R_i^{-1} - R_o^{-1}]}{4\pi k}$$

11. Overall Heat Transfer Coefficient

When we have compound layers (this may also include convection), it is convenient to use the equation $\Phi = - U\ A\ \Delta\theta$ where U is the overall heat transfer coefficient and $\Delta\theta$ is the temperature change across the entire layer. This will be covered and used in the following tutorials.

WORKED EXAMPLE No. 4

A wall with an area of 25 m^2 is made up of four layers. On the inside is plaster 15 mm thick, then there is brick 100 mm thick, then insulation 60 mm thick and finally brick 100 mm thick.

The thermal conductivity of plaster is 0.1 W/m K.
The thermal conductivity brick is 0.6 W/m K
The thermal conductivity the insulation is is 0.08 W/m K

The inner surface temperature of the wall is 18^oC and the outer is -2^oC.
Calculate the heat loss and the temperature at the interface between the plaster and the brick.

SOLUTION
Plaster

$$R_1 = R_4 = \frac{t_1}{k_1 A} = \frac{0.015}{0.1 \times 25} = 6 \times 10^{-3} K/W$$

Brick

$$R_2 = \frac{t_2}{k_2 A} = \frac{0.1}{0.6 \times 25} = 6.67 \times 10^{-3} K/W$$

Insulation

$$R_3 = \frac{t_3}{k_3 A} = \frac{0.06}{0.08 \times 25} = 30 \times 10^{-3} K/$$

Total Resistance

$$R = R_1 + R_2 + R_3 + R_4 = 49 \times 10^{-3}\ K/W$$

$$\Phi = \frac{\theta_h - \theta_c}{R} = \frac{18 - (-2)}{49 \times 10^{-3}} = \frac{20}{49 \times 10^{-3}} = 405\ W$$

(Answer calculated with all memory retained)

Temperature drop over plaster = $\Phi R_1 = 405 \times 6 \times 10^{-3} = 2.43$ K

Hence the temperature at the interface is 18 – 2.43 = 15.57 ^oC

WORKED EXAMPLE No. 5

A steel pipe 120 mm inside diameter has a wall 10 mm thick. It is covered with insulation 20 mm thick. The thermal conductivity of steel is 60 W/m K and for the insulation is 0.09 W/m K.

The pipe carries steam at 150°C and the outer surface temperature of the insulation is 0°C. Calculate the heat loss per metre length. Calculate the temperature at the pipe's outer surface.

SOLUTION

Pipe

$$R = \frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi kL} = \frac{\ln\left(\frac{70}{60}\right)}{2\pi \times 60 \times 1} = 4.089 \times 10^{-4}\text{K/W}$$

Insulation

$$R = \frac{\ln\left(\frac{R_o}{R_i}\right)}{2\pi kL} = \frac{\ln\left(\frac{90}{60}\right)}{2\pi \times 0.09 \times 1} = 0.444\text{K/W}$$

Total resistance =0.445 K/W

$$\Phi = \frac{\Delta\theta}{R} = \frac{150 - 0}{0.445} = 337.2 \text{ W}$$

Temperature drop over pipe wall = $\Phi R = 4.089 \times 10^{-4} \times 337.2 = 0.138$ K

The surface temperature of the pipe is 149.86 °C

This illustrates that a well insulated pipe warms up to the same temperature as the fluid inside and it is quite normal to neglect the temperature drop through the metal wall.

SELF ASSESSMENT EXERCISE No. 2

1. The inside dimensions of a refrigerated box is 2m × 1 m × 1.2 m. It is covered with insulation 50 mm thick and then a liner 5 mm thick. It is required to extract 150 W from the box. Simplifying the problem to a plain flat surface calculate the inside temperature when the outer surface is maintained at 20°C. The thermal conductivity of the insulation is 0.043 W/m K and 0.2 W/m K for the liner. (Answer 4.09°C)

2. A nylon pipe has a bore of 50 mm and a wall thickness of 5 mm. It is covered with insulation 10 mm thick. The thermal conductivity of the nylon is 0.25 W/m K and for the insulation it is 0.1 W/m K. The inside of the pipe carries hot water at 50°C and the outer surface of the pipe is at 10°C. Calculate the heat loss per metre length and the temperature between the two materials. (Answer 70 W and 41.9°C)